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PRELIMINARY SURVEY

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ESTIMATES

BY

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INTRODUCTION

The Anglo-Saxon race, in sundry climates and conditions, and under divers forms of government, is unquestionably pre-eminent to-day in the civilisation of the world.

It is not alone because they are the greatest traders, but because they are at the same time the greatest navigators and engineers of the world that the English-speaking nations hold the proud primacy of race. Whether it be at the first appearance of railways upon the Eastern hemisphere, or the first cable-knot between the old world and the new, or the development of virgin continents, and the carrying of a luxurious civilisation into the heart of nature's wilderness, the Anglo-Saxon is always at the front.

The strengthening of the Anglo-Saxon bond from year to year is more attributable to improved means of communication than to sentiment. The constantly conflicting interests of commerce, the intense rivalries of handicrafts, the minor jealousies of social life, fostered by selfish isolation, produce barriers which would increase and gulfs which would widen; but the iron horse, the ocean greyhound, and the subtler electric fluid are for ever making the men who speak the same tongue shake hands again. To the pioneer surveyor, however, the field available for

new enterprise is rapidly becoming less, whilst the number of surveyors increases. It becomes, therefore, more and more important for those who leave our shores to possess the handiest and most efficient instruments, to know the best and most rapid methods of using them, and to understand the diverse conditions of the countries to which they go.

The members' list of the British Institution of Civil Engineers has now reached the colossal total of about six thousand. These figures alone would serve to show the extent of the demand for foreign employment, for certainly there is not enough home work to go round so many; but, especially in the department of surveying, the Institution list gives but little idea of the number of young men who are issuing year by year from pupilage or college with their eye on our distant dependencies.

It is furthermore a noteworthy fact that, especially for surveyors, although the field of engineering enterprise is becoming greater and greater, the colonial door is closed to young Englishmen, just as soon as men can be trained abroad. Both in Canada and Australia, a diploma is needed to qualify a man to practise as a land-surveyor; the studies for which cannot be easily pursued in England.

In the case of India the Government have met the difficulty by giving their men the special training needed for that country at Cooper's Hill College, but the door is closed to others.

In the Colonies the reason of this is because in the first place Australians are independent in their ideas, but also very much because the young English surveyor is too often an importer of instruments of which he knows little into a country of which he knows less, so they prefer to educate their engineers on the spot.

Things have changed for the better, no doubt, but about twenty years ago it seemed as if the English engineer were educated as much as possible in things he *could not* use, and as little as possible in things which *would* be needed by him in a new country. The writer enjoyed the last two years of the lectures of one of the most celebrated professors of engineering of his day, and purchased the whole of that scientist's textbooks; but it is a significant fact that one of the former students earned his living by explaining after the lecture what the professor had meant to convey to his hearers. Finding a year or two afterwards that he could get the kind of information he wanted in a smaller compass and simpler language the writer parted with his little library of textbooks.

The same drawbacks attended the pupil in the engineer's office as the student in the university.

Men were not then made to keep their levels in adjustment, but allowed to run to the nearest instrument-maker. They were never taught the American method of levelling or curve-ranging, and the road and railway making which they learned was that which was suitable to a country like England, but of little use for the Colonies. The consequence has been that when they arrived there they were thrown upon their own ingenuity, and produced a conglomerate of different types of construction upon different gauges, which has been the reverse of profitable to the investors and without reflecting much credit upon themselves.

On the Canadian Pacific Railway the writer rarely met a young engineer fresh from England who could quickly adjust his level or theodolite or who knew anything of the American system of curve-ranging or had the least notion of telemetry.

It has been the fashion to criticise America for her *cheap* railways, her numerous gauges, her erroneous curve-ranging, and in fact everything that was not the way we do it in England. This has been the language of those who have either not been there or who have not understood the methods adopted there when casually observing them with a biassed judgment on a passing visit.

The American is just an Anglo-Saxon like ourselves, only with a little more liberty and a great deal more scope. He is not at all ashamed to come and learn from the old country what age and experience have qualified her to teach him, but in the handling of a virgin colony, with great undeveloped resources, we may do well to learn of him.

In simplicity of survey practice, uniformity of gauge, types of bridges and of rolling stock, the American engineer may be profitably (though not slavishly) imitated in the work of opening out a new sphere of enterprise such as our recently acquired colonies, and it is to be hoped that, profiting by past experience, English engineers will fuse their ideas into something like uniformity and produce a harmonious construction.

The methods of surveying considered in the following pages are by no means exclusively American. In the class of work formerly called telemetry, but now tacheometry, we have to go to Italians, French, and Germans for most of the original conceptions and the best modern developments. Comparatively few English engineers really practise these methods unless they have learned them abroad, although some are thoroughly proficient in them.

The title of this book, 'Preliminary Survey,' is American, and answers somewhat to our 'Parliamentary Work;' but it covers a wider range, in fact the whole science of surveying

in condensed form with the exception of those minute details where very great accuracy is needed.

The object in view has been to present to the young engineer going abroad a handy *vade-mecum* which with the necessary tables will enable him to carry out a survey in a new country rapidly, correctly, and according to the ideas and requirements of the people. It has also been sought to furnish in the first and third chapters an *aide-mémoire* to the experienced surveyor for his assistance in roughly estimating the cost of the proposed works, and so to guide his decision in the case of alternative routes and situations.

Considerable use has been made of standard authorities on both sides of the Atlantic, but the subject matter is in the main the result of actual experience. The necessary compactness of such a work has made it eclectic. Some methods have been passed over with slender comment, although occupying much space in other textbooks. On the other hand such subjects as tacheometry, computation by diagram and slide-rule, signalling, &c., which are as yet hardly known to the general public except in pamphlet form, are here treated of at considerable length. An attempt has been made to explain the elements of astronomy, as far as they are needed in the simple problems used by the surveyor, in such a manner as will be understood by those having no previous knowledge of the subject, and a great many of the definitions which take up much space in ordinary textbooks have been placed in a glossary. No tables are given which are to be found in the Nautical Almanac or in ordinary mathematical tables, as these have to form part of the surveyor's impedimenta.

The following extract from the statute book of the Dominion of Canada will give a fair idea of what the

pioneer surveyor in any of the colonies should know, both in theory and practice. Both in Australia and India survey practice is carried on very much in the American manner. The subjects enumerated in the Canadian statute are not treated so much in detail in this work, in order to leave space for other subjects, such as tacheometry and curve-ranging, which are equally useful to the railway man.

The author desires to express his acknowledgments for a great deal of useful material to the following gentlemen who have kindly given their courteous permission to use tables, maps, diagrams, and formulæ in works of which they are either the authors or custodians :

James Forrest, Esq., Secretary Inst. C.E. and editor of 'Minutes of Proceedings.'

Captain Wharton, Hydrographer to the Navy, and author of 'Hydrography.'

A. M. Wellington, Esq., C.E., editor of 'Engineering News,' New York, and author of standard works referred to in the text.

John C. Trautwine, Esq. (jun.), editor of Trautwine's 'Pocket Book.'

Other authorities on different subjects have been also referred to, and acknowledged in different parts of the book.

The calculations in chapters three and eight have been very kindly checked by an old friend, Mr. William T. Olive, Resident Engineer on the Manchester Main Drainage ; most of the other figures have been checked in one way or another, but it is possible in a first edition that errors may still remain undetected, and any information as to mistakes in the text, figures, or diagrams will be gladly welcomed by the author.

Qualifications of the Dominion Land and Topographical Surveyor

Excerpt. 49 *Victoria, Chapter* 17. *Royal assent, May* 25, 1883. (*Dominion of Canada.*)

99. No person shall receive a commission from the Board of Examiners authorising him to practise as a Dominion land surveyor until he has attained the full age of twenty-one years, and has passed a satisfactory examination before the said Board on the following subjects; that is to say: Euclid first four books, and propositions first to twenty-first of the sixth book; plane trigonometry, so far as it includes solution of triangles; the use of logarithms, mensuration of superficies, including the calculation of the area of right-lined figures by latitude and departure, and the dividing or laying off of land; a knowledge of the rules for the solution of spherical triangles, and of their use in the application to surveying of the following elementary problems of practical astronomy.

1. To ascertain the latitude of a place from an observation of a meridian altitude of the sun or of a star.

2. To obtain the local time and the azimuth from an observed altitude of the sun or a star.

From an observed azimuth of a circumpolar star, when at its greatest elongation from the meridian, to ascertain the direction of the latter.

He must be practically familiar with surveying operations, and capable of intelligently reporting thereon, and be conversant with the keeping of field notes, their plotting and representation on plans of survey, the describing of land by metes and bounds for title, and with the adjust-

ments and methods of use of ordinary surveying instruments, and must also be perfectly conversant with the system of survey as embodied in this Act, and with the manual of standing instructions and regulations published by the authority of the Minister of the Interior from time to time for the guidance of Dominion land surveyors.

102. Any person entitled to receive, or already possessing a commission as Dominion land surveyor, and having previously given the notice prescribed in clause 98 of this Act, may be examined as to the knowledge he may possess of the following subjects relating to the higher surveying, qualifying him, in addition to the performance of the duties declared by this Act to be within the competence of Dominion land surveyors, for the prosecution of extensive geodetic or topographic surveys, or those of geographic exploration, that is to say :

1. Algebra, including quadratic equations, series, and calculation of logarithms.

2. The analytic deduction of formulas of plane and spherical trigonometry.

3. The plane co-ordinate geometry of the point, straight line, the circle, and ellipse, transformation of co-ordinates, and the determination, either geometrically or analytically, of the radius of curvature at any point in an ellipse.

4. Projections : the theory of those usually employed in the delineation of spheric surfaces.

5. Method of trigonometric surveying : of observing the angles and calculating the sides of large triangles on the earth's surface, and of obtaining the differences of latitude and longitude of points in a series of such triangles, having regard to the effect of the figure of the earth.

6. The portion of the theory of practical astronomy

relating to the determination of the geographic position of points on the earth's surface, and the direction of lines on the same, that is to say:

Methods of determining latitude.

a. By circum-meridian altitudes.

b. By differences of meridional zenith distance (Talcott's method).

c. By transits across prime vertical.

Determination of azimuth.

a. By extra-meridional observations.

b. By meridian transits.

Determination of time.

a. By equal altitudes.

b. By meridian transits.

Determination of differences of longitude.

a. By electric telegraph.

b. By moon-culminating stars.

7. The theory of the instruments used in connection with the foregoing, that is to say, the sextant or reflecting circle, altitude and azimuth instrument, astronomic transit, zenith telescope, and the management of chronometers; also of the ordinary meteorological instruments, barometer (mercury and aneroid), thermometers, ordinary and self-registering, anemometer, and rain gauges, and on his knowledge of the use of the same.

15 GREAT GEORGE STREET, LONDON, S.W., 1890.

CONTENTS

INTRODUCTION

CHAPTER I

GENERAL CONSIDERATIONS

CHAPTER II

ROUTE-SURVEYING OR RECONNAISSANCE

CHAPTER III

HYDROGRAPHY AND HYDRAULICS

CHAPTER IV

GEODETIC ASTRONOMY

CHAPTER V

TACHEOMETRY

CHAPTER VI

CHAIN-SURVEYING

CHAPTER VII

CURVE-RANGING WITH TRANSIT AND CHAIN

CHAPTER VIII

GRAPHIC CALCULATION FOR PRELIMINARY ESTIMATES

CHAPTER IX

INSTRUMENTS

APPENDIX

LIST OF PLATES

PRELIMINARY SURVEY

AND

ESTIMATES

CHAPTER I

GENERAL CONSIDERATIONS

THE following remarks will be more applicable to railway reconnaissance, though much of the principle contained in them is also that which guides the surveyor for trunk roads for military or commercial transportation.

QUALIFICATIONS OF THE SURVEYOR

The man who is first in the field should be a man of wide range of experience rather than a minute technologist. He is usually given much discretionary power as to his location. He has also advisory powers, or rather duties, which are great responsibilities. He is called upon to report upon the scheme from a bare possibility down to a desirable investment. Before engaging his services, the promoters have generally made up their minds that there must be 'money in it,' and they want, like most other people, to obtain a maximum of good showing for a minimum amount of outlay.

The surveyor is generally disposed to favour a new

undertaking, because, however much or little money there may be in it for him, there is likely to be 'work in it,' and he has often to resist the natural tendency to make too good a showing.

There are several considerations which likewise influence him in this direction. Shareholders always expect a sanguine report, and take discount off it in any case; so that a moderate report is to them a bad one. There is a moral certainty that, however carefully a walk-over survey may be made, a revised location will show a material improvement in the line of economy or efficiency, or both, and therefore the surveyor is tempted to make allowance for this in his trial profile. He is, perhaps, well aware that the nearer his profile resembles the surface of a billiard table the better he will please his employers.

When the country is so rough that chaining is out of the question—unless he is able to adopt one of the rapid methods to which these pages are meant to draw attention—a large element of conjecture enters into his calculations, and he is naturally disposed to conjecture favourably rather than critically.

Subject Matter of a Railway Report

The surveyor is generally called upon to advise his promoters—

1. Whether any kind of line is feasible.
2. Whether it is likely to be profitable.
3. What type of railway would be most suitable, and what style of rolling-stock.
4. He is to furnish a plan, profile, and estimate of one or more routes which he considers eligible.

All these points are closely connected. One kind of line is feasible where another is wholly impracticable; a light, cheap railway will often yield a handsome dividend where a heavy line would never emerge from the hands of a receiver.

On the other hand, a *light* railway built to carry *heavy* traffic will probably be wedged out of existence by a higher-class competing line. The style of rolling-stock procurable to handle the business often regulates the location as much as the location rules the rolling-stock. The route is dependent on the topography to a great extent, but the situation of towns with which communication is necessary often overrides the consideration of topography.

It is only experience which can enable an engineer to form rapidly and correctly the general idea of the class of line suited to the circumstances. If, as is often the case, gauge and rolling-stock are fixed factors in the problem, there remain the questions of grades and curves, which must be to a large extent dependent upon the topography, and it is there that the judgment of the surveyor is most needed, both in the limiting and the arrangement of these vital elements of a railway.

With regard to the first point of *feasibility*. This has almost dropped out of the reckoning of to-day. It may be taken for granted that a railway can be constructed nearly anywhere. The only insurmountable difficulties to railway projects are, first, lack of funds; second, opposition of vested interests. It is another thing when we come to the question of—

Whether it will pay. Here the engineer has to study, 1. What the existing traffic of the district is, and how it would be likely to be affected by the introduction of a railway. 2. What is the probability of the traffic being handled by some other means of transit in competition. 3. What rates can be commanded, and whether it will be in the main a through or a local traffic. 4. What is the outlook for development of the business, with any possibly counteracting causes. 5. Probabilities of another competing railway in the future.

All these subjects dovetail themselves into the actual reconnaissance of the route; engineering difficulties give

way to commercial exigencies and *vice versâ*, until the surveyor has evolved his ideas of a line with maximum efficiency at minimum cost which will command the maximum amount of business.

The climate in which the undertaking is to be carried on is a very important consideration. In tropical climates the use of timber is to be avoided where possible, on account of predatory insects and the rapid decay produced by alternations of hot sun and heavy rains. The rainy season regularly changes the trickling rivulet into the mighty river, and these actions of the weather greatly affect the construction of culverts and bridges, and therefore, indirectly, the location. In some places streams are turned into tunnels to save building culverts, and an overflow channel provided at the junction of an embankment and side-hill for abnormal freshets.

In cold climates snowsheds are a very costly item, and the study of the principles of drifting snow will often modify the location.

The general topography also radically affects the location. It rules both gradients and curvature and the type both of gauge and equipment.

If the land falls toward the seaboard, with a heavier export than import trade—such as a mineral railway which only takes back lumber, agricultural produce, and so forth—the gradients can be steeper than would be otherwise permissible ; the rule being then to adopt that which can be surmounted as a contrary grade by the light traffic.

The method of 'bunching,' or concentrating the severe gradients in order to handle them specially, is a very important one. The best policy for a new country is to *carry long trains as far as possible with one engine*, and then to divide them on a turn-out and take them over the climb in sections, or else to provide an assistant engine for the district.

The following table (No. XXIV. of Mr. Wellington's standard work on American 'Railway Location') shows the

engine ton-mileage required to move 1 ton of net load (ex. engine) 100 miles on a level, except for a rise of 2,400 feet on different grades, worked with assistant engines: according to the average daily experience of American railways.

TABLE I.—*Traction on Grades.*

Rate of grade on incline	Length of incline	Length of level track	Engine ton-mileage per ton of net load moved 100 miles		
			While on incline	While on level track	Total
feet per mile	miles	miles			
24	100	—	1·056	—	1·056
30	60	40	0·862	0·210	1·072
80	30	70	0·760	0·369	1·129
100	24	76	0·755	0·400	1·155
120	20	80	0·766	0·421	1·187
150	16	84	0·803	0·442	1·245
200	12	88	0·900	0·463	1·363

'It would be seen that the rate of incline had an inconsiderable influence on the motive power required, for the reason, largely, that the length of the run on which large power was required decreased *pari passu* with the increase of rate, which was not the case with through grades.

'In this table moreover it was assumed that the total length of the road remained uniform at 100 miles, whatever the rate of grade adopted for the high-grade section. This is ordinarily quite out of the question, the lower grade being usually attainable only by adding so much further development within an approximately uniform air-line distance.

'Assuming, for example, that in the above table eighty miles of level track was essential in any case, and that in the remaining air-line distance of twenty miles, any one of the above rates of pusher-grades from twenty-four to 200 feet per mile was obtainable, but only by development—a rather extreme assumption, but sufficient for illustration—the table would thus read:'—

TABLE II.—*Traction on Grades.*

Rate of grade.	Length			Engine ton-mileage per ton of net load moved between the incline		
	Incline	Level	Total	While on incline	While on level track	Total
ft. per mile	miles	miles	miles			
24	100	80	180	1·056	0·421	1·477
30	60	80	140	0·862	0·421	1·283
80	30	80	110	0·760	0·421	1·181
100	24	80	104	0·755	0·421	1·176
120	20	80	100	0·766	0·421	1·187
150	16	84	100	0·803	0·442	1·245
200	12	88	100	0·900	0·463	1·363

TABLE III.—*Adjustment of Gradients for Assistant Engines, according to the Average Daily Performance on American Railways.* (*H. M. Wellington.*)

Ruling grade worked by one engine in feet per mile	Grade at which the same train can be drawn by the aid of					
	One assistant engine			Two assistant engines		
	Of equal weight on drivers	Heavier by 20 per cent.	Heavier by 40 per cent.	Of equal weight on drivers	Heavier by 20 per cent.	Heavier by 40 per cent.
level	24	29	33	46	54	62
10	42	48	53	70	80	90
20	59	66	72	92	104	116
30	76	84	91	113	126	138
40	92	101	109	133	147	160
50	107	117	126	152	167	180
60	122	133	142	169	185	199
70	136	148	158	185	201	216
80	150	162	173	201	217	232
90	164	176	187	216	232	247
100	177	189	201	230	247	261
110	190	202	214	—	—	—
120	203	215	227	—	—	—
130	215	227	239	—	—	—
140	227	239	251	—	—	—
150	238	250	262	—	—	—

Caution. In calculating the increase of motive power due to severe gradients, the wear and tear on locomotives, such as the 'thrashing' of an engine up a steep incline by an inexperienced driver, is an item which, though difficult to calculate, should be allowed for by a large margin.

The assumptions in the above table are that the rolling friction on the level is 10 lbs. per ton; for lower frictions the gradients are proportionally lessened. The gradients are compensated for curvature.

A good method of overcoming steep gradients is by the Abt rack system. The special feature about it is that sections of mountain line can be worked thus without changing gauge or altering the rolling stock. The locomotives do not depend on adhesion, therefore they can be much lighter just where the construction of bridges is the most serious item.

The Abt system is also specially adapted to short branch mountain lines.

The notes and memoranda that the surveyor wants are to give him a general but accurate idea of the alternative advantages of the different schemes that arise, and it is with that object that the chapter on Graphic Calculation has been added; condensed to its utmost limit.

In selecting a route and deciding upon the class of line for a railway scheme it should be considered—

First: If a competing line, how to obtain a pronounced superiority to the existing one, either on the score of efficiency or economy.

Second: If the first in the district, how without wasteful expenditure to secure *primacy*. That is to obtain a line which will so handle the existing and prospective business as to *hold its own*, and that the best, of the business.

In order to ensure that his line will be suitable to the future mode of working it, the surveyor should be acquainted both with the ordinary and the special types of rolling-stock that are to be used. In new countries it is essential to

economy that the engines and cars should be flexible, not only as regards side-play, but also, if one may coin the term, 'up and down' play.

It is furthermore necessary that level crossings should be permitted over all country highways, and, when on the level prairie, over existing railways also.

Station buildings should be very primitive, and the booking performed on the train.

It was stated by Mr. J. C. Mackay at the Institution of Civil Engineers in 1886, that 'the present railways of the Cape Colony had been constructed on a lavish scale with rails weighing 60 lbs. per yard, and expensive stations, some of them costing over 20,000*l.*, while the railways alone had cost 8,000*l.* per mile. This great expenditure had been incurred *for the sake of conveying one train per day*, in some cases only one train every other day, and the consequence was that the revenue did not pay one half the interest on the loan, after deducting working expenses, and the working of these lines was obliged to be carried on in such a manner that the *bullock waggon competed successfully with the railways.*

'At Kimberley, with its 1,500 passengers and 350 tons of goods per mile of railway per annum, a line with rails of 60 lbs. per yard, and expensive rolling-stock and stations, had been adopted.'

The writer is not in a position to verify at the moment the accuracy of these figures, nor to state to what extent they may be modified by subsequent development of the districts; but as they stand, reflecting most adversely upon the judgment of the promoters and the engineers, it should be added as a qualification that they only serve to show one side of the question, but that which needs to be most emphasised for a new country—*the danger of putting old-country ideas upon young-country shoulders.* The counter-evil of putting down poor lines where there is business for good ones, probably to pad the promoters' pockets,

has plenty of illustrations both at home and abroad, and the engineer is too often compelled against his judgment to make both his location and construction suit the 'spirit of the times.'

The former evil of wasteful or even *premature* expenditure is one which greatly checks the inflow of fresh capital into a country. A receiver is a perfect scarecrow to fresh enterprise. Purely speculative railway-making is as great a hindrance to *bonâ fide* undertakings as jerry-building in its smaller sphere; whether it come in the form of too good or too bad a line of railway.

A single line with properly arranged passing places, rails 30 to 40 lbs. per yard, engines of 15 to 20 tons, in easy country, can be built for 4,000*l.* per mile, including the equipment, in almost all parts of the globe; provided that the line starts from the seaboard, or from a place in rail-connection with the seaboard.

This line, properly located, is capable of handling 1,000 tons of freight per day, and is, therefore, even with low rates, in a position to yield a handsome profit to the investors. Putting net receipts at ¼*d.* per ton-mile, it would return 9½ per cent. on the cost of construction with that volume of business.

Approximate Rule for finding the Amount of Traffic required to pay five per cent. on the Cost of Construction of a Railway.

Assumptions. Tariff, ·75*d.* per ton-mile for passengers and freight. Passengers reckoned at two tons each. Expenses ·50*d.* per ton-mile. Net receipts, ·25*d.* per ton-mile. 365 days to the working year.

Traffic in tons per diem = cost in £ per mile × ·1315.

Example: On a road costing 4,000*l.* per mile, the traffic needed in tons per diem = 4,000 × ·1315 = 526. The amount of traffic required varies inversely as the net value of the receipts per ton-mile.

Therefore for any other net value such as ·53*d.* per ton-mile, the amount of traffic is found by the slide-rule in the following manner:

Place the given value ·53*d.* on the upper scale of the *slide* under the ·25 on the rule. Find the required multiplier 0·62 on the rule opposite to the 1315 on the slide. Leave the brass marker at 62 on the rule, and make a 1 of the slide coincide with it. Then the result, 248 tons, will be found on the rule opposite to 4,000*l.* cost, or 186 tons opposite to 3,000*l.* cost and so on.

Converse Rule

To find the percentage on cost of construction when the argument is:

1. The tonnage of freight per day (average of 365 days).
2. The net profit per ton-mile in decimals of a penny.
3. The cost of construction of one mile in £ sterling.

Multiply the tonnage by the profit. Divide the product by the cost of construction. Multiply the quotient by 152·1. Result is the percentage.

Example.

Data are:	Daily tonnage	542
	Net receipts per ton-mile .	·47*d.*
	Cost of construction per mile .	4650*l.*

By Slide-Rule, using lower scales of Slide and Rule

Place a 1 of the slide over the 542 (tonnage) on the rule. Place the brass marker at 47 (receipts) on the slide. Place 465 (cost) on the slide at the marker. Place the marker at the right-hand 1 of the slide. Place the left-hand 1 of the slide at the brass marker. Read the percentage

8·34 on the rule opposite to the constant 152·1 on the slide.[1] (The operation can be done in about 35 seconds.)

Mr. Robert Gordon in his paper on Economic Construction of Railways, 'Min. Proc. Inst. C.E.,' gives a note on the problem of the maximum capacity of a single track, quoting from Mr. Thompson of the New York, Pennsylvania, and Ohio Railroad ('Railway Gazette,' 1884, 1–43). He says that for trains running an average of twenty miles per hour, the most economical speed for freight, the maximum is reached with stations 3·7 miles apart, when with an allowance of six minutes' detention for each train crossed, and eight minutes extra for each passenger train passed, it is found that the limit is reached when the time of detention equals that of running in the 24 hours, which gives sixty trains per day both ways. For fifty trains and over, the track should be doubled between the termini and the next stations. In practice it is found that grades limit the number of cars run in a train, so that, if forty loaded cars be the ordinary number on the level, only twenty are taken over an undulating country by a single engine. Actually, on Mr. Thompson's division, 98 miles in length, the Standard engine takes nineteen cars, the Mogul twenty-three cars, and the Consolidation thirty-three cars each per train.[2]

The type of railway is affected first of all by grade. There

[1] For the explanation of the principle of the slide-rule see pp. 242, &c., also 361, &c.

[2] *The total annual expenses on railroads in the United States* usually range between 65 and 130 cents (2*s.* 8½*d.* and 5*s.* 5*d.*) per train-mile, that is, per mile actually run by trains. Also between 1 and 2 cents (½ and 1*d.*) per ton of freight and per passenger carried one mile. When a road does a very large business, and of such a character that the trains may be heavy and the cars full (as in coal-carrying roads), the expense per train-mile becomes large, but that per ton or passenger small; and *vice versâ*, although on coal-roads half the train-miles are with empty cars.—Trautwine's 'Engineer Pocket Book.'

See also, at p. 21, Table of 'Earnings of American Railways.'

can be no question that there are many light narrow-gauge railways which are earning a good return on the capital which could not have kept their heads above water as standard-gauge railways. It is true, on the other hand, that a very large mileage of narrow gauge is every year being converted into standard gauge, in order to avoid breaking bulk. Surveyors are not gifted with prophecy to know what the changes of the next fifty years will be, but a fair amount of experience will enable them to tell whether the line will always remain a feeder, or whether it is likely to form part of a trunk line.

The narrow gauge enables the engineer to adopt curves of very small radius, but then he must keep to a flexible type of locomotive and car which involves lighter loads and less speed; consequently, less business done for the same labour and superintendence.

A theoretically perfect railway is an air-line on a dead level between two points, and yet, apart from its cost, in nine cases out of ten it would not be the best line.

A great deal of the sinuosity of a well laid-out railway in a new country is productive. It carries the trains to where the business is or where it is going to be.

Sometimes the local traffic is the major part of the business. It generally commands a higher rate than through traffic, and it would be a serious mistake to straighten a line to catch through traffic of small bulk carried at cut-rates and by so doing to lose a steady monopoly of a lucrative local business.

Bad gradients are worse than sharp curves; the latter can be to a great extent mitigated in their discomfort by well-made carriages; in their resistance by flexible locomotive frames; and in their danger by careful signalling. But gravity no skill can dispose of, and bad gradients have killed many a promising line.

Considerable opportunity exists in every line of railway for *arrangement* of the curves and gradients so as to make

them as little objectionable as possible. The first question for the surveyor is whether or not the construction is to be homogeneous. If, for instance, he has to traverse level prairie for fifty miles, then cross a range of hills 3,000 feet high within an air-distance of another fifty miles, then another stretch of prairie of fifty miles, he has to consider whether it would be advisable to develop his mountain section for better gradients or whether he should arrange the line for a different class of locomotives in the three sections. All this affects the location.

The two main points to be borne in mind with regard to curvature are the speed at which trains will have to run and the kind of rolling-stock which will have to be carried. The following Tables IV. to IX. from Mr. Wellington's book already referred to will be found useful in fixing the limit of curvature and gradient to be adopted on a new line. The American curve-nomenclature is explained in Chapter VII.

TABLE IV.—*Resistance on Curves.*

Type of engine	Weight in lbs.	Length of wheelbase		Resistance on 4° curve at 10 miles per hour		
		Rigid	Total	Total	lbs. per ton	lbs. per degree
American .	101,000	7′ 6″	21′ 10¾″	1963	39·0	9·75
Ten-wheel .	123,000	12′ 5″	23′ 8″	1750	28·4	7·1
Consolidation	136,000	14′ 6½	21′ 1″	1850	20·0	5·0

Note.—Consolidation engines are made to run round a Wye (see p. 240) with curves of 136 feet radius without any trouble.

Average Curvature per mile on some of the Railways in the Rocky Mountains.

Name of Railway	Length of section. miles	Average degree per mile
Colorado-Central	34	420
Virginia, Truckee	22	278
Union Pacific	65	59
Texas Central	143	24·7
Southern Pacific	142	63·6

TABLE V.—*Curvature and Grades on Sections of Eastern Trunk-roads.*

Name of road	Miles	Curvature per mile	Per cent. curved	Degrees per mile	Per cent. level	Rise per mile	Rise and fall per mile	Ruling Grades
New York Central .	296·6	0·78	19·8	15·1	34·8	1·8	3·8	21–23
Boston and Albany .	202·0	1·55	56·0	72·5	0·0	9·3	13·7	80
Concord and Portsmouth	40·0	1·41	37·2	93·0	3·8	19·0	15·3	80
Ulster and Delaware	73·2	3·60	41·0	80·5	13·6	24·1	11·6	160–142
Montrose Penna . .	27·2	6·35	49·0	24·1	3·9	38·4	9·6	95–74
Summary of N. Eastern States . . .	5,372	1·88	35·5	55·9	22·8	4·7	13·0	

The column 'Rise per mile' gives the average excess of rise over the fall in one mile. The next column gives the feet of rise and fall. Thus, if a road rose 500 feet and fell 200 in 100 miles, it would be given above: Rise per mile 3·0, Rise and fall 2·0. The first quantity is an unavoidable necessity, due to difference of level at the termini.

From the same work, on the authority of Mr. M. N. Forney, locomotive expert, it is stated that:

TABLE VI.—*Curve Limits for fixed Wheel Bases.*

					Feet rad.
Axles	3	feet apart	will roll	in a curve of	67
,,	4	,,	,,	,,	91½
,,	5	,,	,,	,,	133
,,	6	,,	,,	,,	174½
,,	7	,,	,,	,,	251
,,	9	,,	,,	,,	479
,,	10	,,	,,	,,	643½

CENTRIFUGAL FORCE

On any three curves having radii as 1, 2, 3, the centrifugal force at any given velocity is as 3, 2, 1; but the coefficient of safety against overturning or disagreeable effect is as $\sqrt{3}$, $\sqrt{2}$, $\sqrt{1} = 1·73$, 1·41, 1·00.

TABLE VII.—*Giving for various curves the inferior and superior limits of speed within which the centrifugal force is more or less objectionable or dangerous.*

Curve		Maximum and minimum limits of speed in miles per hour	
Degree	Radius in feet	Minimum. Having no disagreeable effect	Maximum. On the point of overturning the vehicle
2	2,865	41·39	130·89
4	1,433	29·27	92·55
6	955	23·90	75·57
8	717	20·70	65·44
10	574	18·51	58·54
12	478	16·90	53·43
14	410	15·64	49·47
16	359	14·63	46·28
18	319	13·78	43·58
20	288	13·09	41·39
22	262	12·48	39·46
24	240	11·95	37·78
26	222	11·61	36·72
28	207	11·06	34·98
30	193	10·69	33·80
40	146	9·25	29·27
50	115	8·28	26·18
60	96	7·56	23·90

Rule. For the centrifugal force in lbs. per ton of 2,000 lbs. $C = \cdot 023348\ V^2 D$, where C=centrifugal force in lbs. per ton, V=velocity in miles per hour, and D=degree of curvature from which the following table is made.

TABLE VIII.—*Curve Limits at Different Speeds.*

Speed in miles per hour	Degree of curvature				
	1°	5°	10°	15°	20°
10	2·33	11·67	23·35	35·02	46·70
20	9·34	46·70	93·39	140·09	186·78
30	21·01	105·07	210·13	315·20	420·26
40	37·36	186·78	373·57	560·35	747·14
50	58·37	291·85	583·70	875·55	1,167·4
60	84·05	420·26	840·53	1,260·8	1,681·1
70	114·4	572·03	1,144·05	1,706·08	2,288·1

The centrifugal force varies directly as the degree of curvature.

The heavy division lines mark the assumed maximum limit of speed for safety, when the centrifugal force is $=\frac{1}{4}$W.

On the 4 per cent. grades of the Mexican Railway, reversed curves of 150 feet radius were worked for a year with ordinary locomotives.

Narrow-gauge railways have rarely been constructed with curves sharper than 24° in the United States, but there are a few as sharp as 30° in Colorado.

In the writer's location of 3 ft. gauge railways in the Sandwich Islands; curves of 40° (146·2 ft. radii) were the minimum, but many of them were more than a semicircle; previous railways had been constructed with 75 ft. radii in that district, but in most cases without necessity. The trains run at ten miles an hour with perfect safety, though it can hardly be called comfortable. There is practically no danger of the trains overturning because the loss of speed due to curvature keeps them well within the limits of safety. For rules on the same subject applicable either to feet or Gunter's chains, see p. 264.

There is chronic trouble to railway managers from curves of unnecessary sharpness, put in either to save the trouble of a second revision, or from lack of experience on the part of the surveyor.

It is true that rolling-stock can be made to go round almost anything, but not without suffering from it.

It is a curious fact that single-line railways which have

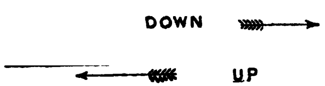


Fig. 1.

their crossing stations arranged in the usual way, as Fig. 1, wear the locomotive wheels more on one side than the

other. This arises from the trains always entering the stations faster than they leave them. Probably, if the crossing stations were made as shown in Fig. 2 this would not take place. It is a preferable arrangement, except

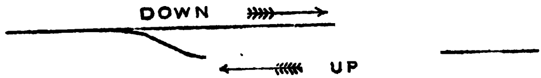


Fig. 2.

where the through express trains run past the station without stopping, in which case the usual arrangement is to be preferred unless very flat curves are used in the switch on Fig. 2.

Effect of Increased Distance from Development, and of Curvature upon Working Expenses

The following notes and Table IX. are from Mr. Wellington's book, being deduced from extensive American statistics.

Fractional changes of distance increase or decrease expenses by only 25 *to* 40 *per cent. of the average cost of operating an equal distance.*

600 *degrees of curvature will waste about* 50 *per cent. as much fuel as the average burned per mile run.*

The lowest probable limit of curve-resistance at ordinary speeds in ordinary curves is about $\frac{1}{3}$ lb. per ton per degree of curvature. With worn rails and rough track it may be as high as $\frac{2}{3}$ lb. per ton.

Curve-resistance per degree of curve is very much greater on easy than on sharp curves, so that when, for example, the resistance is 1 lb. per ton on a 1° curve, it may be 6 lbs. to 8 lbs. per ton on a 10° curve, and not more than 15 to 18 lbs. on a 40° or 50° curve.

The almost uniform increase in cost in the first three main divisions of the line is principally due to grades and curves, which get worse as the line stretches inland.

TABLE IX.—*Running Expenses on Pennsylvania Railroad affected by Curvature.*

Average cost per train mile in cents	Repairs	Fuel	Stores	Total
Eastern division .	6·42	6·93	1·15	14·50
Middle division .	8·87	7·00	1·07	16·94
Western division .	9·25	7·59	1·39	18·23
Mountain and Tyrone . . .	6·60	7·04	0·83	14·47

COMPENSATION FOR CURVES

Some of the various rules used in compensating steep gradients for curvature are given by Mr. Robert Gordon in 'Min. Proc. Inst. C.E.,' vol. lxxxv. :

'The best American practice invariably allows compensation when the curve falls on a gradient by lessening the inclination as the sharpness of the curve increases. Some difference of opinion exists amongst the authorities as to the amount of reduction required, but the average given is 0·05 per 100 ft. per degree of curvature.

'This practice varies, however, and Mr. A. A. Robinson, who has had great experience on steep gradients, gives as follows :—

TABLE X.—*Compensation for Curves.*

				Per 100 feet per degree
'Rate of maximum grade	0 to 1 in 166·0	compensation		0·06
,, ,, ,,	1 in 166 to 1 in 62·5	,,		0·05
,, ,, ,,	1 in 62·5 to 1 in 33·5	,,		0·04

'Mr. Blinkensdorfer gives 0·03 to 0·07 in the same limits ; while Mr. Wellington allows 0·06 on all maximum curves. The practice also of widening the gauge on curves varies much. Some engineers allow only the same play of ½ inch that is given on straight lines ; while others increase it ½ inch and more on curves. But opinion is unanimous in requiring a tangent between reverse curves, and sharp

curves are eased off at both ends. In some cases gradients also are eased at the approaches.'

The following tables are taken from Mr. Trautwine's 'Pocket-Book:'—

TABLE XI.—*Table of Annual Expenses on some United States Railroads.*

Name of company	Per mile of road	Per train mile	Per cent. of receipts
	$	cents	
Lehigh Valley, 1872	—	—	65½
Pennsylvania Central, 1869	32,000	—	70¾
Philadelphia and Reading, 1859	—	—	54¼
,, ,, 1868	17,200	144	71
Connecticut, average of all the R. R., 1861	3,781	95	57
Massachusetts, average of all the R. R., 1861	3,785	85	60
New York State, average of all the R. R., 1867	13,856	—	76
New York Central, average of all the R. R., 1867	15,620	170	77¾
English R. R., averages for 1856-7-8	—	66	50
Scotch ,, ,, ,,	—	56	44
Irish ,, ,, ,,	—	52	40

TABLE XII.—*Statistics of several United States Narrow-gauge Railroads for* 1884. (*Poor's Manual.*)

—	Gauge, feet	Length, miles	Rolling-stock: Locomotive	Passenger	Mail, &c.	Freight	Cost of road and equipment per mile	Gross annual earnings per mile of road	Annual expenses per mile of road	Expenses ÷ gross earnings
							$	$	$	
Bridgton & Saco Riv., Me.	2	16	2	2	1	16	12,167	1,112	834	·75
Profile and Franconia, N. H.	3	14	3	7	—	6	15,430	1,346	640	·48
Camden and Mt. Ephraim, N. J.	3	6	—	—	—	—	13,645	2,868	2,642	·92
Bradford and Kinzua, Pa.	3	39	5	5	2	69	14,922	1,793	1,717	·96
Denver and Rio Grande	3	1,685	239	115	72	5,676	35,000	3,519	2,573	·73

TABLE XIII.—*Items of Total Annual Expenses for Maintenance and Operation of all the Railroads of the United States in* 1880. (*Poor's Manual.*)

—	$ per mile	Per cent. total	Per cent. of earnings
Repairs of road, bed, and track	451	11·23	6·82
Renewals of rails	197	4·89	2·97
Renewals of ties	122	3·04	1·85
Repairs of bridges	102	2·55	1·55
Repairs of buildings	87	2·17	1·32
Repairs of fences, crossings, &c.	17	·42	·25
Telegraph expenses	41	1·01	·62
Taxes	152	3·77	2·29
Maintenance of road and real estate	1,169	29·08	17·67
Repairs, &c. of locomotives	249	6·19	3·76
Repairs, &c. of passenger, baggage, and mail cars	120	2·99	1·82
Repairs, &c. of freight cars	257	6·40	3·89
Repairs, &c. of rolling-stock (including renewals and additions)	627	15·58	9·47
Passenger train expenses	137	3·41	2·07
Freight train expenses	330	8·21	4·99
Fuel for locomotives	374	9·31	5·66
Water-supply, oil, and waste	70	1·74	1·06
Wages of locomotive runners and firemen	310	7·72	4·69
Agents, and station service and supplies	451	11·23	6·82
Salaries of officers and clerks	139	3·46	2·10
Advertising, insurance, legal expenses, stationery and printing	123	3·06	1·87
Damages to persons and property	40	·98	·60
Sundries	250	6·22	3·78
Running and general expenses	2,224	55·34	33·64
Aggregate annual expenses	4,019	100·00	60·78

TABLE XIV.—*Gross Annual Earnings per Mile, per Passenger Mile, and per Ton Mile, of some of the Principal United States Railways in* 1880.

—	Length in miles	From passengers per mile of road	From passengers per passenger mile	From freight per mile of road	From freight per ton mile
		$	$	$	$
Pennsylvania R. R.	1,806	4,700	·0242	15,615	·0089
New York Central	994	6,651	·0200	21,794	·0086
Central Pacific	2,447	2,237	·0303	4,577	·0249
Chicago, Burl., and Quincey.	1,805	1,532	·0240	7,202	·0111
Philadelphia and Reading	780	3,429	·0201	17,200	·0161
Union Pacific	1,215	2,624	·0320	7,154	·0199
Atchison, Topeka, and Santa Fé	1,398	1,144	·0606	3,974	·0209
Average of United States	87,801	1,641	·0251	4,740	·0129

TABLE XV.—*Annual Earnings and Expenses of the above roads in* 1880.

—	Length in miles	Gross earnings per mile	Expenses per mile	Expenses ÷ gross earnings
		$	$	
Pennsylvania R.R.	1,806	20,315	12,267	·585
New York Central	994	28,445	17,969	·609
Central Pacific	2,447	6,814	3,340	·470
Chicago, Burlington, & Qu.	1,805	8,734	4,454	·497
Philadelphia and Reading	780	20,629	11,754	·568
Union Pacific	1,215	9,778	4,507	·426
Atchison, Topeka, & Santa Fé	1,398	5,118	2,408	·458
Total United States	87,801	6,611	4,019	·608

ESTIMATES

A few estimates of roads, railways, and tramways will now be given, which will to some extent fix the ideas on what must necessarily be subject to very great diversity, according to the circumstances of each case.

The surveyor would do well to make rough shots at his estimate on his first walk-over, so as to guide him in his

choice of alternative routes, and even the most approximate 'aide-mémoires' are often of great service.

The tendency of even experienced men is to over-estimate rough country and under-estimate easy country. A big gulch or cañon is apt to scare most men, and swaggering viaducts float across their mind's eye, which often by patient reconnàissance melt down to one or two reverse curves and a 'bit of a trestle.'

An ordinary American railway of about 100 miles in length in moderately easy country will require about 15,000 cubic yards of earthwork per mile.

Mr. Trautwine's estimate, made quite a number of years ago, is near enough for a rough shot to-day.

It is as follows for a single line in United States:—

Gauge 4′ 8½″. Labour $1·75 = 7*s.* per day.

	$
Grubbing and clearing (average of entire road) 3 acres at $50	150
Grading 20,000 cubic yards of earth at 35 cents .	7,000
Ditto 2,000 cubic yards of rock at $1	2,000
Masonry of culverts, drains, abutments of small bridges, retaining walls, 400 cubic yards at $8 .	3,200
Ballast 3,000 cubic yards broken stone at $1 . .	3,000
Cross-ties 2,640 at 60 cents delivered	1,584
Rails (60 lbs. to a yard) 96 tons at $30 delivered .	2,880
Spikes	150
Rail-joints	300
Subdelivery of material along the line . . .	300
Laying track	600
Fencing (average of entire road) supposing only one half of its length to be fenced	450
Small wooden bridges, trestles, sidings, road-crossings, cattle-guards, &c.	1,000
Land damages	1,000
Engineering, superintendence, officers of Co., stationery, instruments, rents, printing, law expenses, and other incidentals	2,386 [1]
	26,000

[1] This amount is only extended to units to bring the total to a lump sum.

Add for depôts, shops, engine-houses, passenger and freight stations, platforms, wood sheds, water stations with their tanks and pumps, telegraph, engine-cars, weigh scales, tools, &c.; also for large bridges, tunnels, turnouts, &c. (Trautwine.)

Mr. R. C. Rapier, of the well-known firm of Ransomes and Rapier, in his 'Remunerative Railways' gives an estimate for the equipment of a metre-gauge single line railway, 40 miles long, including 40 lb. rails, wooden sleepers, seven engines weighing 15 tons each on six wheels, turn tables, tanks, water-cranes, weigh-bridges, sheerlegs, signals, 35 passenger carriages and brake vans, 150 freight waggons of different kinds, workshop-fittings, and stores. The total for 40 miles 86,708*l.*, or for 1 mile 2,168*l.*

The same author gives another estimate for the equipment of a forty-mile 3 ft. 6 inch gauge-railway with 45 lb. rails, eight engines weighing 18 tons on six wheels, and a similar list of materials to the metre-gauge, somewhat increased. The total for forty miles 98,840*l.*, or for one mile 2,471*l.*

Where it is necessary, as in England, to put up gatekeepers' houses, and sometimes signal-interlocking gates at level crossings, it is often as cheap, having regard to future expenses of operating the line, to go over or under the road.

Gatekeeper's house.

	£	s.	d.
Wages 15*s.* per week = 5 per cent. on capital of	780	0	0
House	220	0	0
	£1,000	0	0

Even where there is no help from the ground the earthwork can generally be got under 20,000 cubic yards.

Alternative bridge.

	£	s.	d.
20,000 cubic yards earthwork at 9*d.* . . .	750	0	0
One bridge	250	0	0
	£1,000	0	0

TABLE XVI.—*The following Table is a short Extract from one given by Mr. Burnett in the 'Min. Proc. Inst. C. E.' Vol. lxxxv. of the New South Wales Government Railways, 4' 8½" Gauge.*

Name of section		Length	Ruling gradient	Minimum curves	Level of lowest and highest points	Particulars as to cuttings, embankments, permanent way, &c.		Cost of way and works		Remarks
								Total	Per mile	
				chains				£		
Great Western Main line	Penrith to Bathurst	111	1 in 30 and 1 in 33 for 17 miles, remainder 1 in 40	8	88 3,658	Formation width in cutting, 18 ft; slopes 1 to 1 in earth. Rock cuts ⅛ or ¼ to 1. Bank 30ft. wide top. Slopes 1½ to 1. Ballast 6 in. deep at middle, 8 in. at sides, 10' 3" wide top.	Rails double-headed, 75 lbs. per yard, fished and carried in chairs, 25 lbs. each, fixed to iron-bark, blue gum, or boxwood sleepers, 9 ft. × 10 in. × 5 in. placed 3 ft. centre to centre.	1,878,747	16,926	Single line. Very heavy works. Total excavation, principally in rock, was 5½ mill. cub. yds.
	Bathurst to Dubbo	133	1 in 40	12	2,153 3,150	Formation width 15 ft. Slopes as above. Ballast as above, but 8' 7" wide top.	Rails single-headed, 70 lbs. per yard (Vignoles), fished and fixed direct to the sleepers, 8 ft. × 9 in. × 4½ in. placed 2' 8" centres	1,007,461	7,575	Single line
	Dubbo to Byerock	177	Easy gradients and curves	—	—	Ditto	Ditto	1,034,509	5,845	ditto
Inland branches to main lines	Wallerewang to Mudgee	85	ditto	—	—	Formation width 15 ft. with slopes as above. Ballast 6 in. deep middle; 8 in. sides; 8' 7" wide at top.	The lighter permanent way, as above, but steel rails.	932,317	10,968	ditto
	Junee to Jerilderie	232	ditto	—	—	Ditto	Ditto	1,320,738	5,908	ditto
	Werris Creek to Narrabri	97	ditto	—	—	Ditto	Ditto	552,013	5,691	ditto

A railway 87 miles long was completed for the Nizam of Hyderabad about the year 1885, for under 6,000*l.* per mile. It was promoted by the Indian Government, and built under the supervision of the Government engineer. The gauge was 5 ft. 6 inches, the rails were 66¼ lbs. per yard, of steel, and the sleepers steel also. The price included equipment and fencing.

ROADS

The Grand Trunk carriage road of the Bengal Presidency cost approximately 500*l.* per mile. The work was done mainly by starving poor during famine time, under the administration of Lord William Bentinck about 1835. The width was 40 feet, metalled in the centre 16 feet wide with either broken stone or a natural concretion of carbonate of lime, called 'Kunkur,' rammed by hand (there being no steam rollers in those days). The cost of laying the metal at an average lead of one mile was 162*l.* 6*s.* 0*d.* per mile, and cost of repairs and maintenance of ditto 33*l.* 12*s.* 0*d.* per annum. Total cost of maintchance was 50*l.* per annum. (Gen. Tremenhcere, 'Min. Proc. Inst. C. E.' vol. xvii.)

The main roads of South Australia are described by Mr. Charles T. Hargrave in 'Min. Proc. Inst. C.E.,' vol. l. and he gives the average cost with a metalled way 18 feet wide and a 60 feet right of way as follows:—

	£	s.	d.
Earthwork in cuttings &c., 800 yards at 1*s.*	40	0	0
Culverts, nine at 15 cubic yards each; 135 cubic yds. at 12*s.*	81	0	0
Wheelguards and posts; 18 sets of 2 posts and one guard at 40*s.* per set	36	0	0
Forming the metal bed; 80 chains, 8*s.*	32	0	0
Bottom metal or soling, 4″ thick, 15 cubic yards per chain for 80 chains; 1,200 cubic yards at 4*s.*	240	0	0
Metal 2½″ thick, 22 cubic yards per ch. for 80 ch.; 1,760 cubic yards at 7*s.*	616	0	0
Blinding (*i.e.* thin top dressing of gravel) 7 cubic yards per ch: for 80 ch.: 560 cubic yards at 1*s.*	28	0	0
Rolling eight days at 25*s.*	10	0	0
Carried forward	£1,083	0	0

Brought forward . . .	1,083	0	0
To this must be added fencing (not always done) 80 ch. at 8 rods = 640 rods at 5*s*. . .	160	0	0
And if land has to be purchased; 8 acres per mile at 5*l*.	40	0	0
	£1,283	0	0

For the first six miles out of Adelaide during ten years, 300 to 400 cubic yards of metal were needed per mile per annum. At fifty miles from the city only slight repairs were needed.

Annual cost of clearing culverts and weeding 3*l*. per mile for each side of road. On 326 miles constructed the cost of maintaining the metalled portion was 122*l*. per annum.

The price of labour was 5*s*. 6*d*. to 6*s*. per day; masons and carpenters 8*s*. to 9*s*.

A temporary road for conveyance of railway materials into the bush costs from 50*l*. to 100*l*. per mile.

Stone-Crushers

'Blake's or Blake-Marsden Stone-Crushers' vary in size and cost, from a $10'' \times 8''$ machine breaking stone of that size to the extent of $3\frac{1}{2}$ cubic yards per hour, nominal horse-power 3, total weight including screening apparatus 5 tons 6 cwts., price 157*l*., up to a $30'' \times 13''$, breaking stone of that size to the extent of 14 cubic yards per hour, 16 horse-power, weight 16 tons 2 cwt., price 440*l*.

Road-Rollers

A 15-ton 'Aveling and Porter Roller' costs about 650*l*. It will roll on an average 1,100 square yards per day. The cost of rolling, including all charges, is somewhere between $\frac{1}{2}$*d*. and 1*d*. in England; and the cost of binding material about 3*d*. per square yard.

The two last estimates are gathered from Mr. Boulnois' Municipal Surveyor's Handbook.'

Tramway Estimates

The following estimates of tramway construction in the United States are from the report of the committee of the American Street Railway Association at the Minneapolis Convention in October 1889, upon 'The Conditions necessary to the Financial Success of Electricity as a Motive Power.'

They are for a single line ten miles long equipped complete with fifteen cars.

Cable-road

	£
Road-bed, rails, conduit cable	140,000
Power station	25,000
Cars	3,000
Total for ten miles	£168,000

Electrical Overhead-wire Construction

	£
Road-bed and rails	14,000
Wiring	6,000
Cars	12,000
Power station	6,000
Total for ten miles	£38,000

Storage Batteries

(*also termed Secondary Batteries or Accumulators*)

	£
Road-bed and rails	14,000
Cars	15,000
Power station	6,000
Total for ten miles	£35,000

'In the above cases of electrical construction the motor car would be capable of pulling one or two tow-cars if necessary. These figures your committee has no doubt will

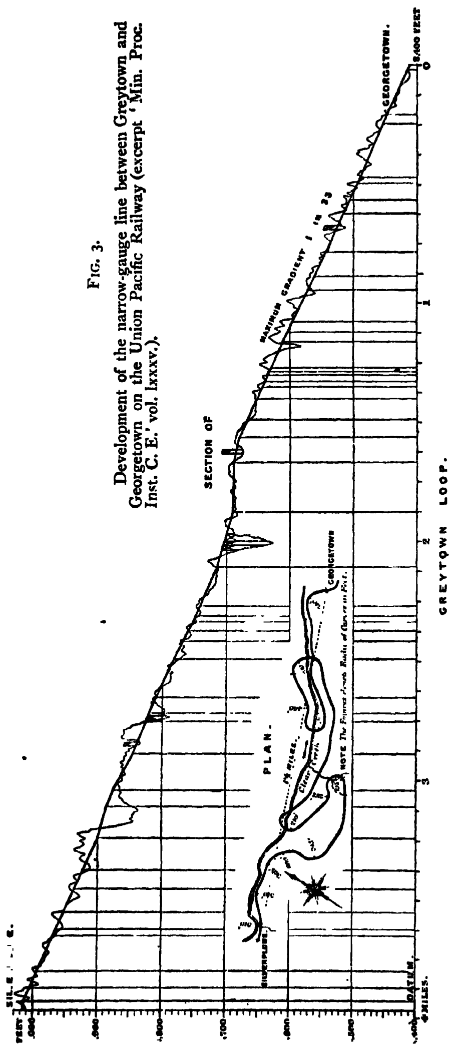


FIG. 3.

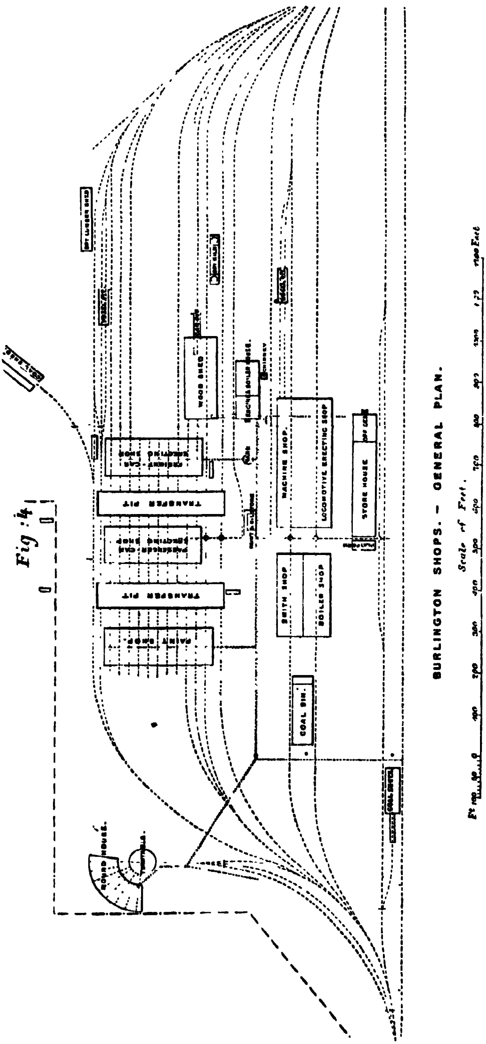
Development of the narrow-gauge line between Greytown and Georgetown on the Union Pacific Railway (excerpt 'Min. Proc. Inst. C. E.' vol. lxxxv.).

be found to be calculated within a reasonable limit of cost.'

Figs. 3 and 4, excerpt 'Min. Proc. Inst. C. E.' vol. lxxxv., will give some idea of the way difficulties of ascent are overcome by curve-development, and the ground plan of Burlington shops will show the way the lines are laid out for the engine depôts in America.

The actual measurement for the preliminary estimate is much facilitated by diagrams, some of which will be found in the chapter on 'Graphic Calculation.' These operations follow upon the actual survey, whether it be merely a route-survey or a telemetric survey, and are therefore described later on.

Fig: 4



Ground plan of the engine and carriage shops of the Chicago, Burlington, and Quincy Railway at Burlington, designed by Mr. Rhodes, C. E. (excerpt 'Min. Proc. Inst. C. E.' vol. lxxxv.).

CHAPTER II

ROUTE-SURVEYING OR RECONNAISSANCE

BEFORE going into the subject of 'Route-survey' in detail, the first steps of American pioneer railway-making will be briefly described.

The Nipissing division of the Canadian Pacific Railway will furnish a fair illustration of American location. It is on the north shore of Lake Superior, and it was there that the writer gained his first experience of that kind of work. A network of lakes and rivers leads the Indian, on his fishing and hunting expeditions, clear across the watershed of Northern Ontario down to Hudson's Bay. The first pioneering was done by Mr. W. R. Ramsey, with an Indian guide and one or two white men. Taking only the aneroid and prismatic compass, they followed up the course of several rivers with a canoe, often having to carry it over long 'portages,' and undergoing considerable hardships, until they emerged at the junction point with another survey, proceeding from Port Arthur on the other side of the great lake. A sketch-map was made for the approval of the general manager, showing the topography to a small scale, and the length of line.

The next party, headed by Mr. Ramsey, consisted of the usual preliminary location party :—

1 Transit-man	2 Axe-men
2 Levellers	2 Chain-men
2 Rod-men	1 Slope-man
1 Picket-man	1 Cook

The *modus operandi* is then as follows :—

The leader keeps ahead of the party, revising his previous survey with aneroid and compass, and keeping the whole of the operations under control. Every morning he indicates the direction to the transit-man, who has a line cut through the bush and chains it, driving stakes at every hundred feet. The leveller follows on the transit-man, making the profile (or section as we call it in England), whilst another takes the side-slopes of the hills with a clinometer. The field-work is plotted in tent at night. The profile is greatly facilitated by printed profile paper, which dispenses with all scaling.

Under favourable circumstances a mile or a mile and a half a day can be surveyed in this manner, but on the division here referred to only an average of ten miles a month was possible with one party. At times there were four parties in the field to distribute the work. Nearly all of it was gone over three or even four times, always saving money on the construction. A hundred pounds extra in survey will often save a thousand pounds in construction.

The present chapter is devoted to methods which do not require chaining, but more on the American methods of location will be found in Chapters VI. and VII.

When the country through which a road or railway is to be made is not well known—specially when the inhabitants are unfriendly, but in almost every case of projected work in a new country—it is necessary to obtain, cheaply and speedily, one or more surveys of alternative practicable routes or of the general topographical features of the area.

A chained survey would be out of the question, and even a telemetric traverse would generally be too expensive. A fairly accurate map is required, which will be made by one man at the rate of ten to twenty miles per day, and which will serve the projectors with sufficient information to guide them in approving the route favoured, in capitalising their enterprise, and in preparing their prospectus.

The class of work most suitable for this object has been

more thoroughly studied and practised, though with a different object, by military than by civil engineers, and the best instruments and books extant are from those sources. Failing time or money for chaining or triangulation, it is necessary to have recourse for linear measurement to records of pace of men, gait of horses, or speed of river steamers taken with as much care as possible. Military movements are made with trained regularity, and no one who has not studied the subject would believe what great precision is attained in making maps of marches the distances of which are laid down simply and solely from either time-records or pace-measurers.

These maps, which are called reconnaissances, route-surveys, or flying surveys, vary in accuracy, from a rapid field-sketch relying upon a trained eye for estimation of distance and a knowledge of perspective for the filling in of detail, to a survey correct within a margin of from one to five per cent.; based upon true trigonometrical and traverse principles and preserved from cumulative error by astronomical observations similar to those which determine the position of a ship at sea.

The term 'a mere sketch' is often applied rather contemptuously to what may be a most valuable and perhaps the only record of the topography of an important position.

Military engineers know best the value of a good sketch. They have to do their work under fire or in danger of surprise, and a man who can dash off a sketch of the enemy's position, the points of vantage, and the best line of attack, will by so doing provide in a few moments information which may decide the final issue of the struggle.

Surveyors have more time at their command, and their disposition is rather to depreciate methods which overstep the bounds of rigid mechanical exactness. The standing orders of our English Parliament for deposited plans of public works call for a high standard of accuracy, and justly so in a country like this, possessing already excellent

maps published by the Ordnance Department. It was due to the existing vested interests that they should be protected from the attempts of speculative promoters to interfere with their property without fully and clearly representing in all its bearings what the effect of such interference would be. The standing orders, however, permit of the framing of memorials of opposition containing frivolous allegations which as such are frequently overridden by the Examiner.

Surveyors who get their training in England do not consequently have much opportunity of practising either route-survey or field-sketching, and so, specially in regard to the latter, do not know if they have a talent for it or not. Artists are born, not made, but there are few engineers who are so wanting in 'eye' that they cannot greatly increase their efficiency as pioneers by a course of study and practice in sketching. Even when the route survey is carried out with all possible care so as to approach to the purely mechanical system, a knowledge of sketching will be most useful in filling in the adjacent country. Lake, wood, bluff, cañon, or river will, by a few dashes of the pencil, give the promoters at home something more for their minds to feed upon than a system of straight lines with the bearings written up. It should of course be evident from the map, when part is sketched and part scaleable, which is the reliable portion.

The art of free-hand drawing can hardly be dwelt upon in a work of this kind. A very few lessons in perspective combined with a thorough course of practice in the field is the only way to obtain the needed skill. Even in the course of telemetric survey, the sketching of detail is a great help both in the plotting of contours and as an accessory to the finished map.

Photography is also becoming a useful aid to the surveyor. The little 'Detective' camera recently introduced is only a little larger than a carriage clock; it is carried in

the hand without needing a tripod or other paraphernalia. It is directed and focussed by observing a reflected image of the view on the top of the camera. It has also the advantage over previous inventions of the kind that the views may be removed one by one instead of in batches.[1]

The scales adopted in route-surveying vary from six inches to the mile, to thirty or more miles to the inch. If to a small scale, it is usual to plot in miles of latitude and longitude, so as to correspond with the daily astronomical observations.

The route survey is essentially a traverse, checked where possible by triangulation with compass angles and range-finder, or simply by rays drawn upon the sketch-board or plane-table.

Surveying with the Sketch-Board or Plane-Table

Those who have become proficient in the use of the plane-table are generally enthusiastic in favour of it, and there can be no doubt that for a topographical survey of a large area, based upon the known stations of a primary triangulation, it is both rapid and accurate. During the season of 1886–7 the U.S. Geological Survey mapped in this way an area of 56,000 square miles with a staff of 160 men, at an average cost of 12*s.* per square mile.

[1] A camera should be preferred which does not require the removal and changing of the slide every time. Abrahams & Co. make the 'Ideal' camera, which is quite suitable for the purpose. If the surveyor is on one side of a wide cañon and a train is passing along the other side, he can get a good representation of the whole hillside by taking views of the train all along its passage, the steam from the engine will make a good landmark and especially just as the train goes into tunnel. The views can afterwards be pieced together; the length of the train will serve as a measure of relative distances, and considerable information be placed on record by a few seconds' field-work. The slides can be preserved and developed at leisure or sent home for development, but the developers are provided in soluble pellets for foreign use.

The author's practice in surveying for railways has led him to the persuasion that the plane-table is more useful as an adjunct than as a universal instrument. He uses the modification of Captain Verner's military sketch-board plane-table either buckled on the wrist or mounted on its light tripod, and in this form finds it one of *the most valuable* portions of the surveying outfit.

When the country is very magnetic, the needle becomes unreliable, and the plane-table may have to be used for the whole of the field-work.

Under favourable conditions, it is possible to do pretty much the same class and quantity of work with the plane-table alone as with the prismatic compass and field notes.

With the plane-table, the work is all done in the field, which has the advantage of exhibiting the errors liable to arise with either class of work, and so to enable them to be eliminated on the spot. It has the disadvantage of occupying time in mapping during daylight, which might be wholly given to fieldwork, and the mapping done either at night or when the sun is too powerful, or when too wet to go out. It has the further disadvantage of disclosing to unfriendly natives the object of the traveller's journey, being much more conspicuous than a prismatic compass. It has the advantage of affording to the skilful surveyor a complete method of triangulation, producing results of an accuracy (even over an extended area) very little short of that obtained with good theodolites ; its horizontal angles being determined by rays aided if desired by the telescopic sight-rule, and independent of magnetic variation. It has the disadvantage of being with difficulty checked when carrying forward a continuous traverse, especially so when the bases have to be short, as in a crooked road with high hedges. There are generally some salient points on which rays can be taken as independent checks, but without them every mistake in angle is perpetuated.

For an extended triangulation a measured or calculated base is required as in geodetic survey, whether working with the plane-table or compass; and where the needle can be relied upon, the angles can be taken with the same accuracy with either, but in the case of a continuous traverse, the compass has the advantage of giving the magnetic bearing every time independently of the previous work.

When the plane-table is used as a universal instrument, a stadia telescope is attached to the sight-rule, which has to be transported in a separate case. The class of work performed by it under these circumstances answers to the telemetric survey with the transit, but when passing rapidly through a country, it is much more cumbersome to transport a heavy plane-table with sight-rule and telescope than to shoulder a theodolite.

A few remarks will now be made upon the method of triangulation with a large-sized plane-table over an extended area, after which the subject of the plane-table will be only considered in the form suitable to preliminary railway or road survey with the smaller instrument described in Chap. IX.

TRIANGULATION WITH THE PLANE-TABLE

Where a base is known by the exact geographical positions of two towns or villages a few miles from one another, and commanding a good view of surrounding country, an almost unlimited tract may be correctly surveyed from it. When this is not the case, the two extremities of the base have each to be located astronomically, as described on Chap. IV., and the distance between them calculated by problem on p. 150. The accuracy of the succeeding triangulation will depend fundamentally upon this astronomical work. The latitude should be easily obtainable within a sixth of a mile, and by careful repetitions to about 100 yards of the earth's surface, so that if the watch be not reliable to give Greenwich time to a second, the more the

base approximates to a true meridional line the better for the results.

To obtain practice in England the plane-tabler should locate two suitable points for his base from the six-inch Ordnance map of the district, which gives parallels and meridians to every single second or about thirty yards. They should be selected as far apart as possible, say from two to seven miles.

A common military plane-table 30 inches × 24, of one-inch panelled deal, will map the whole county of Yorkshire on a scale of four miles to the inch. It can afterwards be compared with the shilling edition of W. H. Smith & Son's reduced Ordnance maps on that scale.

The paper should be pasted on the board by its margins, well damped so as to stretch as tight as a drum when dry. The parallels and meridians should then be drawn as described on p. 75, and the base line AB laid on from the data taken from the six-inch Ordnance map. The table is then carried to station A, and, with the sight-rule touching the base line, the board is turned on its axis until the opposite base-station B is brought into the exact line of sight. The compass, which should be of the portable trough-needle type, is then placed in one corner of the map, and carefully turned to and fro until the needle is at rest at the centre of its run. A fine pencil line is then drawn round the compass box with the letter N. (mag.) opposite the north point of the needle.

With the plane-table firmly clamped, rays are then taken from A to all prominent points in view; they should not be drawn right across the paper, but merely fixed by a fine pencil line in the margin of the map about half an inch long, with marks denoting the point of observation and the point observed, thus A/A_1 to correspond with descriptive entries in the Field Book, shown overleaf.

When all possible rays have been taken from A, the instrument is shifted to B, and 'set in meridian' by a back

FIELDBOOK

Date	Time	Station	Object	Latitude			Longitude			Aneroid	Variation of compass	Remarks
				°	′	″	°	′	″			
1890 Jan. 1	3 P.M.	A	B							640	$19\frac{1}{4}$° W.	
			A_1									Spire of Hockley Parish Church
			A_2									Clump of Beeches near Hockley

ray on A. The compass-box is placed on its delineated position on the paper, and if the sight-rule has been correctly adjusted, and there be no local magnetic deviation or sufficient diurnal variation to account for it, the needle will still be at rest at the centre of its run. If not, its position should be measured by the graduation on the compass-box as an addition to or deduction from the initial variation, and an entry made of the variation at B.

In selecting the next base C by intersection of two rays from A and B, a point should be chosen which, in addition to advantages of commanding position, should form as nearly as possible with AB an equilateral triangle; the reason being that in any triangulation the more acute the angles, the less reliable must of necessity their intersections be and an equilateral triangle is the only one in which *no* included angle is less than 60°.

When primary points have thus been located all over the map, the filling in of roads and other detail is done either with the prismatic compass and passometer, as shown in the example on p. 54, or, if more accuracy is required, with compass and tape. This subsidiary survey is plotted on tracing paper, and pricked through on to the map. This method is preferable to traversing in the detail with the plane-table itself as described on p. 41 because it saves the map from getting soiled, and is an independent check (when the tape is used) upon the other work.

The main practical points to be observed in using the table are to set it up firmly upon its tripod ; to level it perfectly true with a circular bubble ; to be very exact with its setting in meridian ; to keep the pencil finely pointed ; to draw the rays with the utmost nicety, and above all things not to get hurried.

ROAD AND RAILWAY WORK

The work required for preliminary road and railway survey resolves itself into two classes : the first a reconnaissance of such rapidity that even stadia measurements take too long, and, except at intervals, there has to be as little time as possible devoted to erecting, levelling, and adjusting of instruments of any kind ; the second, a telemetric location survey, which will give the levels sufficiently close to plot a profile from them, and this is quicker and better done by the transit-telemeter than by the plane-table.

The 'Verner' sketch-board is described on p. 318, to which the reader's careful attention is directed; its use will be first explained when accompanied by a prismatic compass on reconnaissance, and then some few further illustrations will be given of its more extended application when enlarged and developed into the regular surveying plane-table for the sake of those who may wish to make the most of it.

VARIATION OF THE COMPASS BY THE PLANE-TABLE

Before starting, the variation of the compass should be ascertained either by an observation of the solar azimuth, as explained on p. 132, or, failing suitable instruments for that purpose, by equilinear shadows on the sketch-board, as shown by Fig. 5. Erect the instrument on its wooden tripod, having the roll of mapping paper tightened up into its place. Fix the brass stile in the hole provided for it near the compass-box, and adjust the board level and stile vertical by the clino-

meter or otherwise. Draw the centre line of the table across the paper from headpiece to tailpiece, and 'set' the table so that the needle shall be at rest in line with this centre line. Check once more the adjustments by clinometer, and the instrument is ready.

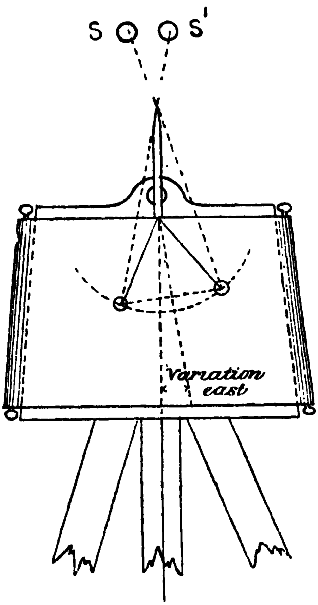


Fig. 5.

About an hour or two before noon place a mark with a pencil at the extremity of the shadow cast by the stile, and from the base of the stile as a centre and with a radius equal to the length of the shadow describe a circle right round the board. When the sun begins to drop again in the afternoon, watch the shadow until it once more touches the circle. Mark it, and bisect the chord drawn between the two shadow-poins in the circle, and draw a line to the base of the stile from the centre of the chord. This will be very nearly the astronomical meridian and the angle between it and the centre-line will be the variation of the compass. The error, as explained on p. 135, arises from the altered declination of the sun during the lapsed time, which varies from 0 to 1 angular minute per hour. The error may at all times be neglected for this class of work. When the weather is uncertain it is best to take two or three points, say at 2 hours, 1½ hours, and 1 hour before the meridional passage, in order to ensure getting the sun in the afternoon again.

Sketching and Plane-tabling

The sketch-board is used upon two distinct principles. When buckled on to the wrist, the correctness of the alignment depends on the little compass in the head-piece, checked as much as possible by bearings taken with the prismatic compass. No back and forward rays can, of course, be taken. The board has to be '*set*' at every point of observation by the 'working meridian,' which is a line drawn across the compass-box with an index at its end, by which the compass-box can be turned in any direction. The circle round the compass is graduated into divisions of ten degrees each, and the index is placed in such a position that when the needle is under it the board will be directed in the general line of route. When once fixed the index is never touched unless the survey begins to run off the paper. It is a fiducial line by which to adjust or 'set' the board whenever a sight is taken; and considerable practice is required to hold the board steady on the wrist with the needle truly under the index whilst the sight is being taken. It is also quite an awkward business to keep the edge of an ordinary sight-rule or ruler at the station-point whilst it is being rotated to take a sight. It was to meet this difficulty that the needle-point sight-rule, described in Chap. IX., was contrived, which has proved a great convenience, and much better than merely sticking a needle in the station. Most surveyors use elastic bands for keeping the ruler in position, but a thin string with a spring underneath is neater.

The 'working meridian' is fixed upon before starting by means of any existing map, or, failing all such data, by inspection of the ground. For instance, if a route-survey be required from London to Birmingham we draw a pencil line between the two cities upon an ordinary atlas, and, running up the line to the intersection of a parallel, we find the astronomical bearing to be roughly 50° N.W. Supposing also that the variation of the compass has been just deter-

mined, as already described, to be 20° W., the magnetic bearing of the air-line between the two cities will be 30° N. W. This is called the 'Line of Direction,' and is marked as such on the headpiece, corresponding with the direction in which the paper has to be fed forward upon the rollers. The words 'Line of' correspond with a due northerly direction; that is to say, when the index is placed in line with the 'Line of

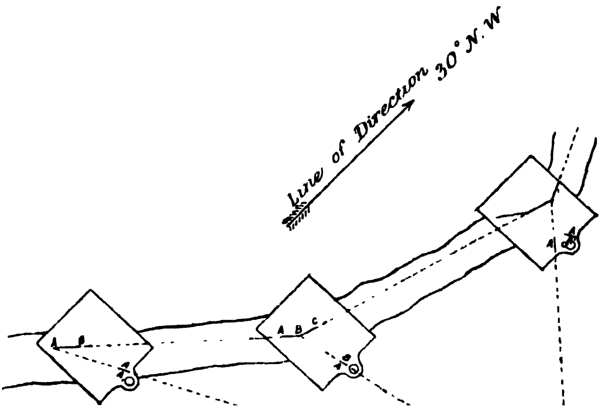


Fig. 6.

Direction,' and the needle is brought under it with its north pole towards the words 'Line of,' the board will be held in position for running due north.

We want to run a course of 30° N.W., and therefore fix the index 30°, that is, three divisions to the right of the words 'Line of,' so when the needle is brought to rest under the index, the 'Line of Direction' will point to Birmingham.

The width of the paper being ten inches, it will take in

20 miles on either side of the air-line to a scale of four miles to the inch, and this would take in the whole map if we were following one of the main highways.

If the line should run off the paper a 'cut-line' must be drawn and a fresh start made in the middle of the paper with the same line of direction. This method of using the sketch-board is illustrated by Fig. 6. The magnetic bearing of the line of direction should be written up before commencing, so that the index, if accidentally shifted, may be replaced.

The prismatic compass is of great use in checking the angles taken in this manner with the sketching-board. The bearings of at least the main lines of the traverse can be taken, and whenever important intersections are wanted upon salient points they should be likewise checked with the prismatic compass. It is quite possible to have too many instruments, but a compass clinometer, as described in Chap. IX., is no inconvenience and most valuable. However skilfully the board is held in line, the slightest jog to the arm may, imperceptibly to the sketcher, twist the table several degrees. The value of the sketching-board is not lessened, but on the contrary much enhanced, by assisting it with the prismatic compass. The same amount of detail can be filled in, but with increased accuracy.

Sketching with the Aid of Maps

Where existing maps of any kind are available, they should be made all possible use of. An enlargement from an atlas, however imperfect, should be laid down to scale, and unless the map is thoroughly reliable, like an Ordnance map, it is best to draw the enlargement in pencil, and to plot in ink, or *vice versâ*, so that the divergences may be at once apparent. To copy detail from the existing map would only confuse, but a check of alignment and distances is of great assistance.

It frequently happens that during a rapid traverse a case

arises demanding more careful treatment than can be given with the board on the wrist. A metallic telescoping tripod is buckled to the bag of the sketch-board, or, when riding, is attached to the saddle. It is only sixteen inches long when shut up, and gives no trouble. By it the sketch-board can be used as an ordinary plane-table, but being light and vibratory it is only used in emergencies; for continuous plane-tabling the wooden tripod should always be employed.

In this form, the work can be done in fairly easy country at the rate of twenty miles a day. It is something more than a sketch and something less than a survey, but seeing that it takes hardly any more time to a practised man than if he simply took notes, it has the great advantage of graphic representation over mere literary description.

Accurate Plane-tabling

The second method of using the sketch-board mounted on its wooden tripod differs in no way from the ordinary plane-table except that the instrument is smaller.

The principle of this kind of surveying is the geometrical law of similar figures, by which when a single side is known all the rest are determined by their positions in the figure. It is a graphic triangulation resting on the same mathematical theory as telemetry. The method of setting up the table has been already explained.

The U.S. Geological and Coast Survey have covered immense tracts of country with plane-table work and use instruments of large size—24 inches by 30 is the maximum. They are fitted with elaborate joints for levelling them true, and furnished with sight-rules carrying stadia telescopes with vertical arcs for measuring angles of inclination. The figure on p. 42 of a traverse by the first method will also serve for the second. The preliminaries are all the same, the only difference being that the board is 'set' at each new station by a backsight on the previous station with the sight-

rule. The precision attainable in this way is very great. Check angles should still be taken with the prismatic compass, especially where the bases are short, but the rays are generally more accurate than the compass-bearings, particularly so if there is local attraction to the needle. The compass-bearings, both with the needle in the headpiece (which is set to the 'working meridian' precisely as before) and those by the prismatic compass, are simply checks and nothing more.

In taking a sight, the ray should be projected at the edge of the paper about half an inch long; it should be marked with the signs of the back and fore station thus: A/B if taken from one main station to another, or B/B_1 &c. if taken from a main station to some outside point. The signs should

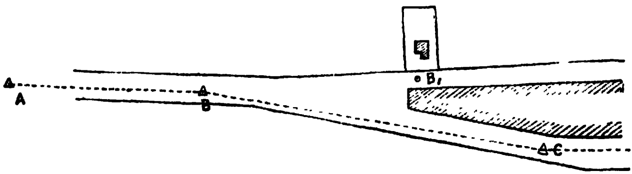


FIG. 7.

also be entered in a book of description (see p. 38) specifying what the points represent, and accompanied by little sketches, especially when the points are not very sharply defined, such as distant villages, hill-peaks, or river-bends, so that when arriving at the next station and taking the backsights of intersection, the memory may be assisted as to the precise spot viewed on from the last station. These little sketches are sometimes made upon the map, but it is not such a good plan, as they are liable to come in the way of succeeding rays.

The flag on Fig. 6, p. 42, is shown as an out-station, but the board can be moved to its vicinity and backsights taken to all the previous stations on flag poles. The coincidence of

these rays with the foresight will prove the accuracy of the work and close the traverse.

The whole principle of plane-tabling is here embodied. The stadia work, levelling and contouring, which are added by means of the accessory instruments to the larger tables, do not form an essential part of the instrument itself, and will be treated of under telemetry.

AUXILIARY PLANE-TABLING

The method of using the plane-table as an adjunct to the tacheometer for filling in detail will be next described. In Fig. 7 a tacheometer traverse is being carried along a main road, and a building with enclosure is proposed to be filled in with the plane-table, being situated upon a cross road, and it being desired to avoid a deviation with the tacheometer. A point B_1 is fixed in the vicinity of the building by a sight with the tacheometer, if possible, within 100 feet of the further corners. The plane-table, worked by an assistant, is then set up at B_1, and aligned by a ray on B. The line BB_1 is then laid down on the paper without any reference to the compass.

The distance BB_1 is not absolutely necessary, but, being known, it is better to plot it, and so locate B on the paper to enable a checksight to be taken upon it if the table has to be moved to another sub-station. If all the corners are within 100 feet, they can be taped without moving the table and laid off to scale on rays taken with the sight-rule. If the table has to be moved to another sub-station in order to command the whole of the detail, it can be triangulated as already described, but it is generally quicker to locate several points with the tacheometer as plane-table stations than to sub-triangulate with the plane-table.

LOCATION OF THE INSTRUMENT

The subject must not be dismissed without touching upon the well-known three-point problem for locating the

instrument, although it should not be used when a more direct method is possible.

It frequently happens that, when surveying with the aid of a map, it becomes necessary to determine the position of the instrument on the ground from one or more points indicated on the map and transferred from it to the plot. If only one or two points are known, the problem cannot be solved without the compass.

Case 1. When one point is known. Maps are generally constructed having the astronomical north at the top of the sheet. Ordnance maps and ordinary atlases are made thus, but not so the parish maps. When this is not the case, the

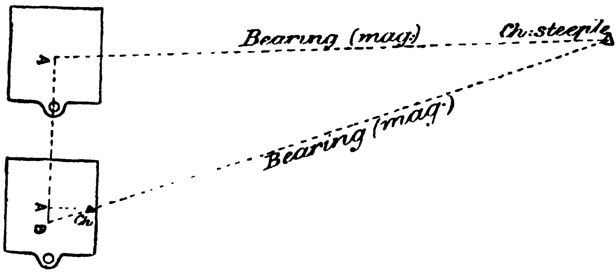


FIG. 8.

position of the astronomical or magnetic meridian or both must first be determined on the ground from the known point in the manner already described.

When the geographical alignment of the map is known, and the variation of the compass has been ascertained, the magnetic meridian should be marked on the map at the known point, a bearing taken to it with the prismatic compass, and plotted backwards from it, i.e. $180° \pm$ bearing. A base line is then run at right angles if possible, but if the ground will not admit of it, as nearly square as possible, and the bearing taken. The base line is measured, and from its extremity another bearing is taken to the known point.

Then laying down the two bearings from the point towards the observer's position on the plot, take the measured distance by scale and place it with the dividers, in its proper bearing, where the extremities will coincide with the two rays from the known point. If the base cannot be run square, the bearing of the base has to be run out with a parallel ruler. It is also convenient to lay off the base in an even number of feet or yards as 100 or 1,000, for then the distance can be read off from a table of tangents.

Case 2. When two points are known, the prismatic compass is used in the same way, but a base is dispensed with. The two known points are plotted from the map in their proper position upon the plane-table, their bearings

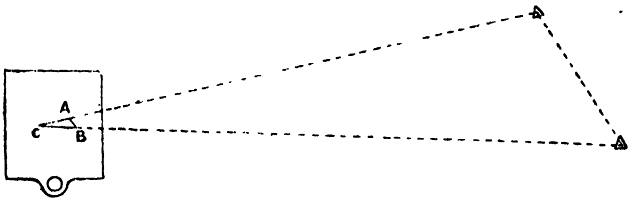


FIG. 9.

taken and plotted backwards as before from magnetic meridians drawn through the known points. Their intersection is the locus of the instrument.

Case 3. When three points are known the plane-table can be located by the sight-rule alone.

Let A, B, C be the three known points and *a*, *b*, *c* their position on the plot. It is required to locate upon the plot the point of observation and to set the table so that the plot shall have its lines parallel to those in nature, or, as it is termed, the table be 'in meridian.' It is obvious that rays through A*a*, B*b*, or C*c* will intersect somewhere, and by Euc. vi. 2 we know that in any triangle, *x*AB, *x*BC, or *r*AC, when the sides AB, *ab*; AC, *ac*; BC, *bc*, are proportional they are parallel. Therefore, if by adjustment

of the table we find the position in which the three rays A*a*, B*b*, C*c*, intersect in one point, the plot will be parallel and the table in meridian, and the point of observation correctly located on the plot.

When the instrument is not 'in meridian,' a 'triangle of error' is formed as shown on the figure, the elimination of which adjusts the table. There is one exception, when the

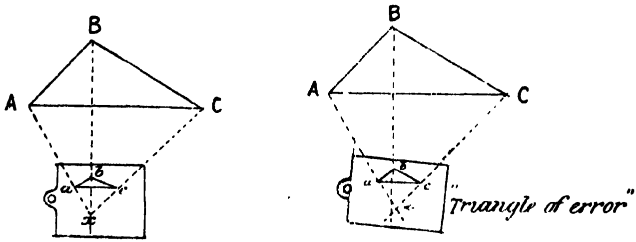


FIG. 10.

point of observation is situated in the circumference of a circle described about the three known points, the triangle of error disappears for any position in that arc. This is at once seen by being able to rotate the table without causing any error, and some other point must be chosen from which when located the required point can be determined. See more on the three-point problem by station-pointer in Chapter IX.

The Plane-table as a Range-finder

Fig. 8, p. 47, illustrates the principle of range-finding described on p. 338. The plane-table will, carefully handled, determine this range or distance of an object with as great accuracy as with some optical range-finders. Military engineers do not use it for this purpose because the enemy is fond of making a target of it.

Measurements of Distance

Setting aside the chain, the methods available for determining the distances upon route survey are of three kinds. *First*, mere judging by the eye and hand, for which some data will be presently given of value to the sketcher. *Secondly*, the ancient method of pace-counting, to which may be added the trocheometer for wheeled vehicles, the time-measurement of horses' gait, and the patent-log records combined with time measurement when steaming on a river. *Thirdly*, the telemetric or optical measurement of distance, which, including the subject of military range-finding, is treated of in the chapter on Tacheometry, also in the chapter on Instruments. It belongs in its practice more to the route survey, though in principle it is to be classed with tacheometry.

First. Judging by the eye is a faculty which is an important part of the capabilities of the sketcher. A good shot will sometimes come pretty near the distance by looking at it as a range for a fowling-piece; a cricketer by the distance he can throw a ball. By holding the hand across the field of view at arm's length, so as to cover the height of a man, a horse, or a man on horseback either with the palm or with one or more fingers, we have a measurement somewhat on the stadia principle and of some assistance in guessing.

The *second* method, of pace-counting &c., may be brought to a very fair degree of accuracy for the purpose of reconnaissance, and we will first touch upon the passometer or pace-measurer. It is in appearance like a watch, the mechanism consisting of an escapement carried by a loaded lever, which is shaken by the shock of the step and returns by means of a light spring; the escapement actuates a train of wheelwork, and moves an index on the dial to record either the number of paces or the actual

mileage. If the latter, an adjustment is provided to make the index read correctly to any given length of pace and instrument is called a pedometer. For long bases, the mileage indicator is as useful as the other; but for short distances, and varying inclinations, the counting index is preferable as no instrumental adjustment is needed, but scales of paces are drawn upon the board to their ascertained value under different conditions of travel, with their respective designations. The needle-point sight-rule is provided with a broad chamfered edge upon which, when using only one or two scales of paces, they can be gummed with stamp-margins so as to have the working-scale at all times on the paper and avoid dividers. This is dispensed with by using the Mannheim slide-rule as sight-rule and scale : see p. 245.

The passometer does nothing more than count the paces ; the accuracy of the measurement depends upon the regularity with which the walker can pace. A course of training in this is indispensable, but can be easily gained during times of recreation. The chief points are to walk naturally without trying to step yards or any other specified distance, to hold the body erect, and to maintain the same speed.

The best way to arrive at reliable data of pacing is to walk over a piece of road where the mile-stones are correctly indicated. A piece of railway will do very well for flat walking if the sleepers and ballast are avoided, keeping to the side of the bank or cutting. A turnpike road is better. For hill walking a six-inch Ordnance map will give the contour lines crossing the public roads, from which the gradients can be calculated.

The time of one's walk has a great deal to do with the length of step. By educating oneself into the same rate of time—uphill or downhill, fresh or tired—the length of pace will be much more uniform than it is ordinarily. Gradients as steep as 1 in 40 do not then make any difference on the average length.

The following trials were made up and down a road varying from 1 in 20 to level, but all uphill one way, and down the other.

—	Distance, feet	Number of paces			Value in feet of 100 paces		
		1st time	2nd time	3rd time	1st time	2nd time	3rd time
Uphill							
A to B . .	2,900	1,091	1,161	1,132	266	268	256
B to C . .	2,080	783	705	730	266	295	285
C to D . .	820	284	285	282	289	287	291
Total .	5,800	2,158	2,151	2,144	269	269·5	271
Downhill							
D to C . .	820	286	296	284	287	277	288
C to B . .	2,080	726	725	741	287	287	281
B to A . .	2,900	1,156	1,121	1,163	251	261	249
Total .	5,800	2,168	2,142	2,188	267	270·5	265

Average of the three times . . . . 2·687 feet per pace
Range of one time from average . . 1·4 per cent.
Maximum range of shortest distance from average . . . . } 10·9 per cent.

These trials were under unfavourable conditions as regards gradient, and are given to show results attainable by an unpractised walker. See also closing error on p. 55. They demonstrate very clearly the *tendency of pacing towards uniformity over long distances even when there may be great variations over short lengths.*

The distance was only a little over a mile, but this fact becomes more apparent on daily journeys of ten to twenty miles, in which the total error can easily be kept within from 1 to 2 per cent.

When particular exactness is needed, and an assistant is present, it is advisable every two or three miles to check the rate by taping a stretch of 300 paces or so, in order to detect changes due to fatigue, rough or slippery roads, &c.

A series of scales may be constructed for use on various gradients, but it is less confusing to work to one scale and make a marginal note in the fieldbook to guide in making the corrections when plotting, or calculating the latitude and departure.

The plotting may be done by the protractor, but the principle of working to latitude and departure is more exact, and the calculations are done as quickly with the slide-rule as if the angles were laid off with the protractor. There is a harmony, moreover, between this process and the daily astronomical observations, both being a reference to rectangular co-ordinates.

The analogy of traversing with navigation should be thoroughly studied; even down to curve-ranging as will be shown later on. In the traverse for route-survey the work is nothing more than the dead-reckoning; only instead of suffering from liability to error in under-currents, slip of screw, and what not, it is troubled with magnetic aberration, irregularities of pace, and 'personal error.' The astronomical observation comes in to help out the land surveyor with the addition of the frequent sighting of landmarks whose geographical position is known, and the number of which becomes every year greater and greater. The astronomical work is sometimes performed with the Hadley's sextant, but the surveyor finds a greater range of usefulness in the transit theodolite.

Surveying with the Sextant

The Hadley's sextant is a favourite instrument with travellers, who learn its use from one of the ship's officers when getting to their destination, and then employ it for traversing on land. It will not take angles any more correctly than they can be plotted direct upon a plane-table; it is necessary to correct the angles when they are taken between points at considerable difference of level, whereas the plane-table gives the horizontal projection at

once. It is much slower in sighting, and needs careful sketches and entries in the fieldbook to avoid mistakes. Its great advantage is its portability, and an immense deal of good work may be done with it, but only the principles of adjusting it are given in the chapter on instruments, its use being very simple.

Closed Passometer Traverse

In order to exhibit the degree of accuracy attainable with the passometer and prismatic compass alone, the closed traverse illustrated by Figs. 11, 12, was made in the course of

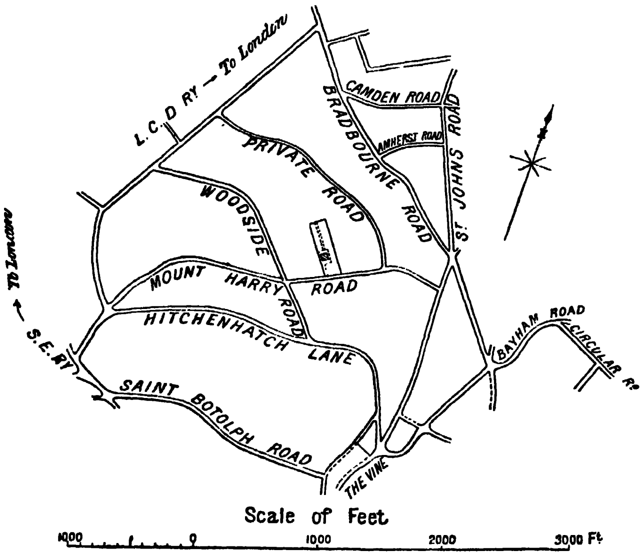


Fig. 11.

daily walks and visits to friends, and without making corrections for sloping ground. It has to be remembered that not only does the pace vary in length according to the slope,

but a deduction has to be made for the horizontal projection of the distance. As going uphill tends to shorten the step, and increase the number of them to the mile, the error is aggravated by the projection. Going downhill it is diminished. The table for deductions due to projection is

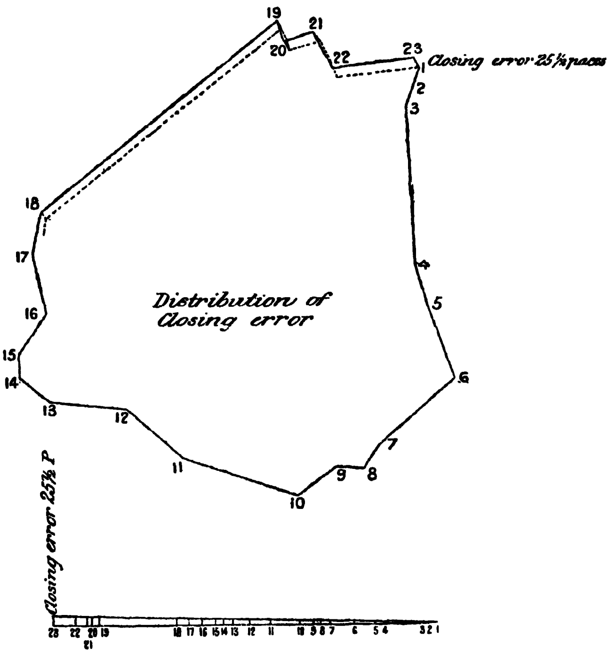


Fig. 12.

given in the chapter on chaining, but the pedestrian must make his own table of pace-variation from actual experiment.

The roads of Sevenoaks are both hilly and tortuous, and therefore represent an unfavourable case for a route-survey.

Hardly any check angles could be taken on account of the obstructions to view. It will serve to show a degree of accuracy which can be at least equalled in more favourable situations. The 'closing error' of $25\frac{1}{2}$ paces has been purposely left in the plot, and the process by which it should be distributed over the bases is placed upon a separate figure. The closing error does not represent the maximum error; it is an average one, arising in large measure from the sloping ground; but there is also a slight twist in the plot, so that in one place where both pace and angle error assist one another, there is a divergence of fifty paces from the truth in the neighbourhood of one of the cross roads. The total closing error amounts to $\frac{3}{4}$ per cent. of the periphery, the maximum error to about $1\frac{1}{2}$ per cent. On a day's march of twenty miles, the error at the next solar observation would be barely detected; but as each observation is independent, cumulative error of pace-measurement is removed within the instrumental limits of about $\frac{1}{6}$ mile of latitude, or, supposing an exact chronometer, of longitude either, but failing the chronometer, the error in longitude may be much more, as already explained in Chapter I.

Distribution of Closing Error

This must not be confounded with fudging, which means correcting by guesswork. Distribution of error is to a great extent dissipation of its amount all over the plot. The process is almost self-explanatory on Fig. 12. The total periphery is laid out on a straight line marking each station, and an ordinate is laid off at the extremity equal to the closing error. The extremity of this ordinate being connected with the other end of the line, a triangle is formed the ordinates to which at every station represent (on the assumption of the error being gradually cumulative) the correction to be applied at each point in a direction parallel to that of the line between the two divergent ends of the traverse.

Where the error is small in proportion to the total periphery this process reduces it to an unscaleable quantity on any single base line.

Scale of Paces

The construction of a scale of paces is as follows. Let us suppose that we wish to produce a map upon a scale of six inches to the mile. A chained base of 1,000 feet is paced and repaced until the average has been found to be for instance 357 paces. We then lay down our scale of miles, say three inches for half a mile, in furlongs and chains, and calculate by slide-rule the value of 1,000 paces in furlongs and chains.[1]

Taking this amount from the mile scale we lay it down as our scale of 1,000 paces and subdivide it as follows: (Fig. 13, p. 59). From one end of it, we erect a perpendicular, and selecting some convenient decimal boxwood scale such as a 10ft. to the inch, we adjust it so that one end is at the extremity of the pace-scale, and some multiple of ten (in this case the 40) on the perpendicular; we then draw a line forming the hypotenuse of a triangle and tick off every four divisions of the boxwood scale so that we have subdivided our hypotenuse into ten equal parts. All we have to do

[1] The two proportions are as follows:
Let x be the number of paces in a mile, and y the number of chains in 1,000 paces.

$$1{,}000 : 5{,}280 :: 357 : x; \text{ or } x = \frac{357 \times 5{,}280}{1{,}000}$$

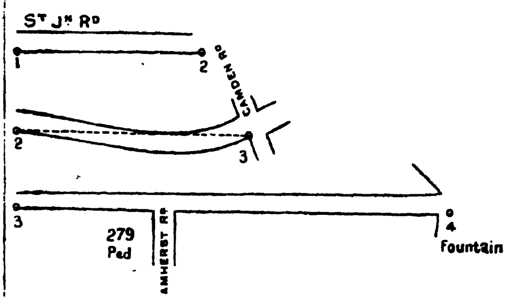
$$x : 80 :: 1{,}000 : y; \text{ or } y = \frac{80 \times 1{,}000}{x}$$

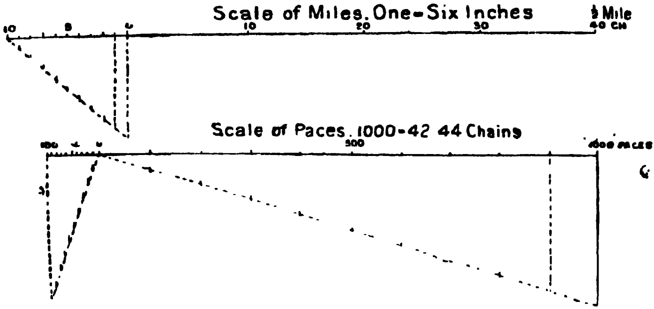
Using the lower scales of rule and slide. Place the right-hand 1 of the slide over the 528 of the rule. Place the brass marker at the 357 on the slide. Without displacing the marker, bring the 80 of the slide to the marker and read off the result, 42 ch. 44 links, on the slide opposite the left-hand 1 of the rule.

Fieldbook for Compass-Passometer Survey.

Number of station	B.S.	F.S.	Mean bearing	Magnetic Azim.	Distance	Latitude N.	Latitude S.	Departure E.	Departure W.	Aneroid Bar.	Compass Bearing	Clinometer Slope + −	Total distance by passometer
	deg.	deg.	deg.	degrees	paces								
1	—	199	199½	S. 19½ W.	70	—	66·0	—	23·4				70
2	20	195½	196	S. 16 W.	39	—	37·5	—	10·7				109
3	16½	178	178	S. 2 E.	418	—	41·8	14·6	—				527
*	*	*	*	*	*	*	*	*	*				*
*	*	*	*	*	*	*	*	*	*				*
Summation of traverse . .						1,316·1	1,294·1	1,162·2	1,175·2				
						1,294·1			1,162·				
Latitude and departure of closing error . . .						22·0			13·0				

Remarks and Sketches
FIG. 13.





SCALES TO FIG. 13.

then is to rule down perpendiculars to the pace-scale and it will be divided into spaces of 100 paces each. See also direct scaling by slide-rule, p. 245.

FIELDBOOK

The fieldbook, on opposite page, requires but a very brief explanation. The backsights are taken to equalise minute errors of observation, to locate and remove them when important, and to detect magnetic deviation.

To reduce the meridional bearings to azimuths from north or south point, the following table may be used.

TABLE XVII.—*Reduction of Azimuths*

From $0°$ to $90°$, azim.	=	bearing unaltered :	N. E.
From $90°$ to $180°$, azim.	=	$180°$ − bearing :	S. E.
From $180°$ to $270°$, azim.	=	bearing − $180°$:	S.W.
From $270°$ to $360°$, azim.	=	bearing − $360°$:	N. W.

[The author's pocket altazimuth has both graduations. See Chapter IX.]

The error at closing is seen to be 22 paces to the north and 13 to the west. We will represent it thus $\overset{13}{\diagdown d|}_{22}$ Then (1) to find the direction and magnitude of the line itself we have tangent angle $d = \frac{13}{22}$. By slide-rule as before = ·591.

Keeping the brass marker at the ·591 we shift the tangent scale to its initial position and find under the marker the angle 30° 35′ N. W.

(2) To find the length of the closing error $L=\sqrt{22^2+13^2}$ For 22^2 bring the brass marker to 22 on the lower scale ot this rule, the upper index will then correspond with 484 on the upper scale. Similarly with the lower index at 13, the square 169 is found on the upper scale. Adding the two together=653; direct the upper index to 653 on the upper scale and the square root 25·5 is read off from the lower. It will be seen that involution and evolution are performed by simple inspection without using the slide, and this forms one of the most important uses of the slide-rule.[1]

(3) *To get latitude and departure.* Place the right hand extremity of the sine-scale of the slide under the distance, and read off the latitude from the rule above the complement of the angle, and the departure opposite the angle itself. Thus in the first entry the distance=70. Place the extremity of the sine-scale opposite a 7 on the upper scale of the rule. The reduced bearing is $19\frac{1}{2}°$, of which the complement is $70\frac{1}{2}°$; opposite these two angles on the slide we shall find the departure and latitude respectively.

Check-sights are very useful in such work as this to correct twists The house shown on the plot was filled in entirely by angles. When engaged in filling in new roads to an old but accurately triangulated map such as an old Ordnance Survey the errors are localised by first plotting the work in the usual way and then superimposing a tracing of it upon the Ordnance Map; the errors of the new work are thus narrowed within the limits of the nearest reliable points and the whole made very nearly as correct as the rest of the Ordnance Map.

[1] This operation can be also performed by placing the sine-scale with the angle 30° 35′ under the 13 of the rule, and the answer will be found opposite the right-hand 1 of the slide.

Profile

The profile or section is produced from readings of the aneroid barometer at every station. The distances are laid off and the heights ruled up as in ordinary levelling. In the case of a closed traverse, there will generally be a 'closing error' of levels which has to be equalised or distributed similarly to the closing error of the traverse. Some of the sources of error are explained in the chapter on instruments, but they are usually cumulative and approximately uniform. They can only be treated as such, and the total periphery of the base being laid off upon a horizontal line representing the true datum, the amount of the closing error is laid off on a perpendicular at its extremity either above or below according as the last reading is less or greater than it ought to be. A 'false datum' is then drawn from the starting point at the end of the perpendicular, and the levels are scaled from the false datum, which is afterwards erased.

Contours

The contours are laid on by the pocket-altazimuth, check-angles being taken along the bases and in other directions. The pocket altazimuth is fully explained in Chapter IX. When we know the elevation of the point of observation and the slope of the ground in any direction we can plot the contours from the cotangent of the angle (that is the tangent of the complement) which represents the horizontal distance, corresponding to one foot of difference of level. Thus let the slope be 10° of depression. To find cot 10°. Place the tangent-scale in its initial position and brass marker at 10°. Reverse the slide and place the right-hand 1 of the slide at the index. Read the answer 5·67 on the slide opposite the left-hand 1 of the rule. If we want contours at every 10 feet the horizontal equivalent will be ten times this, *i.e.* 56·7 feet. Inasmuch as the aneroid readings are in

feet it is best to have a feet scale upon the plot as well as the two already mentioned for setting out the contours.

This is by no means the only way of contouring. The contours of the Ordnance Survey are taken with the level, and dots are placed on the map where the staff was held. They are either plotted from cross sections or from field

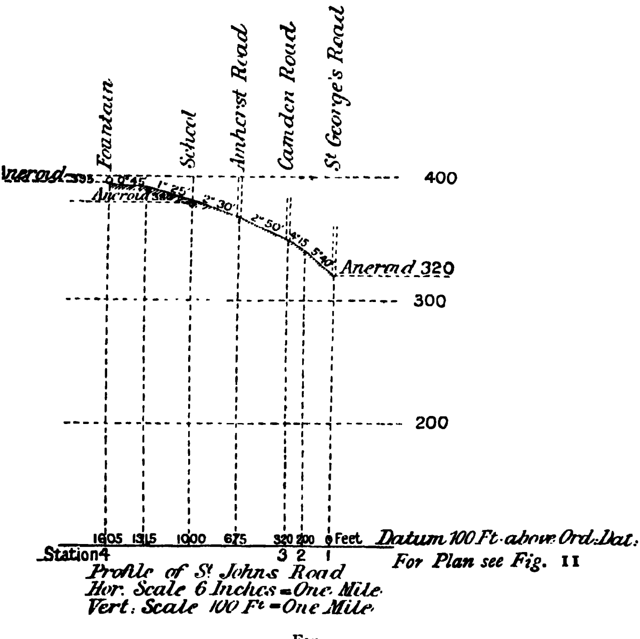


FIG. 14

tracings on which the location of the level points are established by tape measurements from hedges, buildings, &c. The cross section is the most laborious and hardly the more exact method of the two. If the contours are needed before the plan is plotted, it is unavoidable, but when the principal contours are at a hundred or even fifty feet interval it be-

comes a very tedious operation in hilly country. By the second process, the level is kept at nearly the same collimation going round the hill until it comes round to the same point again, or leaves the region of the survey. Telemetric contouring with the level (for which see p. 197) is sufficiently accurate for all ordinary purposes and is independent of any plan. Hill-sketching is often done in the form of contours by the eye, instead of hachures, which convey but little idea of the topography; such contours by being close or far apart show at once the relative steepness or flatness of the ground.

The examples of profile and contouring given, Figs. 14, 15, and 16, with the fieldbook are, one from the traverse, Fig. 11, the other from a walk through Knole Park. The instruments used were the aneroid barometer, pocket altazimuth, and passometer.

The aneroid was first examined between Ordnance benchmarks, with results as follows :—

	Ordnance	Aneroid
Ordnance Benchmark at 'The Vine' to Ordnance Benchmark Railway Bridge London, Chatham, and Dover Railway	187·8	185
Ordnance Benchmark at 'The Vine' to Ordnance Benchmark Railway Tavern	200·8	205
Ordnance Benchmark at Railway Bridge to Ordnance Benchmark at Railway Tavern	13·8	20

On the profile, the discrepancies between the aneroid readings and the elevations calculated by the altazimuth were so small that they were not distributed. In the contouring, on the other hand, the aneroid had to be used with less time allowance for settling, and needed considerable correction from the altazimuth. The profile was taken whilst walking up the hill with a friend without detaining him beyond two or three minutes. The only entries made in the fieldbook at the time were the station column, vertical angle, pedometer, aneroid, and remarks.

If a cumulative error of the aneroid is discovered from benchmarks, or from returning to the starting point on a closed traverse, it must be eliminated as already described before entering its readings in the column provided for the purpose. When in the field its readings can be entered in

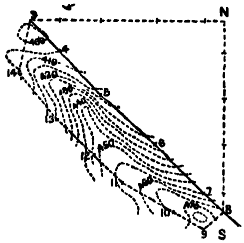


FIG. 15.

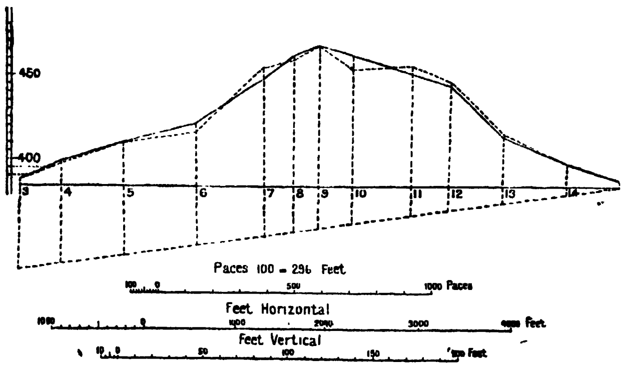


FIG. 16.

the column for remarks. When this is done, the aneroid levels *rule* the profile ; the altazimuth angles are only relied upon for the portion of profile between the aneroid readings.

The advantage of using a clinometer or altazimuth in conjunction with the aneroid, especially in the form recom-

Form of Fieldbook for Altazimuth and Passometer Profile, checked by Aneroid Barometer.

Station	Distance, paces	Total distance, passometer	Distance, feet	Total distance, feet	Clinometer angle		Difference of level		Total difference of level		Elevation by clinometer	Aneroid	Aneroid reduced to datum	Remarks
					+	−	+	−	+	−				
1	0	0	0	0	5° 40′						320	1,305	320	Scale adjusted by slide-rule for uphill, 533 paces = 1,605 feet. Aneroid adjusted by Ordnance bench-mark, cor. Railway Tavern. Station 1 is corner of St. George's Road.
2	66	66	200	200	4° 15′		19·7		19·7		340	—	—	—
3	40	106	120	320	2° 50′		9·1		28·8		349	—	—	Camden Road.
—	118	224	355	675	2° 30′		18·5		47·3		367	—	—	Amherst Road.
—	109	333	325	1,000	1° 25′		14·2		61·5		382	1,365	380	Mr. Norman's School.
—	104	437	315	1,313	0° 45′		7·7		69·2		391	—	—	—
4	96	533	290	1,605	—		3·8		73·0		393	1,380	395	Fountain by five roads.

mended in Chapter IX. is, first, rapidity, and secondly, accuracy of detail. The aneroid could be used throughout, but is not reliable for small differences of level ; for instance, on a stretch of almost dead level, the aneroid might show jumps of a few feet here or there which did not exist at all, but the altazimuth is graduated to show a difference in vertical angle of 10′, which equals 1 foot in 333. On the other hand, the clinometer angles on a long stretch will soon run into a very serious error if unchecked by the aneroid. For more on the aneroid see Chapter IX.

The use of the slide-rule for the calculations cannot be over-estimated, it is simply invaluable. To begin with the scales. It is tedious enough to transfer with dividers from a scale of paces to a scale of feet, but when it comes to three or more scales of paces for uphill, level, and downhill at varying inclinations, the work would be interminable, and recourse has to be had to percentages of addition or deduction as already described. But with the slide-rule it simply means a gentle tap upon the slide, and the new scale is there ; no calculation whatever is needed. Take, for instance, the scale for the slope of St. John's Road, on which 533 paces measured 1,605 feet. We use the upper scales of the slide and rule, and bring the 533 of the slide to the 1,605 of the rule ; the distances in feet corresponding to any number of paces can then be read off directly from the slide. If then the rate alters to 543 paces to 1,611 feet, we have nothing more to do than to slide forward to that ratio. For rise and fall the operation is almost instantaneous, being analogous to that for latitude and departure (see Chapter V. p. 173.)

Contouring with Pocket Altazimuth and Aneroid

The accompanying diagram of contouring (on p. 64) will illustrate first of all the adjustment of aneroid error already alluded to. The traverse was made with the passometer precisely as on Fig. 12, but the correction of the closing error

was done by the slide-rule, and as this is by far the most satisfactory method, as well as much quicker, it will now be explained.

The northings and southings being added up as before, the difference of latitude amounted to 1,812 feet south and 1,764 feet north. Placing the difference of the two, 48, on the upper scale of the rule opposite to 1,812 on the slide, the correction for the first of the northings, 141, on the base 9–10 will be given on the scale opposite 141 on the slide, and equals 3·7.

The departures being corrected in the same way, the plotting of the traverse is proceeded with as before, by rectangular co-ordinates from a N. and S. line, and an E. and W. line, the total latitude or departure being scaled for each station from the starting point afresh, so as to avoid the errors certain to arise in scaling from point to point.

When the traverse is complete, the total periphery is laid out on a horizontal base as before described. It was found in this case that on returning to station 3, where the traverse closed, there was a discrepancy of fifty feet in the aneroid readings, arising from change in the weather (see Fig. 16).

As the instrument read too low at closing, the error was laid off above the true datum, and the aneroid levels scaled up from the false datum. The profile from the aneroid is shown by a dotted line. The difference of level by clinometer angles was then worked out by slide-rule as before, and came to a total difference of 14 feet more than the aneroid. The rises and falls were then reduced in the same way as the errors of latitude and departure by the slide-rule. The maximum difference of level existed at the two extremities of the work, and amounted by the aneroid to 80 feet. This was checked by a second visit with the aneroid to the two extreme points, and found to be correct. The rises and falls were then squared with this total by placing the total error of 14 feet in ratio on the slide-rule with 80, and the error subdivided over the whole traverse propor-

tionally to each rise or fall. The full line on the profile shows the final correction, and the erratic nature of aneroid readings, which is never got rid of even in the most delicate instruments. The line 10–11 for instance is shown by the aneroid as an ascent, whereas it was really a descent, and if it had not been for the altazimuth would not have been detected. To be perfectly sure of the main difference of level by the aneroid, the surveyor should repeat his visits to the two principal positions several times. On extended survey he should have several aneroids, and despatch an assistant on horseback with an aneroid to go to and fro between two objective points until a fair average is obtained. When long base lines are measured by the passometer, say over 400 paces, the clinometer angles are of little value, and the aneroid alone has to be followed.

The contours are plotted in two different ways: First, on lines the extremities of which are known in elevation; secondly, on lines of which the slope alone is known by the clinometer.

Let us take the base-line 3–4, the ends of which were found to be 393 feet and 404 feet high respectively. The contours being required at every 5 feet, we want to know where the 395 and 400 contours cross the line. The distance was 369 feet, and the difference of level 11 feet. Placing the 11 on the rule opposite the 369 on the slide, we look for 2 and 7 on the rule, and find opposite to them on the slide their horizontal equivalents, namely 67 and 235, which we tick off upon line 3–4, measured from 3, and so on. This plan is adopted for all the base lines of the traverse, and also for any points which have been triangulated. In the present case a series of points *a*, *b*, *c*, *d*, *e*, &c., were determined along the rising ground in the middle of the plot by intersecting compass-rays, and the slope taken with the clinometer. The distances were then scaled from the plan, and the difference of level calculated as before with the slide-rule. The points were all trees of sufficient prominence to be identified from

successive stations, but these are among the least eligible of points, being badly defined. Low cottages, whose height above the ground varies but little, are amongst the best points to choose. The contours were then filled in on the rays from the primary stations to these points, and also on lines connecting the points themselves.

It is convenient to tick off the distance on a slip of paper, marking each tick with its elevation ; then apply the distance to the scale, and mark off from it the distances of the contours without using dividers.

The second method of plotting contours, on lines of which the direction and slope alone are determined by the altazimuth, is for the outlying ground in order to show the trend of the contours after they leave the closed traverse. The process is as simple with the slide-rule as the former one. We take a bearing in the direction in which we desire the slope (sometimes we require several from the one station), using either a tree or similar mark, which we can describe for identification in the fieldbook, or else send out a flag.

We then take a vertical angle to the same point and book it likewise.

When we have plotted the rest of the traverse and contours between fixed points we lay off radial lines from the respective stations in the directions of the independent compass-shots.

Then with the slide-rule we place the scale of tangents under the upper scale of the rule, and note the percentage of the slope (*i.e.* 100 times the tangent). Thus if the slope be $3° 25'$ we find the percentage to be 5·97, which is the same thing as the tabular tangent of ·0597 to a radius 1. It is also the same thing as saying that on that slope every horizontal stretch of 100 feet has a rise of 5·97 feet. We then reverse the slide to show the scale of numbers, and, placing the 5·97 of the rule opposite the 100 on the slide, we can read off the horizontal equivalents for each difference of level. Thus, supposing the elevation of the point from

which the line was taken was 473, and the slope +, we should want for 5ft. contours horizontal equivalents for 2ft., 7ft., 12ft., and so on, which we find at 33, 117, and 201. They should be plotted on with the paper-slip from the scale as before.

When a slide-rule is not available, the foregoing explanation will serve equally well for the use of tables, which is of course much more lengthy. A set of scales of horizontal equivalents should be made for every degree of slope. To save pricking off odd distances, the scales are constructed from the tabular cotangents of the slopes which are the horizontal equivalents to a difference of level of 1 foot. They can also be laid off graphically by erecting a perpendicular scale at one end of the profile in the manner shown on Fig. 16, p. 64, and drawing horizontals cutting the base-lines at the required elevations.

Distance-Measurement other than on Foot.

Hitherto only pedestrian operations have been dwelt upon, and for several reasons the surveyor should keep to his feet where he can. He is more independent and can better give his attention to his work when he has no animal to look after ; he can use the sketch-board as a plane-table, and take steadier sights with the prismatic compass. But it often happens that he has to ride and to depend on his animal for the measurement of his distances. It is better to have a horse who *walks* well than a fast one. A well-trained cavalry horse will, according to Captain Verner, R.A., walk with much greater regularity than a man. Horses will fall into an even pace much better when two are together. The walking gait is measured over a chained base. The trot is counted similarly ; each rise in the saddle being a 'trot.' As a rough approximation three yards may be counted to each 'trot.' A canter requires exceptional horsemanship to be used to any extent for distance-measurement.

When steaming on a river, the speed is measured by revolutions of the screw or paddle, by the patent log, or by time only. In any case the speed of the current has first to be measured before starting, and periodically afterwards. A good method is by patent log combined with timing for short distances.

Mr. George Kilgour, M.I.C.E., made a survey for 200 miles in length with a steam launch between the first and second cataracts of the Nile in five days, for the Soudan Railway. He adopted time-measurement of speed, for which he had three scales, full speed, half speed, and dead slow; the value of the scales was determined by an accurately measured base on the centre line of the vessel, at the two ends of which, near the bow and stern, two lines of sight were placed square with the centre line, by which the time was registered in which the vessel passed some well-defined point on the bank, such as a cocoanut tree, at the three rates of speed. The survey was made from the deck with a plane-table kept constantly in the meridian by the compass, and astronomical observations were taken at night.

Distance-measurement by the Range-finder.

The most recent improvements in this class of instrument are described in Chapter IX. but a few further remarks will here be made upon their use.

There are cases when they can be used for measuring all the bases of a traverse. All that they require is some well-defined point to view on and an accurately measured sub-base at station. When the bases themselves are not measurable in this way there are sometimes well-defined points close to those bases, from which the distances to them can be accurately determined and so serve equally well. All these instruments require considerable practice in order to obtain reliable results every time. Those which have no lens power require a good and quick eye and a steady hand.

The most important feature about the range-finder is that it is a rapid means of measuring inaccessible distances; and across rough country where pacing is either impossible or very inaccurate, it is as reliable as on level ground. A range-finder is always valuable as a check on salient points off the centre line.

Unless exceptionally proficient in its use it is not advisable to take long shots except as rough checks. It is better to keep the bases within 1,000 feet, if possible. An error of one foot in sub-base means fifty feet in distance. Two Weldon range-finders are better than an optical square and a range-finder because then they can be used either in the manner explained in Chapter IX. or with the two acute angles or with one right angle and one acute.

Distance-measurement by a Two-foot Rule

The following method needs no instruments beyond a two-foot rule and a decimal scale. One of the old-fashioned carpenter's rules, with a brass slide to it, graduated on the one side with the logarithmic scale and on the other with inches and tenths, will answer the purpose. Or if a Mannheim rule is at hand with its bevelled millimetre scale it will give a closer reading, or, best of all, a ten-inch slide-rule of the author's pattern described in Chapter IX. with a fifty scale on its bevelled edge. This will give an even dividend-number and as close a reading as it is possible to have.

Dividends are given for all three scales below.

The two-foot rule is held with the eye at the centre of the joint, and the legs are spread so as exactly to intercept a known sub-base. It is convenient to measure distances up to 300 feet with a sub-base of five feet or ten feet in the form of a pole, with a pair of discs or crossheads painted black and white, held by an assistant either on foot or on horseback at the point whose distance is to be measured.

For longer distances, a base of fifty feet should be run

out with a tape by two assistants, who will hold a flag or a disc at each end of the base. The line of sight to the centre of the base should be at right angles to it. This is easily done with a single assistant by a sighting-piece at the middle of the pole, but when a base is to be taped it is easier to set it at right angles to a line of sight to one or other of its extremities. In the latter case, if the angle is small the error will be hardly scaleable; but to be exact the aperture of the legs of the rule should be measured on the square from one extremity to the line of the other leg produced.

All that is required is to divide the dividend in the table according to base and scale by the measurement of the aperture of the legs of the rule. The only advantage in this case of a slide-rule is to perform the division, but that is so simple that it can be done mentally when using an English scale.

DIVIDEND-NUMBERS

	Base 5 feet	Base 10 feet	Base 50 feet
Scale of inch and tenths	600	1,200	6,000
Scale of millimetres	1,524	3,048	15,240
Scale of inch and fiftieths	3,000	6,000	30,000

Example. Viewing on a ten-foot base, measured the aperture of a two-foot rule twenty-seven fiftieths of an inch; required the distance.

Ans. $\frac{6000}{27} = 222{\cdot}2$ feet.

Good guesses at distance can be made by similar means without anything more than a base which is itself guessed at. Perhaps some reader will pooh-pooh such guesswork as this, but no one who knows what it is to be without any assistant for measurement and to need some help to the mere guess at distance in perhaps a very deceptive piece of country will under-value such a method as this when they

have tried it. In even slightly civilised countries, people still build with some system, and mud cabins, log huts, or snake fences go pretty much in sizes.

In countries where they build in brick, one-storey and two-storey houses bear considerable uniformity of height per storey. Then a man on horseback, or a cow or any other animal, may be taken for a guessed base. Even trees in old wooded countries, though varying individually up to any size, form as a forest a line of approximately equal height which can be ascertained for different species and used as a base. The suggestive fact is that any height of that kind is much easier guessed at than a distance, because in the one case there is something to go by, in the other nothing.

Mapping

We come now to the various contrivances for producing a correct graphic representation of the fieldwork upon paper.

The only absolutely true map is a terrestrial globe, but as we cannot carry globes about with us we have recourse to the principles of projection, which are quite numerous in variety, but are all of them artificial representations of a spherical, or more properly spheroidal, surface upon a plane.

If the survey extends over a large area, it becomes necessary to adopt some method of projection by which, in the first place, the distances are reduced to sea-level, and in the second place, the meridians converged or distorted so as to allow for curvature.

When the survey is a continuous traverse of a railway route this is not necessary. It is not the object of the railway surveyor to know the sea-level dimensions ; he needs the actual length of his road wherever it may be. The difference in length between a degree of latitude at sea-level and at 528ft. ($\frac{1}{10}$ mile) elevation is only about nine feet. At an elevation of 5,280 feet (one mile) it would be about

92 feet, and the distance-measurement practicable on route-survey does not come nearer than that.

Neither does the surveyor want a distorted map, but one to which he can apply a scale throughout; he therefore does not need to take account of the earth's curvature, but plots his traverse on a horizontal plane. When the area over which a triangulation extends is not large, the surveyor is still able to adopt one of two methods of plane construction.

In the first the meridians and parallels of latitude are all parallel straight lines at right angles to one another.

In the second the parallels of latitude are straight lines, but the meridians are converging straight lines, or, if great accuracy is needed, curved lines.

Limits may be given merely to fix the ideas within which to use the first or second method, of say 1,000 square miles for the first and 100,000 for the second.

When the survey is in high latitude the spheroidal form of the earth much more affects the map than near the equator, in which region a belt could be projected all round the globe by the first method without sensible error.

Taking as an illustration of the extreme limit given for the use of the first method, 1,000 square miles; this area would be contained in a square of which the side was not quite half a degree; let us lay down two plots of squares of which the side is 2°, which will embrace an area of about 16,000 square miles, or sixteen times the limit given, and examine from it what the error would amount to in using the first method, at a mean latitude of 32°.

First. By mean longitude. The length of a degree of latitude at 32° is 68·90 statute miles (see table, p. 175), and the length of a degree of longitude at that latitude is = 58·70 miles (see table). In the centre of the paper draw a horizontal line to represent the middle parallel of latitude, and through its centre erect a perpendicular to represent the central meridian 5° of longitude, and lay off upon it 68·90 miles above and below, and through the ends draw parallels to

represent 31° and 33° of latitude. Lay oft on the middle parallel 58·70 miles on each side of the centre, and for the first method rule up verticals to represent the meridian of 4° and 6° longitude. The whole figure will then represent four equal rectangular quadrilaterals.

For the second method lay off on the 33° parallel two lengths of 58·09 miles, and on the 31° parallel two lengths of 59·32 miles, being the length of a degree of longitude at the respective latitudes (see Table XXV. p. 175), and complete the figure.

The error of contraction at 31° will be seen to be = 1·2

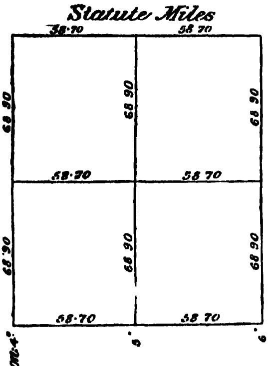


FIG. 17.

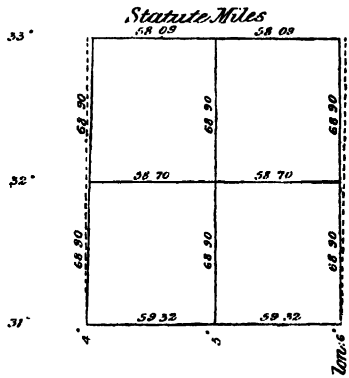


FIG. 18.

miles, and the error of expansion at 33° = 1·22 miles by the first method. No error exists on lines running due north and south. A diagonal through one of the quadrilaterals is subject to an error of about half a mile, but on a diagonal clear through the figure there will be hardly any error, because the contraction in the upper almost exactly balances the expansion in the lower.

At the same latitude a similar figure bounded by a ½ degree of latitude and covering an area of 1,000 square miles would

have an error of expansion at the top of the sheet of about fifty yards and a similar error of contraction at the bottom.

This matter of possible error has been gone into numerically to fix the ideas before commencing to plot a survey without wasting time in deciding upon whether to use projection or not.

The second method, of straight converging meridians, may be used with quite sufficient accuracy up to a latitude of 65° for stretches of 100,000 square miles, and there is not much work done above this latitude anywhere.

It may be said, therefore, that the method of plane construction meets all the ordinary requirements of the surveyor, but in case he may be called upon to reduce extensive surveys to atlas scale it may be as well to explain the principles of

Conical Projection

A globe may be conceived to be wholly contained inside a cylinder or partly contained inside a hollow cone.

For purposes of projection the cylinder must have a diameter equal to that of the globe, but the cone must be

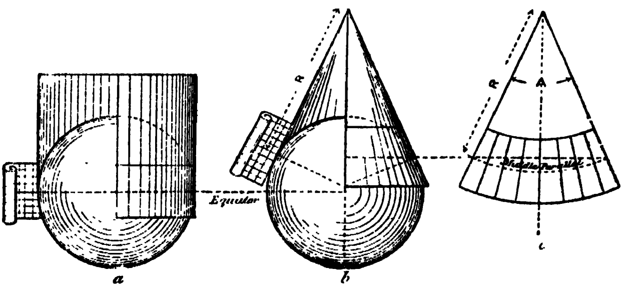


Fig. 19.

of such dimensions that its sides will be tangential to the radius of the sphere at the point of contact. Supposing the earth to be thus contained; a belt of a few degrees on either side of the equator might be conceived to be un-

wrapped or developed on the cylinder without sensible error. This answers to plane parallel construction.

Similarly a belt of the cone may be developed as shown on Fig. 19 *b*, with converging meridians and with curved parallels. Maps of continents are drawn in the atlas upon this principle, and being of large extent the apex of the cone is determined, and the radial parallels of latitude drawn direct from it, with trammels.

Example. It is required to project by conical projection a belt of 10° longitude, say from 20° to 30° east, whose middle parallel of latitude is 50°. The width to be 10°, *i.e.* from 45° to 55°; the scale 100 miles to the inch. Draw the scale of miles at the foot of the paper. Draw a horizontal base-line in the middle of the paper, fix its centre, and draw a perpendicular through it from top to bottom. The base will represent the chord of the middle parallel, and the perpendicular the central meridian 25° longitude E. Calculate the length of the chord by following formulæ.

Radius of Cone = rad. of earth × cotan lat. $= 3{,}950 \times \cot 50° = 3{,}314 = R$.

Central angle = total longitude × sin latitude $= 10° \times \sin 50° = 7°{\cdot}66' = A$.

$$\frac{\textit{Chord}}{2} = R \sin \frac{A}{2} = 3{,}314 \times \sin 3°{\cdot}83' = 221 \text{ miles.}$$

$$\textit{Versed sine} = R - R \text{ cosin } \frac{A}{2} = 7{\cdot}4 \text{ miles.}$$

Lay off $\frac{\text{chord}}{2}$ on each side of the centre, and 221· less 7·4 on the perpendicular, and 7·4 below the centre. Then from the apex draw the middle parallel through the extremities of the chord and the versed sine, and join the apex to the two ends of the chord for the two extreme meridians 20° and 30° east, upon which, on either side of the middle parallel, lay off distance, = the latitude for each degree (see table on p. 175), and describe the arcs of the remaining parallels.

Then for the meridians draw the bottom chord to the parallel 55°, subdivide it into ten portions, and draw radial lines to the apex.

If the radius of the cone be inconveniently long for plotting, lay off the extreme meridians by protracting an angle at each end of the chord to middle parallel equal to $\frac{180° - A}{2}$. In this case the angle would be 86·17°. Draw top and bottom chords to parallels 45° and 55°, and subdivide each into ten equal portions, through which divisions draw the converging meridians.

Stereographic Projection

is that in which the great circle of a sphere is assumed as the plane of projection, and one of its poles as the projecting point. In terrestrial maps it is used for representing a hemisphere. The plane of projection is termed the primitive. Projections of great circles drawn through the pole of projection are straight lines, and all others are circles. The centres of all great circles passing through any point in the plane of the primitive are situated in a straight line called the locus.

In the stereographic projection of the eastern hemisphere, the primitive is usually the great circle of longitude passing through the 20th western and 160th eastern meridians, and the pole of projection is on the equator at 70° east longitude.

The principal use of this projection to the surveyor is for astronomical problems, such as that of 'graphic latitude,' p. 158, or the chart of circumpolar stars on p. 393. The explanation of this projection is given in detail in Chambers's 'Practical Mathematics,' and in Heather's 'Instruments' in a more general manner.

Mercator's Projection

is a development of the earth's surface by elongation of the meridian, so that a ship's course will always appear as a straight line, and is the projection used in the published Admiralty chart. They are, however, constructed on the principle of

Gnomonic Projection

for which see p. 89.

CHAPTER III

HYDROGRAPHY AND HYDRAULICS

WHEN a company is formed to develop the resources of some new country like Africa, it often falls to its lot either to make new harbours or improve natural ones, to canalise rivers or to deepen them ; and the pioneer surveyor has to be prepared at least to make a hydrographic chart of a considerable degree of accuracy, to measure the discharge of streams and rivers, or possibly to undertake trial borings, soundings, or even dredging on a small scale for the purpose of estimating the cost of a proposed work.

This chapter will be occupied with short descriptions of methods adopted in carrying out such operations, keeping strictly to preliminary work.

HYDROGRAPHY ON LAND

Some of the naval surveyor's methods differ but very little from that of the land surveyor. He chains base-lines, plants permanent trigonometrical stations, triangulates with the theodolite, and reduces his work to sea-level. It does not come within our province to follow up such lengthy methods as these. We will content ourselves with the subject technically called coast-lining, which signifies the mapping of the shore-line, either from the ship, aided or unaided by points on shore, by boats, by traverses on foot, or by combinations of all the above methods ; together with a short paragraph on Boat-survey of rivers.

Coast-lining on Foot

This, where practicable, is the best way of rigorously determining all the small indentations, creek-mouths, directions of small streams, &c. Boating tends to oversight in these particulars, whilst from the ship itself only the general outline can be obtained. The first thing is to obtain fixed points on the shore visible from one another from which to fill in the intervening detail. These may be salient points in nature, such as prominent rocks, trees, or houses, or else they may be beacons, cairns, or flagpoles, whitewash marks, or any other artificial stations.

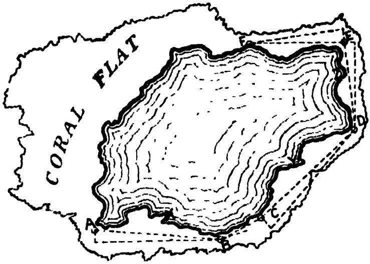
Their position is determined by astronomical observation, and the base is measured by difference of latitude on a meridional line or by a meridian distance. These problems are described in Chapter IV.

The plane-table is not much used by naval men, but it might be made a very useful instrument, both on board ship and on shore. They do, however, use an ordinary field-board for plotting in details of coast-line from primary points, or 'fixes' as they call them, plotted previously on the board. The intervening work between the primary points is put in by telemetry; they use a ten-foot pole with a pair of discs as a base, the angle of which is measured by a sextant or micrometer.

The pole is maintained square with the observer by a directrix at its middle, or else it is swayed slowly in a horizontal position until the observer has measured the maximum angle.

Another plan, where the ground is sufficiently open to admit it, is to use a 500 feet lead-line run out square from one end of the line to be measured. Fig. 20 is taken from Captain Wharton's 'Hydrographical Surveying,' and represents a method largely used by Lieutenant W. U. Moore in the survey of the Fiji Islands, and performed by two officers only. Starting at A officer No. 1 moves on to B, leaving

officer No. 2 at A with the lead-line, who fixes a flag at A and runs out his line either with an optical square or a '3, 4, and 5' line, and plants another flag at its extremity. Officer No. 1 then measures the angle between the flags $= 500 \times \cot \theta$. Each officer then takes sextant angles to salient points previously determined along the shore, so as to make intersecting rays for subsidiary 'fixes.' They then travel to meet one another, filling in detail by bearings with the prismatic compass, and distances with the micrometer and ten-foot pole or sextant and ditto. The writer never having done this kind of work cannot speak positively, but he thinks it probable that the plane-table would be found preferable for the filling in of detail between the extremities of the base. The subsidiary fixes form independent checks to the detail, and if a plane-table were used errors could be more easily eliminated at the time.

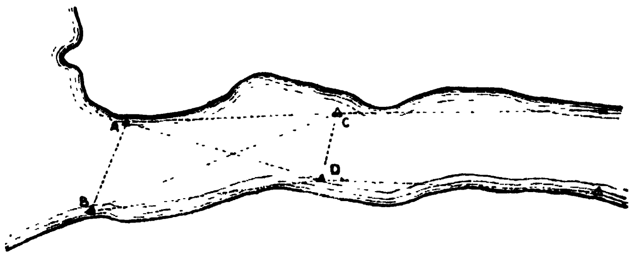


SURVEYING WITH BOATS

Rock-bound coasts, or rivers with banks covered with dense jungle, are best surveyed with boats. The use of boats in coast-lining does not much alter the methods already mentioned; we will therefore confine ourselves to boat-survey of rivers. A steam pinnace is used whenever one is obtainable, and provided with a compass and patent log. The latter is attached to the gunwale, and the fan towed astern. The prismatic compass stands on a tripod in the stern. The velocity of the current is measured from time to time by anchoring in midstream. A current-

meter should be added to the equipment as being more adapted for that purpose than the patent log, especially with slow currents.

When a large-scale plan of a wide river has to be made, several boat-parties (Fig. 21) work in concert, four if possible; two on either side of the river, triangulating their way up from point to point. Two starting-points are established at the mouth of the river, whose relative position and distance has been determined with the utmost possible accuracy. Two of the boats remain at these points, and the other two move up the river to convenient stations, and each of the four boats takes sextant angles to the other



three. Here the point C is fixed from line AB by the angles BAC and ABC, and the point D by the angles ABD and BAD; the angles ACB, ACD taken from C and the angles BDC, BDA taken from D, form independent checks. The shore-line may be sketched by the boats A and B when moving up stream to take the places of C and D. If the sounding has to be done thoroughly, the boats should return by a diagonal course AD, BC, or else the boats C, D, when first moving up the river, can fill in the shore-line with the patent log and compass angles, and also take soundings along the lines AC, BD, leaving the boats A, B, only the diagonal soundings AD, BC to take.

Surveying from the Ship

A great deal of very good charting is done without any connection with the shore, except the landing of a boat to make sound signals. This class of work is termed 'Running Survey.' It is the most rapid though the least accurate of any of the methods which are considered worthy of being called surveying. The mere sketch independent of anything more than guesses at distance is not now resorted to, although many islands exist in the Pacific, as well as portions of the mainland of Australasia, and other continents, where the charts are still, to a large extent, sketchy, and it is the important duty of the Hydrographical Department of the British Navy to diminish year by year the 'terra incognita,' which presents its ofttimes hidden dangers to our world-covering mercantile marine.

Running survey is often assisted by known points on shore. If there are, for instance, three sharply defined peaks whose geographical position has been accurately determined, the work is both facilitated and improved. For by the station pointer, see p. 367, the ship's place is located at any time as long as they are kept in view. As the ship moves forward, her path being thus clearly defined upon the chart, subsidiary cuts are made by angles with the sextant to salient points upon the coast and the detail sketched in. From fifty to a hundred miles of coast-line can be thus put in in a day.

When no assistance of this kind can be obtained from the shore, the base is obtained by sound-signals. The vessel is maintained as nearly as possible in the same position, whilst a party on shore and a party on board alternately fire guns within sight of one another, so that the time between flash and report may be taken by the chronometer on either side. During this operation angles are being taken by the sextant to all the important points, which can be seen both from the shore station and from

the ship. When the length of the base is determined by the sound signals, the landing party returns on board, the various peaks are located on the chart by intersections, and henceforth serve as known points from which to identify the ship's place at any time as she travels forward.

Sound travels at the rate of about 1,090 feet per second at a temperature of 32° F. and increases its velocity at the rate of 1·15 feet according to some authorities, and 1·25 according to others, for each degree Fahrenheit of rise in temperature, or *vice versâ* for fall of temperature.

The object of signalling from both ends of the base is to counteract the effect of the wind, which greatly affects the accuracy of the result. It is not, however, always resorted to.

The firing is done by prearranged signal, such as the dipping of a flag, in order that no time may be wasted in looking out.

Chronometer watches beat five ticks to two seconds. English lever watches nine ticks, and Geneva watches ten ticks.

When watching for the warning signal with a telescope, the watch should be tied to the ear with a handkerchief, and the counting commenced thus: nought, nought, nought, until the flash is seen, then one, two, three, to ten. At each ten one of the fingers is put down, so that the ten fingers will represent one hundred beats or forty seconds with a chronometer=about eight and a quarter miles, which is longer than bases are usually measured in this way.

The following example is from Captain Wharton's 'Hydrography,' p. 64.

'In meaning the result the arithmetical mean is not strictly correct, as the acceleration caused by travelling with the wind is not as great as the retardation caused in the opposite direction, as in the latter case the disturbing cause has clearly acted for a longer period. The formula used is

$$T = \frac{2\,t\,t'}{t + t'}$$

where T is the mean interval required, t the interval observed one way, t' the interval observed the other way. The mean interval thus found multiplied into the velocity of sound for the temperature at the time will give the required distance.

'As an example let us suppose A and B the two ends of the base to be measured.

'At A have been observed—

46 beats with watch beating 5 beats to 2 seconds.
47 ,, ,, ,, ,,
46 ,, ,, ,, ,,
Mean 46·33 beats = 18·532 seconds.
81 beats with watch beating 9 beats to 2 seconds.
82 ,, ,, ,, ,,
83 ,, ,, ,, ,,
Mean 82 beats = 18·222 seconds.
Mean at A = 18·376 seconds.

'At B have been observed—

85 beats with watch beating 9 beats to 2 seconds.
87 ,, ,, ,, ,,
88 ,, ,, ,, ,,
Mean 86·66 beats = 19·258 seconds.
47 beats with watch beating 5 beats to 2 seconds.
47 ,, ,, ,, ,,
48 ,, ,, ,, ,,
Mean 47·33 beats = 18·932 seconds
Mean at B = 19·095 seconds.

Then working, $T = \frac{2\,t\,t'}{t+t'}$; = 18·728 seconds.

'Temperature 80° F., at which velocity of sound = 1145·2 feet per second × 18·728 seconds = 21,448 feet.'

The temperature must be taken in the open air with the thermometer shaded from the direct rays of the sun, but not in too cool a spot or it will not give the true temperature of the free air.

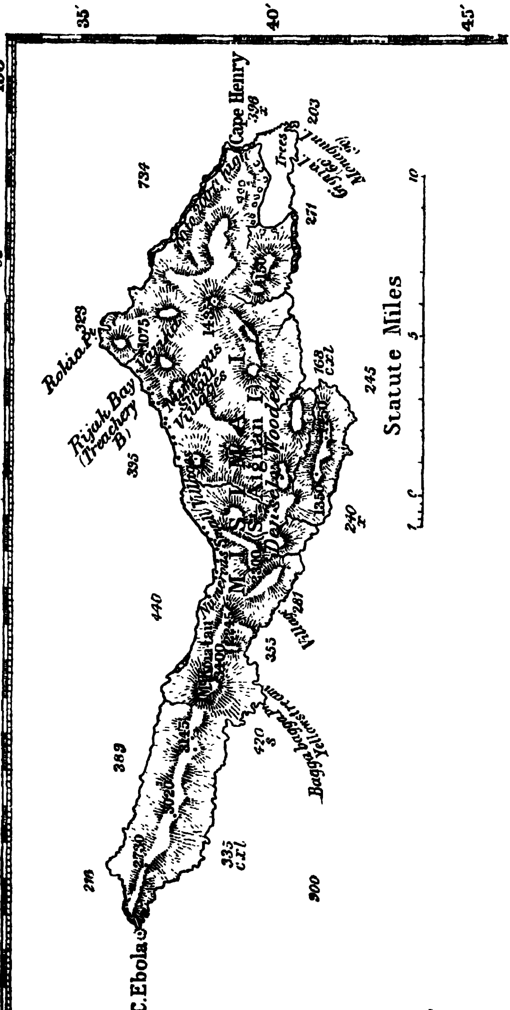

Cape Henry
Rohia Pt.
Rijah Bay
(Treachery B)
Numerous Small Villages
Densely Wooded
Statute Miles
C. Ebola
153°
35'
40'
45'
50'
55'

FIG. 22

The small island shown on Fig. 22 is a copy from a recently made chart kindly lent by Lieutenant Vernon Brooke Webb, one of the hydrographers of H.M.S. 'Dart' on her expedition to New Guinea. It will serve to illustrate the rapidity of running survey ; the time actually occupied was nineteen hours, no special hurry having been made over the work. The distance covered is about sixty miles. The method was running survey, aided by several fixed stations on a neighbouring island, from which the ship's place could be located whilst on one side of the island by the station-pointer. The vessel steamed at about four miles an hour, towing the patent log all the time, and taking soundings where shown. These being pretty deep occupied about ten minutes each. On the further side of the island all the distance-measuring was done by patent log. The hills were put in by sextant angles, both for position and elevation. The rivers were shot in with compass angles. The inhabitants being cannibals, the island was not explored inland. The graduation is a little displaced in order to bring the margin within the sheet.

The following are the notes appended to this chart :—

Current ; ↠→, ebb ; ←, flood ; kn, knots ; H.W.F. & C ix^{h} o^{m}., springs rise 5 ft., neaps, 3½ ft ; B., bay ; C., cape ; Cr., creek ; D., doubtful ; H^{d}., head ; H^{r}., harbour ; I., island ; L., lake ; P., port ; P^{t}., point ; R., river ; Rk., rock.

bk., black ; cl., clay ; crl., coral ; f., fine ; g., gravel ; h., hard ; m., mud ; r., rock ; s., sand ; sh., shells ; st., stones.

Figures on the land show the heights in feet.

Bearings to the marks and views are magnetic.

Soundings in fathoms.

Magnetic variation in 1890, nearly stationary.

Gnomonic Projection

The Admiralty charts are now all constructed upon this projection and published on the Mercator's projection.

Gnomonic projection is very similar to conical projection, but the plane of projection is not the development of a conical zone ; it is a true plane touching the sphere only at one point, viz., the middle point of the central parallel. The convergency of the meridians is measured by the difference *of true bearing of one point from the other at the extremities of the map.*

It is the only projection in which ***all*** great circles are represented by straight lines, so that spherical distances from point to point are scaleable all over the chart when it has been properly graduated.

This does not mean that one scale may be used throughout, but, unless in very high latitudes, considerable portions of the chart are practically to one scale and the graduations at the side give the scale at any point.

The actual length of any projected line measured on a meridian from the middle parallel is proportional to the tangent of the latitude.

It will be seen from the following illustration, that at a scale of 10 miles to the inch in 45° latitude, a stretch of 250 miles square, or 62,500 square miles, would not differ in measurement from a plane construction with converging meridians ; because the lengths of the tangents at that scale are sensibly equal within 2° on either side of the middle parallel to the developments of the spherical distance. The difference at 2° between tangent and arc is ·0000222, and if we consider ·01 inch as the limit of scaleable quantity, we get 37·5 feet as the limit of radius at which error is appreciable, which is equal to about $8\frac{3}{4}$ miles to the inch.

The natural scale, or the proportion which the chart lineally bears to the actual size in nature, is obtained by dividing the number of inches in the nautical mile at the latitude by the number of inches corresponding to one mile on the chart. The result will be the denominator of a fraction whose numerator is one.

Thus, supposing the scale to be 1·8 inches to a mile in

latitude 3°, we divide 72,552 (the number of inches in a mile, see Table XXVI., p. 175) by 1·8; this gives $\frac{1}{40306}$ as the natural scale, which should be noted on all sheets that are not graduated.

ILLUSTRATION OF LIMIT UP TO WHICH PLANE CONSTRUCTION DOES NOT SENSIBLY DIFFER FROM GNOMONIC PROJECTION.

Scale 10 miles per inch $= \frac{1}{633600}$. At a mean terrestrial radius of 20,888,628 feet the length of radius to scale = 32·968 feet. The length of 1° from central parallel on central meridian = ·57546 feet. The length of 2° = 1·15127 feet.

But the mean length of a degree of latitude = 69·05 miles, and at 10 miles per inch = 6·905 inches or ·5754 feet. Therefore the projection is sensibly equal to the development of the spherical distance within at least 3° of latitude.

As it is not likely that the surveyor would be called upon, unless with proper notice for obtaining his special map equipment, to carry out so extended a survey as to need the exactitude of the above method, it will not be further dwelt upon.

He will find a thorough description of the gnomonic projection in Capt. Wharton's 'Hydrography,' Murray's, Albemarle Street.

SYMBOLS IN CHARTING

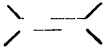
The annexed specimen list of symbols, taken by Capt. Wharton from the Admiralty Manual, shows the authorised delineation, and is reproduced here almost verbatim.

The Days of the Week.

Sunday	Sun's day	Sun ☉
Monday	Moon's day	Moon ☽
Tuesday	Teut's day	Mars ♂
Wednesday	Woden's day	Mercury ☿
Thursday	Thor's day	Jupiter ♃
Friday	Friga's day	Venus ♀
Saturday	Saturn's day	Saturn ♄

The following Signs are used in the Fieldbooks.

Cutting  Embankment 

Objects in line, called transit	φ
Station where angles are taken	▲
Zero, from which angles are measured . . .	⊕
Single altitude sun's lower limb	$\underline{\odot}$
,, ,, ,, upper limb	$\overline{\odot}$
Double altitude sun's lower limb in artificial horizon	$\underline{\underline{\odot}}$
,, ,, ,, upper ,, ,, ,,	$\overline{\overline{\odot}}$
Sun's right limb	$\odot\vert$
Sun's left limb	$\vert\odot$
Sun's centre	Φ $\ominus$
Right extreme, or tangent, as of an island . .	$\succ$
Left ,, ,, ,, . .	$\prec$
Zero correct	Z K
Windmill	♉
Water level	w. l.
Whitewash	W. W.
Bridge	

In Colouring.

Sand	Gamboge, dots black.
Low water, sand edge . . .	Gamboge, dots carmine.
Mud, dry low water . . .	Neutral tint, edge of fine black dots.
Coral, dry low water or any rocky ground covering and uncovering	Burnt sienna and carmine mixed for wash ; same darker for edging.
Cliff	Dark neutral tint.
Roads	Burnt sienna.
Fathom lines up to five fathoms .	Either a faint wash of cobalt all over the area included within the fathom line, or a narrow edging of the same colour inside the dots of the fathom line.

Signalling with Heliostat or Heliograph

These instruments are described in Chapter IX. Their use is also sufficiently explained to dispense with anything further here.

The following description of the methods used is nearly verbatim from the 'Military Handbook of Signals.'

The Morse code is that which both military and naval officers use. It has nothing but a combination of dots and dashes for its basis.

The duration of the flash is what constitutes it a dot or a dash, and practice is required both to give uniform durations and to read the signals. An unpractised signaller at the receiving station can take down the flashes by dot and dash on paper and afterwards read them off. In giving the signals, the transmitter should count, say, ten for a dash and three for a dot, or less when in good practice. Counting the duration of the dot as a unit, the pause between each letter should be three, and between each word six units.

Table XVIII.—*Alphabet.*

A	• —	N	— •
B	— • • •	O	— — —
C	— • — •	P	• — — •
D	— • •	Q	— — • —
E	•	R	• — •
F	• • — •	S	• • •
G	— — •	T	—
H	• • • •	U	• • —
I	• •	V	• • • —
J	• — — —	W	• — —
K	— • —	X	— • • —
L	• — • •	Y	— • — —
M	— —	Z	— — • •

Numbers.

1	• — — — —	6	— • • • •
2	• • — — —	7	— — • • •
3	• • • — —	8	— — — • •
4	• • • • —	9	— — — — •
5	• • • • •	0	— — — — —

Punctuation.

Full stop • • • • • •
Preparative and erasure, a continuous succession of dots.

The preparative sign is to call attention. To answer it the received either gives the

General Answer

— a single dash, or else he gives the code letter of his name or station.

When intended as an erasure to signify that a mistake has been made by the transmitter, it should be answered by the erasure signal.

The Break signal I I • • • • is used between the address and text of a message, and after the text if the name and address of the sender are to be signalled.

The Completion signal V E • • • — • but sent as a group, not two letters, denotes the completion of a message.

'Repeat' I M I given continuously • • — · • •

The Figure signal F I • • — • • • means that figures are intended.

The Figure Completion signal F F • • — • • • — • means that figures are done.

Indicator • — • — is sent at commencement and conclusion of message. It is answered by the same signal.

All right	R. T.
Go on	G.
Move to your right	R.
Move to your left	L.
Move higher up or further off . . .	H.
Move lower down or closer . . .	O.
Stay where you are	S. R.
Separate your flags	S. F.
Use blue flag	B.
Use white flag	W.
Use large flag	L. F.

Use small flag	S.
Your light is bad	L. B.
Turn off extra light	T. O. L.
Wait	M. Q.
Say when you are ready	K. Q.
I shall signal without expecting answers	K. K. K. K.

In America the signal for all right is O. K., supposed to have been invented by a Mr. Joshua Billings.

Some of the above signals refer to flag and light signals. The lime-light apparatus is not described in this work, but the flag system will be here explained.

FLAG-SIGNALLING

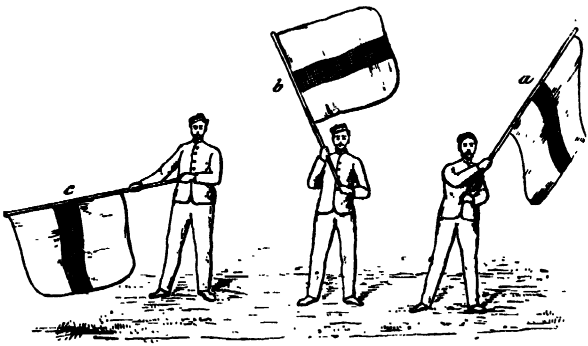


FIG. 25. FIG. 24. FIG. 23.

The Morse alphabet is used, but the dots and dashes are made by movements of the flag and not simply by exposing it.

Either large or small flags are used. They are both made of the same material, a sort of muslin, and of two colours, white with a blue horizontal strip for use with a dark background and dark blue for use with a light background.

The large flags are three feet square mounted on a pole, five feet six inches long, one inch diameter at the butt and tapering to half an inch at the top.

The small flags are two feet square mounted on a pole three feet six inches long, three-quarter inch diameter at the butt and tapering to half an inch at the top.

ARMY FLAG DRILL

Large flags. The signaller may work from left to right or from right to left, or may turn his back to the station to which he is signalling, according to the direction of the wind, so that the flag may be waved from the normal position against the wind.

To make a dot. Wave the flag from the normal position *a*, Fig. 23 to a corresponding position, *b*, Fig. 24 on the opposite side of the body, and without any pause back to *a*, Fig. 23, keeping the left elbow close to the side.

To make a dash wave the flag from *a*, Fig. 23, to *c*, Fig. 25, so that the point of the pole nearly touches the ground, still keeping the left elbow close to the side and straightening the right arm ; make a short but distinct pause in this position and then return to *a*, Fig. 23.

When signalling a letter, say R, • — • the flashes representing it should be made in one continuous wave of the flag, taking particular care that no pause is made when at the normal position. Thus to make R wave the flag from *a*, Fig. 23, to *b*, Fig. 24, back to *a*, Fig. 23, and without any pause down to *c*, Fig. 25 ; slight pause at *c* (see instructions for making a dash), back to *a* (Fig. 23) ; then without pause to *b*, Fig. 24, and back to the normal position *a*, Fig. 23. A pause equal to the length of a dash should be made at the normal position *a*, Fig. 23, between each letter of a word or a group of letters. When the word or group is finished the flag pole is lowered and the flag gathered in with the left hand.

A slight pause should be made at the normal position before commencing a word or group. In receiving a

message the flag should be lowered and gathered in until required for answering.

In order to keep the flag always exposed while moving it across the body to form the flashes the point of the pole should be made to describe an elongated figure of 8 in the air.

With clear sending and under favourable conditions these flags can be seen and read with the ordinary service telescope at distances up to twelve miles.

Uniform unbroken backgrounds are always better than broken ones. Bare earth rocks and trees form the darkest backgrounds, and these appear darker or lighter in proportion to their distance in rear of the object projected on them. If far off the background will be lighter, if immediately behind the object it will be darker. Sky forms the lightest background and then water and distant land; green or stubble fields form an intermediate class of backgrounds against which light or dark objects appear almost equally visible. As dark backgrounds appear darker in proportion as they are nearer to the object, so white backgrounds, such as chalk cliffs, whitewashed walls, &c., appear lighter under the same conditions. The position of the sun should always be taken into consideration in applying the foregoing observations. All backgrounds become lighter when the sun is opposite to them and darker when it is behind them. An exception to this rule is, however, found in the light mists which rise from valleys towards evening or the smoke of habitations, which both form a lighter background than the surrounding country, whatever be the position of the sun. The most favourable conditions for flag-signalling are a clear atmosphere and a clouded sun.

Standard of Efficiency

For the standard of efficiency the *minimum* rates of reading correctly from and sending a test message with the different instruments are as follows.

Large flag at the rate of . .	9 words per minute		
Small flag ,, ,, . .	12	,,	,,
Lamp ,, ,, . .	10	,,	,,
Heliograph ,, ,, . .	10	,,	,,

The degree of accuracy is tested by the percentage of letters read correctly. A man is reckoned as—

Very accurate who can read	97 per cent. correctly		
Accurate	95	,,	,,
Fairly accurate . . .	93	,,	,,
Inaccurate	90	,,	,,
Very inaccurate . below	90	,,	,,

Tides and Currents

The soundings on charts are given in fathoms of depth at low water of ordinary spring tides because those are the times of highest high water and lowest low water. For the theory of tides the reader must study works of a different character from the present one. Some definitions of terms are given in the glossary. No satisfactory data can be obtained from a casual investigation ; observations should be made at different times of the day, month, and year, and the results registered in a systematic manner. The pioneer surveyor is not supposed to have as much time as this at his command, and we shall therefore confine ourselves to practical details for a preliminary examination of tidal phenomena.

Datum

A fiducial point should be chosen on shore as near as possible to the position where the register is to be kept, but safe from any encroachment of the sea. A hard rock point or a benchmark on a house or tree will do. If there is only a sandy beach sometimes a piece of old iron pipe 6″ diameter is obtainable, which filled with Portland cement concrete makes a very durable datum.

TIDE GAUGE

If a pier is available, a planed board is lashed to it marked in feet and tenths, the former being painted red, white, and blue alternately so as to be read at a distance. If there is no pier, a pile must be driven and stayed. To the side of the pile the planed board is attached, or else a simple maximum and minimum device is constructed of a casing made with four boards and containing a float maintained in position by guides working in slits and having indicators one above a guide, the other below the opposite guide. These indicators have to be set twice in the twenty-four hours and are then self-registering. Or else if left for any length of time they will leave on record the maximum and minimum during that period.

The registration is made from simple inspection of the tide-board, the zero of which has been previously levelled with a levelling instrument from the datum point on shore.

There are many elaborate and expensive self-registering tide-gauges for use on permanent works of construction or important hydrographical work.

A very beautiful repertory of tidal instruments was exhibited by Sir William Thomson at the Institution of Civil Engineers in 1881 in connection with his lecture on that subject. These comprehended a tidal harmonic analyser, a tide predicter, and a model of the No. 3 Clyde Tide-gauge. Wave disturbance is almost entirely annulled in the floater; clockwork mechanism records by either an ink or a pencil marker the movement of the tide upon a revolving diagram like the steam-engine indicator. The abstruse calculations of the harmonic analysis are replaced by those of an automatic integrator, and the time, not merely of high or low water, but the position of the water-level at any particular port, is predicted for any time of day of any future year.

The velocity of currents at sea are generally taken by a ship at anchor with a current-meter. They are also ascertinued by observing a drifting boat from two points upon the shore. When far out at sea they are calculated by comparing the day's run, ascertained astronomically with the figures of the patent log, making allowance for instrumental errors.

An elementary explanation of tidal phenomena, together with high and low water at the principal ports of the United Kingdom and tidal constants for minor ports, is given in Whitaker's Almanack, which should form part of the travelling library of every surveyor.

The velocity of river currents is best taken by current-meter, unless sufficient time is available for obtaining the data required by Kutter's formula.

Floats are either surface-floats, of wood or wax or vertical tin tubes loaded at bottom. In large rivers the average of the vertical floats, which gives the approximate mean velocity at that section, has been found to be about ·9 of the surface velocity.

Hydraulics

The numerous problems of more or less complexity occurring in hydrostatic and hydraulic science are peculiarly suitable to treatment by the graphic method, and by the use of large-scale diagrams may be solved with a greater degree of mathematical precision than is actually necessary with any formula, by reason of the element of uncertainty which in hydraulics must always attach to the resisting power of the conduit. For the approximate estimates which the preliminary surveyor has to prepare, the small-scale diagrams on Plate I. Fig. 26, and Figs. 28, 29 will be found amply sufficient; any slight inaccuracies due to scale will not produce variations in the final result as great as those between two authorities like Beardmore and Kutter as exhibited

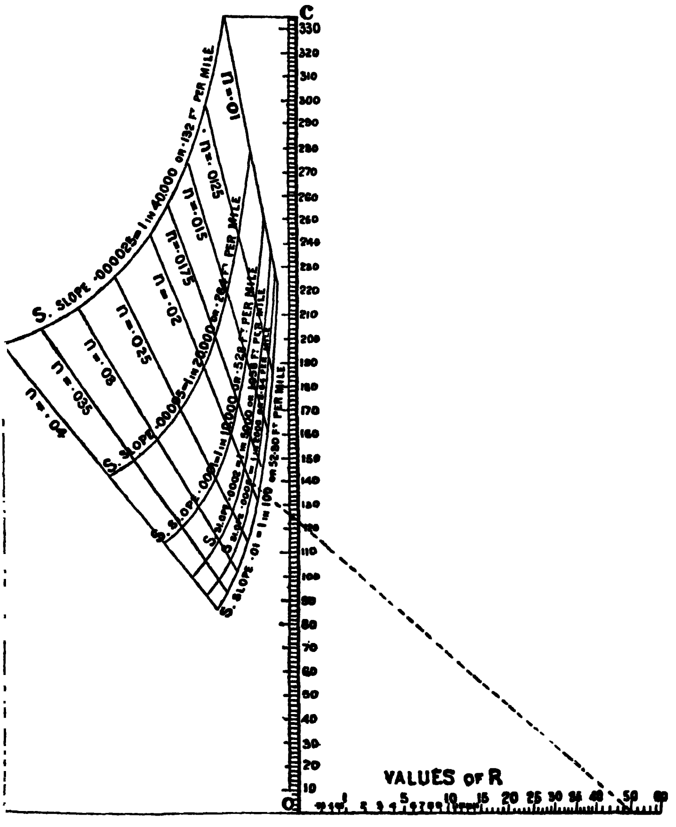


FIG. 26.

Kutter's formula for flow in open channels. Diagram for finding coef. C in formula $V = C \times \sqrt{R \times S}$.

Note.—The values of N are from ·01 to ·04.
,, S ,, ·000025 to ·01.
,, R ,, 0 to 60 feet
Intermediate values may be interpolated by eye.

Note 1.—The coefficient is to be taken from the thickened, right hand side of the vertical scale.

Note 2.—If this diagram is in frequent use with dividers, a piece of dull-back tracing cloth, gummed over it by the four corners, will protect it.

later on. The formula of the latter engineer has been adopted as being the most comprehensive and in accordance with recent experiments. Messrs Ganguillet and Kutter, Swiss hydraulic engineers, have of late years developed the theory of flowing water in open channels, and have furnished a formula of greater complexity than most of the preceding ones, but held by modern authorities to be superior in accuracy. However perfect the formula may be, an element of uncertainty must always exist as far as rivers are concerned, because the coefficient of roughness in the conduit, or *n* as it is called by Kutter, has to be obtained, to some extent by experiment but largely by judgment based upon actual study of the ground or channel. Plate I. Fig. 26 for finding the coefficient C is constructed for English measurement and will embrace all ordinary cases, but where much work is required it will be found advisable to construct a diagram to a large scale, for which full directions are given in 'Trautwine's Pocket Book.'

It is evident that the varying velocities in any conduit of water must arise from the force of gravity modified by the resistances of the surface; from natural roughness, vegetable growths, bends, &c.; the consistency of the fluid, the wind, and it may be other causes.

Kutter's formula, like those of other good authorities old and new, is based upon the theory that the resistances to flow are directly proportional to the area of the surface exposed to the flow (which would be, in the case of pipes running full, the entire internal surface) and to the square of the velocity. The formula stands thus for English measurement: $V=C\sqrt{RS}$. In which V=velocity in feet per second; C a coefficient derived from three independent variable quantities—the resistance of surface, hydraulic mean depth, and slope; R is the hydraulic mean depth; S is the slope or sine of the angle of slope.

The velocity V is the mean velocity of the whole stream,

and is the only reliable means of ascertaining the discharge. Experimental determinations of surface velocity by floats are very useful, but are generally too local to give more than an approximate result. The mean velocity varies in large rivers from ·85 to ·95 of the surface velocity found by floats.

The coefficient C is obtained directly from Fig. 26 by drawing lines from the horizontal scale of R, that is the hydraulic mean depth, to the intersection of the hyperbolic slope curve S with the radial 'roughness' line *n*. This line will cut the ***right-hand side*** of the vertical scale OC at the value of C. The dotted line shown on the diagram is drawn from a hydraulic mean depth of 50 feet to the intersection of $n = {·}02$ with slope of 1 in 2,500 ; the coefficient C= 123 is halfway above the first subdivisions from 120, each of them being 2.

Any intermediate curves or radial lines may be interpolated by the eye with quite sufficient accuracy.

The value of *n* is obtained from the following table.

TABLE XIX. *Artificial Channels of Uniform Cross-Section.*

	Coefficient of roughness, *n*
Sides and bottom lined with well-planed timber	·009
Sides and bottom rendered in cement, glazed ware, and very smooth iron pipes	·010
1 to 3 cement mortar, or smooth iron pipes	·011
Unplaned timber or ordinary iron pipes	·012
Ashlar or brickwork	·013
Rubble	·017

Channels subject to Irregularity of Cross-Section.

Canals in very firm gravel	·020
Canals and rivers of tolerably uniform cross-section, slope, and direction in moderately good order and regimen, and free from stones and weeds	·025
Having stones and weeds occasionally	·030
In bad order and regimen, overgrown with vegetation, and strewn with stones and detritus	·035

Messrs. Trautwine and Hering use ·015 as a value of *n* for ordinary brick sewers instead of ·013 given by Kutter, in consideration of the 'usual rough character of sewer brickwork.' This remark would not apply to the London and Paris sewers or to those of many other large cities, where the brickwork is exceedingly good; but unless properly maintained the coefficient would soon increase.

The hydraulic mean depth R is equal to the

$$\frac{\text{area of wet cross-section}}{\text{length of wet perimeter } abco.}$$

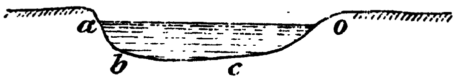


FIG. 27.

In the case of a circular culvert or pipe running full, $R = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$ or $\frac{d}{4}$ where r=radius and d=diameter. When half-full R is also $= \frac{d}{4}$ and the velocity will be the same.

At ·75 of the diameter, *i.e.* $\frac{3}{4}$ full, $R = D \times {\cdot}3$. When $\frac{1}{4}$ full $R = D \times {\cdot}15$. To obtain the hydraulic mean depth of irregular conduits such as rivers, the cross-section must be taken by soundings and the area measured.

The slope is usually termed the sine of the angle made by the average bed of the channel with the horizon. At the small angle of most rivers and sewers the sine is sensibly the same as the tangent, but why it is called the sine is not clear to the writer, because slopes are usually from convenience described in the ratio of base to perpendicular, and not from hypotenuse to perpendicular.

The length of the conduit is always supposed to be measured on the level, and it would seem to be proper to take the ratio from that rather than from the sloping length. The table of tangents corresponding to some leading slopes of feet per mile given on p. 248 will be found sufficient by

the aid of the slide-rule to obtain any required tangent, and the method of interpolating is there explained. Both the hydraulic mean depth and the tangent of slope are often very small decimal quantities of which the square root has to be extracted. Another short table is given, on p. 249, of leading values of squares of decimals by aid of which the operation is greatly facilitated. The diagram only gives slopes up to ·01, that is, 1 in 100. Above that the coefficient C remains the same. The formula upon which the diagram for finding the coefficient C is constructed is for English measure—

$$C=\frac{41{\cdot}6\times\frac{{\cdot}00281}{\text{slope}}\times\frac{1{\cdot}811}{n}}{1+\frac{\left(41{\cdot}6\times\frac{{\cdot}00281}{\text{slope}}\right)\times n}{\sqrt{\text{mean radius in feet}}}}$$

When the velocity in feet per second has been obtained from the diagram and formula $V=C\sqrt{RS}$, the discharge, Q, is obtained in cubic feet per minute by the simple equation $Q=V\times 47{\cdot}12\,D^2$ for circular sewers where D=diam. in feet.

In irregular channels and generally Q=area in square feet × vel. in feet per second × 60.

Example 1. What is the velocity in feet per second and discharge in cubic feet per minute in a circular brick sewer running full; 2 feet diameter; slope S=10 feet per mile= ·00189; R=·25; D=·5?

$RS={\cdot}000945.\sqrt{RS}={\cdot}0307$; C by diagram 4 is found to be (using a coefficient of ·015 for n)=85, and $V=85\times{\cdot}0307=2{\cdot}6$ feet per second.

$Q=2{\cdot}6\times 47{\cdot}12\times 4=490$ cubic feet per minute.

Example 2. What will be the velocity in feet per second and discharge in cubic feet per minute of a V-flume of unplaned timber, sides sloping 60° and bottom board 10″ wide in the clear, the slope being 1 in 50 and the depth of water 9 inches?

The whole calculation can be done in a few minutes by the slide-rule and diagram. The angle of slope of sides gives the horizontal spread from the line of tangents $=5 \cdot 22$ inches, from which we find area $= \cdot 708$ square feet and wet perimeter $2 \cdot 5$ feet; $R = \frac{\cdot 708}{2 \cdot 5} = \cdot 283$ feet; n from Table XIX. $= \cdot 012$; $S = \cdot 02$. C is found from Fig. 26 $= 98$; $RS = \cdot 283 \times \cdot 02 = \cdot 00566$; $\sqrt{RS} = \cdot 075$; and $V = 98 \times \cdot 075 = 7 \cdot 35$ feet per second; $Q = \cdot 708 + 7 \cdot 35 \times 60 = 313$ cubic feet per minute.

The following comparison of the flow of circular sewers running full, calculated from Beardmore's and Kutter's formulæ, has been made with assistance as regards the former from the well-executed diagrams of Mr. W. T. Olive Resident Engineer on the Manchester Main Drainage, published by the Inst. C. E. in their minutes of Proceedings, vol. xciii.

It will be seen that the older formula produces results nearly midway between those by Kutter's rule with the coefficient ·015 and ·01 for roughness, but that in the larger culverts the discharges by Kutter's formula gradually gain upon those of Beardmore, until they are considerably ahead even with the coefficient ·015 for roughness.

TABLE XX. *Comparison of Beardmore's and Kutter's formulæ.*

Diameter in inches	Slope in feet per mile	Discharge in c. ft. per min. (Beardmore)	Discharge in c. ft. per min. for brickwork; $n = \cdot 015$ (Kutter)	Discharge in c. ft. per min. for glazed-ware or iron; $n = \cdot 01$ (Kutter)
6	5	$12\frac{1}{4}$	9·3	15·5
6	50	38	29·4	49
6	150	66	43·4	72·7
24	10	540	490	—
24	50	1,210	1,112	—
96	10	17,100	20,850	—
96	15	21,000	25,633	—

Discharge from Tanks, Pipes, Cisterns, and Weirs.

The diagrams, Plate II., of theoretical velocity due to different heads will need no explanation.

They are furnished for use when no slide-rule is at hand; otherwise it is quicker to work out the velocity by the slide-rule than to scale it on the diagram, and the velocities for fractional values are given with the same rapidity as those of integral values, more accurately than can be scaled from a diagram, and without the interpolation needed with a table.

Example. What is the theoretical velocity due to a head of 15·65 feet? Place the right hand 1 of the slide under the *upper* 15 feet of rule and read the velocity 31·8 feet per second on the *lower* scale of the rule opposite to 8·03 on the lower scale of the slide.

Example 2. What is the theoretical head due to a velocity of 135 feet per second? Place the 8·03 on the *lower* scale of the slide over the 135 of the rule and read the head 282·5 feet on the *upper* scale of the rule opposite to the right-hand 1 of the slide.[1]

Rule 1. For tanks and cisterns flowing into the open air Let V = the theoretical velocity in feet per second due to head, given on Fig. 28.

A=area of aperture in square feet.

D=diameter of circular aperture in square feet.

C=coefficient of friction due to nozzle or opening given by table.

VV=actual velocity of discharge in feet per second.

Q=discharge in cubic feet per minute.

Then $VV=V \times C$

and $Q=VV \times 60A$ or $VV \times 47{\cdot}12D^2$

[1] If the velocity is wanted in miles per hour instead of in feet per second, use the number 5·475 instead of 8·03. For heads from 1 to 10 feet use the left-hand half of the rule, and from 10 to 100 the right-hand half.

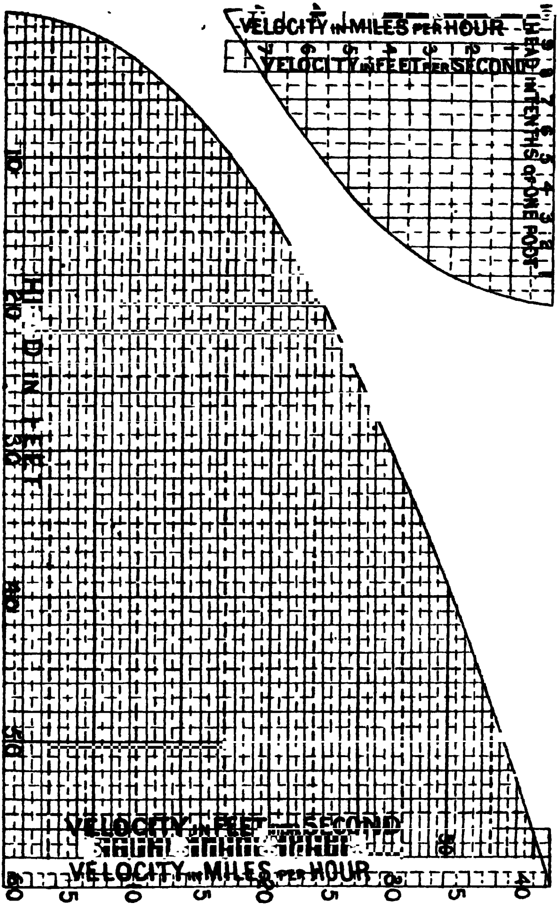


FIG. 28.

Formula $V = 8{\cdot}03 \sqrt{H}$.

$M = 5{\cdot}475 \sqrt{H}$.

Where H = head in feet.

" V = vel. in feet per second.

" M = vel. in miles per hour.

Note.—The vel. in feet per second is the only one which can be obtained from inspection of the diagram: the vel. in miles per hour must be taken with dividers from the outer scale.

FIG. 29.

Theoretical velocity due to different heads.

Note 2.—If this diagram is in frequent use with dividers, a piece of dull-back tracing cloth, gummed over it by the four corners, will protect it.

Values of C for different orifices

Circular or rectilinear openings in thin iron plate in bottom or sides of either vertical or inclined tanks= .	·62
Short tube projecting outwards, bore $\frac{1}{2}$ the length . .	·80
Ditto bore $\frac{1}{25}$ of the length	·70
Short tube projecting inwards	·68
Converging tube, outside of tank ; angle $13\frac{1}{2}°$; velocity at narrow end	·94
Diverging ditto, outside of tank ; angle $5°$; velocity at narrow end	·92

Example. What are the velocity and discharge in feet per second and cubic feet per minute respectively in the following case?

A tank is maintained by a ball cock with a depth of water of $7\frac{1}{2}$ feet. A short cylindrical tube projects from the bottom of $2\frac{1}{2}$ inches inside diameter and 2 feet 1 inch long. The velocity due to 7·5 feet head is found by Fig. 28 to be 22 feet per second. C is an intermediate between ·80 and ·70, $\frac{\text{bore}}{\text{length}}$ of orifice being equal to $\frac{1}{10}$. Interpolating with the slide-rule we find C=·71 and VV=22 × ·71=15·6 feet per second. The diameter $2\frac{1}{2}$ inches=·208 feet and discharge $Q=VV \times 47{\cdot}12\ D^2$ or $=15{\cdot}6 \times 47{\cdot}12 \times {\cdot}208^2$.

Ans. VV=15·6 feet per second.

Q=31·8 cubic feet per minute.

Rule 2. For iron pipes under pressure approx. mean vel. in feet per second=

$$\text{coeff. } m \text{ (Fig. 30)} \times \sqrt{\frac{\text{total length in ft.} + 54 \text{ diam. in ft.}}{\text{diam. in ft.} \times \text{total head in ft.}}}$$

This approximate rule of the late Mr. Trautwine takes into account velocity head, entry head, and friction head. It was not used by him for very long pipes with low heads. The limit given by him was 1,000 diameters, beyond which he neglects entry head and treats the flow by the formula for an open channel. It is not applicable to very high

pressures either, but is suitable to ordinary reservoirs having straight iron pipe conduits.

The diagram Fig. 30 gives values of coefficient m for various diameters of pipes in feet. The horizontal scale gives values of the $\sqrt{\dfrac{D \times H}{L + 54D}}$ for which the ordinates are values of m. D being diameter in feet, H=total head in feet, L=length in feet. The value for ·200 serves for all ratios above it. Intermediate values are to be taken by interpolation. The results are always in excess of those where there is no entry head and all the fall is in the pipe itself.

Rule 3. Discharge over weirs (Eytelwein).

Q=discharge in cubic feet per minute.

L=length of overfall in feet.

H=head in feet.

$Q = 204\ L\ \sqrt{H^3}$

Remark. The head is measured by ascertaining the difference of level between the crest of the weir and the surface of the water before it commences its chute-curve.

The formula is for discharge over a thin plate or a weir with a sharp edge. A slight current towards the weir makes hardly any difference in the result.

Example. Required the discharge over a weir in thin plate length 200 feet. H=1·5 feet.

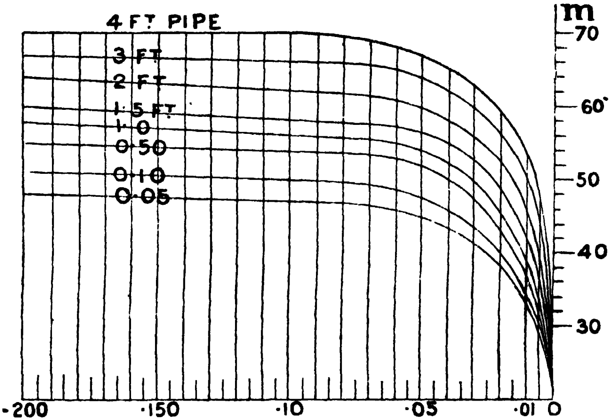
$Q = 204 \times 200 \sqrt{1{\cdot}5^3} = 74{,}954$ cubic feet per minute.

Rule 4. (Approximate—John C. Trautwine) see Diagram, Fig. 31, for C.

$Q = C \times L \times V \times H.$

Where Q=discharge in cubic feet per minute.
C=coefficient per diagram.
L=length of weir in feet.
V=theoretical velocity due to H in *feet per second* from Fig. 28.
H=head in feet measured as in preceding rule.

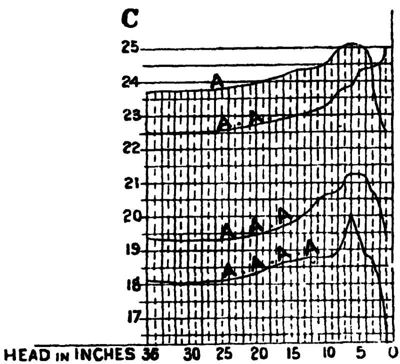
This formula gives results somewhat less than the preceding one for sharp-edged weirs, and is therefore on the



Flow of water in pipes.

Values of $\sqrt{\dfrac{D \times H}{L + 54\,D}}$

FIG. 30.



Discharge over weirs.

FIG. 31.

A = crest 2 inches thick.
A.A. = crest sharp edge.
A.A.A. = crest 3 feet thick, smooth; sloping outward and downward from 1 in 12 to 1 in 18.
A.A.A.A. = crest 3 feet thick, smooth and level.

safe side; it also meets all the ordinary cases by means of the coefficient C with quite a sufficient degree of accuracy for preliminary work.

Example. What would be the discharge over the same weir described in the preceding example only with a crest 3 feet thick, smooth and level?

$Q = 18{\cdot}4 \times 200 \times 9{\cdot}8 \times 1{\cdot}5 = 54{,}096$ cubic feet per minute.

Over a sharp-edged weir it would be 67,000 instead of the 74,954 of Eytelwein's formula, but with a crest 2″ thick it would be 80,000 by Trautwine.

In the case of reservoirs, it is generally easier to determine the discharge over the weir by that of the supply conduit by Kutter's formula.

Horse-Power of Falling Water

$$HP = \frac{62{\cdot}5\ QH}{33000} = {\cdot}001894\ QH.$$

Where Q = discharge in cubic feet per minute, and H = height of fall or head over a turbine or other motor.

Example. Over a fall 16 feet in vertical height, 800 cubic feet of water are discharged per minute.

$$HP = \frac{800 \times 16 \times 62{\cdot}5}{33000} = 24{\cdot}24\ HP$$

Efficiency of Water-Wheels

For undershot wheels	× result found as above			by ·25 to ·33
.. breast wheels	..	,,	..	by ·5.
.. overshot	..	,,	..	by ·5 to ·75.
.. turbines	..	,,	.	by ·75 to ·85.

Horse-Power of a Running Stream

$$HP = \frac{Q \times H \times 62{\cdot}5}{33000} = {\cdot}001894\ QH.$$

Where Q = discharge in cubic feet per minute actually impinging upon the float or bucket; H = theoretic head, due to velocity of stream found by Fig. 28.

Thus, if the floats of an undershot wheel, driven by current alone, be 5 feet $\times$ 1 foot, and the velocity of stream $=210$ feet per minute, or $3\frac{1}{2}$ feet per second, of which the theoretical head is $\cdot19$ feet. $Q=5$ square feet $\times 210=1{,}050$ cubic feet per minute ; $H=\cdot19$;

and $HP=\dfrac{1050\times\cdot19\times62\cdot5}{33000}=\cdot378.$

The wheels only realise about $\cdot4$ of this power, from friction and slip.

The last three rules are from Trautwine's 'Pocket Book.'

HYDROSTATICS

Water pressure is always normal to the surface pressed, *i.e.* in flat surfaces perpendicular to them, and in curved surfaces in line with the radius vector, or, which is the same thing, perpendicular to the tangent at that point.

Whatever the inclination of the surface or the extent of its immersion or distance of its submersion, the following rule holds good.

Rule 1. Pressure in pounds$=62\cdot5$ AD.

Where $62\cdot5=$the assumed weight of a cubic foot of water.

" A=area of surface pressed in square feet.

" D=depth of centre of gravity of surface pressed below the surface of the water.

Rule 2 For pressure in pounds per square inch, multiply the depth in feet by $\cdot434$.

Rule 3. For tons per square foot, multiply the depth in feet by $\cdot0279$.

Rule 4. Total pressure from surface in tons on a section 1 foot wide$=D^2\times\cdot0139$.

For the depth in feet, at which any given pressure exists :

Divide pounds per square inch by $\cdot434$.

" pounds per square foot by $62\cdot5$.

" tons per square foot by $\cdot0279$.

Plate IV., Fig. 32, gives the normal pressure per square

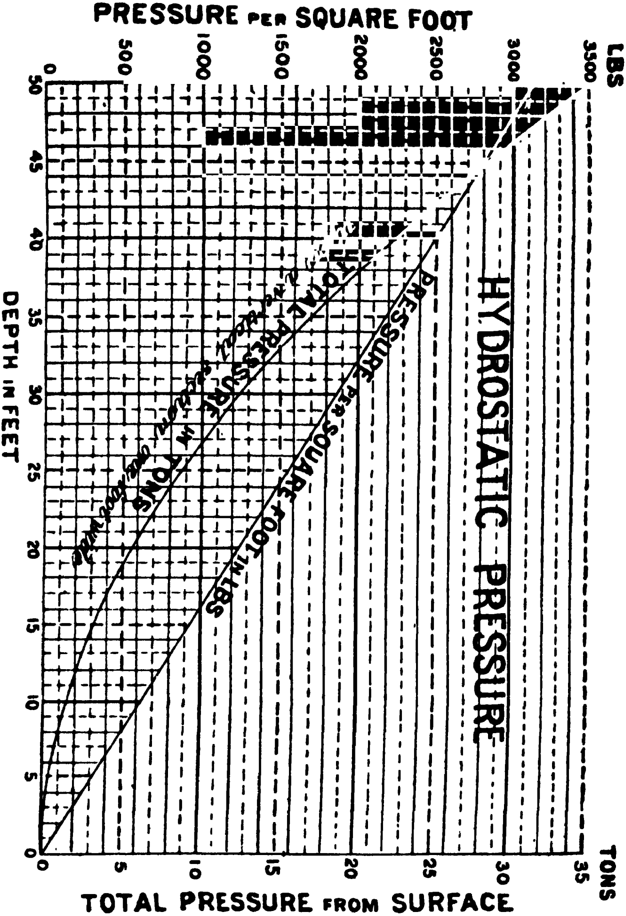


FIG. [illegible]

Note 1.—The left- and right-hand scales have no connection with one another. The lbs. on the left-hand scale refer solely to pressures per square foot. The tons on the right hand refer solely to total pressure from surface.

Note 2. If this diagram is in frequent use with dividers, a piece of dull-back tracing-cloth, gummed over it by the four corners, will protect it.

foot of any immersed surface due to any given depth of its centre of gravity below the surface of the water thus :

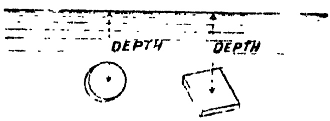


FIG. 33.

It also gives the total pressure upon a plane standing upright in the water, such as a sluice-gate, the plane being one foot wide, thus :

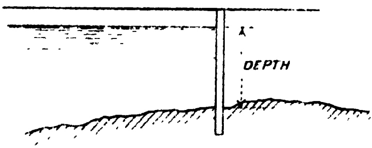


FIG. 34.

In order to combine two diagrams in one, the left-hand vertical scale gives normal pressure per square foot in divisions of 100 pounds each, and measures the ordinates to the straight line. The right-hand vertical scale gives tons total pressure with each subdivision $= \cdot 1$ ton, and measures the ordinates to the curved line.

Example 1. What is the pressure per square inch on a water-main where the head is 300 feet? Ans. $300 \times \cdot 434 = 130$ pounds per square inch.

Example 2. What is the total pressure on a lock-gate 20 feet wide, with a head of 8 feet? Ans. $20 \times 8 \times 8 \times \cdot 01395 = 17 \cdot 85$ tons, or by Plate IV. $= 20 \times \cdot 9 = 18$ tons.

To divide a vertical surface under hydrostatic pressure,

such as a sluice gate, into sections of equal pressure by horizontal lines.

Let N=number of sections, then there will be N − 1 lines, the distance of which *from the top* will be D_1, D_2, D_3, D_{N-1}. Let H=height of water producing the pressure. Then

$$D_1=\frac{H}{N}\sqrt{N}\,;\ D_2=\frac{H}{N}\sqrt{2N}\,;\ D_3=\frac{H}{N}\sqrt{3N}$$

$$D_{N-1}=\frac{H}{N}\sqrt{N\,(N-1)}.$$

Example. Let H=20 feet and N=4.

$$D_1=5\sqrt{4}\,;\ D_2=5\sqrt{8}\,;\ D_3=5\sqrt{12}.$$

$$=10\,;\qquad =14{\cdot}1\,;\qquad =17{\cdot}3.$$

Cost of Dredging

Price of labour 4*s.* per day, no allowance for interest or depreciation of plant.

In from 5 to 10 feet of water, 3*d.* to 6*d.* per cubic yard.

In from 10 to 20 feet of water, 6*d.* to 9*d.* per cubic yard.

These prices include the removal of the material $\frac{1}{4}$ to $\frac{1}{2}$ a mile.

Dredging Plant

The dredging plant used at Leith Docks which remove about 110,000 tons of silt every year cost 13,000*l.*

The dredging plant used at St. Nazaire, Loire, France, to remove 400,000 cubic yards of silt per annum cost 29,800*l.*

The dredging of soft material in small quantity can be done with a bag-spoon (Fig. 35). This is simply a bag, B, of canvas or leather with an iron ring at its mouth. It has a fixed handle, *h*, by which it is thrust down into the mud; another man draws it along by therope *g*. and a third hauls it up when full by the rope *e* (Trautwine).

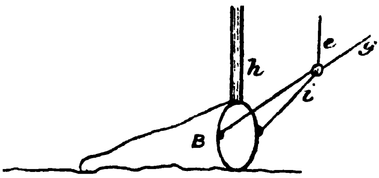


FIG. 35.

BORINGS

in common soils or clay may be made 100 feet deep in a day or two by a common wood-auger $1\frac{1}{2}$ inches in diameter turned by two to four men with 3-feet levers. This will bring up samples (Trautwine). Weale's series, 'Well Sinking and Boring,' contains a detailed description of boring tackle.

CONCRETE

At Aberdeen harbour 16,000 cubic yards of concrete were deposited in jute bags at a cost, including plant, superintendence, and maintenance, of 1*l.* 9*s.* 10*d.* per cubic yard.

13,000 cubic yards of concrete were deposited in mass at a cost of 1*l.* 9*s.* 6*d.* per cubic yard. The concrete consisted of 1 part of Portland cement, 3 of sand, and 4 of shingle.

At Port Said breakwater, concrete in blocks weighing about 20 tons was deposited, forming two breakwaters, one 9,800 feet long, the other 6,233 feet long, at a cost of 1*l.* 5*s.* 1*d.* per cubic yard. The concrete was composed of $3\frac{1}{2}$ of sand and gravel to 1 of shell lime, made in boxes and left two months to set before being used.

DOCK WALLS

The cost of the Hull dock-wall, 43 feet deep to bottom of foundations, cost 19*l.* 9*s.* per lineal foot; the South West India docks, 41 feet deep, 12*l.* 10*s.*; the Penarth Extension,

52 feet deep without excavation, 26*l.* 1*s.*; the cost of the Dublin quay-wall, 47 feet deep, 40*l.* per lineal foot.

The cost of the West India Dock wall per cubic yard was 12*s.* 6*d.*, of the Penarth Extension wall 17*s.*, of the Portsmouth wall 1*l.* 2*s.*, of the Chatham wall 7*s.* 10½*d.*

Cost of Docks per Acre

The Antwerp docks, of about 50 acres, having about 3,700 lineal yards of quay and a basin lock with entrances 59 feet wide, depth of sills at entrance 23 feet, cost about 4,800*l.* per acre of water area.

The Joliette basin at Marseilles, of 54 acres contained by 2,300 lineal yards of quay, cost 640,000*l.* or nearly 12,000*l.* per acre of water area.

At Leith, the old docks cost 28,500*l.* per acre, the Victoria Dock 27,000*l.*, the Albert dock 20,400*l.*, and the Edinburgh dock 24,000*l.* per acre.

The Chicago breakwaters of cribwork filled with rubble, 30 feet wide and 30 feet deep to bottom of foundations, cost 22*l.* 18*s.* per lineal foot.

The New York quay walls of pilework hearting filled with rubble, cut stone face-wall protected at toe by riprap and riprap backing, 43 feet deep to bottom of excavation, cost 49*l.* 16*s.* per lineal foot.

The New York jetties or landing stages for ocean-going steamers of timber un-creosoted cost about 5*s.* 6*d.* per square foot of area.

Graving Docks

Twenty-five of the Liverpool Graving Docks, having an aggregate length of 12,490 feet, cost 940,000*l.*=about 75*l.* per lineal foot of floor length.

Graving docks have been built for vessels of 2,000 tons for under 20,000*l.*

The Somerset Dock at Malta, 468 feet long, cost 150,000*l.*,=about 320*l.* per foot of floor length.

At per cubic yard of capacity they vary from 1*l.* to 4*l.*, the most recent docks at Portsmouth averaging about 2*l.*

The sluicing basin at Honfleur, having an area of 143 acres, cost 200,000*l.*

Warehouses of brick with iron columns and fireproof doors cost from 4*d.* to 8*d.* per cubic foot of total capacity.

CHAPTER IV

GEODETIC ASTRONOMY

THE title of this chapter might convey the idea of a much more comprehensive treatment of the subject than would be possible or desirable in a book of the kind. The rough astronomical calculations which lie within the average surveyor's attainable knowledge or within the power of his instruments can hardly be justly termed astronomy; nevertheless in its most elementary form the subject matter of this chapter will be what it is stated, and an attempt has been made in the glossary and also in this chapter to describe the principles in a manner which will be intelligible to anyone possessing but very little previous knowledge of the subject.

Surveyors who desire to pursue the study of astronomy further will find all they seek in Chauvenet's 'Spherical and Practical Astronomy,' clearly and simply explained.

If a work of a more elementary, and less expensive character is desired, the volume on 'Practical Mathematics' in Chambers's Educational Course, will be found both useful and handy.

Some of the most commonly used formulæ of plane and spherical trigonometry are given in the appendix to the present volume, but the demonstration of them is necessarily omitted; they will nearly all be found, however, in Chambers's 'Mathematics.'

Geodetic astronomy differs but little from nautical astronomy. It is the science of determining by observation and calculation of celestial phenomena the position and course on land. It differs from nautical astronomy only in affording

the facilities of 'terra firma' for some operations which are not practicable at sea, and in being generally debarred from the use of those which depend upon the sight of the natural horizon.

In a treatise on Preliminary Survey the science of geodesy cannot with propriety be treated fully, neither can the analysis of the problems of plane and spherical trigonometry involved therein be closely pursued; but the object of this chapter will be to explain general principles sufficiently to make the formulæ intelligible, and to furnish examples of every problem likely to be useful to the pioneer. A great deal of explanation has been placed in the glossary, which has been arranged alphabetically to be easily referred to, and should be studied before commencing this chapter.

The position of any point on the earth's surface is determined astronomically by finding its relation to the pole and to some arbitrarily chosen meridian, and these two relations are termed latitude and longitude.

LATITUDE

Let us first take the latitude. Everyone has learnt how to find the pole star by the 'pointers'; they perhaps also know that the altitude of the pole star represents roughly the latitude of the place. We can find our place on a map by measuring the distances of latitude and longitude on the circles described upon it for that purpose, but the only fiducial point given us by nature as a starting-point from which to map the world is the north pole. The map of the earth is made from the map of the heavens, but the map of the heavens is first made from the axis of the earth's rotation.

It is the discovery of a stationary point in the heavens called the celestial pole which determines the position of the terrestrial pole and terrestrial equator; and conversely the position of the observer relatively to the celestial pole determines the position relatively to the terrestrial equator.

A moment's thought will serve to grasp this. If I were standing at the north pole, the celestial pole would be overhead, because I should be standing in the axis of the earth's rotation. If I were at the equator, it would appear on the horizon ; consequently if I measured the altitude of the pole star it would be in the first case 90°, in the second case 0° (see Altitude, Glossary). And these are the respective latitudes, measured from the terrestrial equator. It is evident, then, that an altitude of a celestial body taken when passing the meridian will, if we know its distance from the pole, enable us to determine the latitude by a simple subtraction sum.

And now, leaving for the present the subject of latitude, let us touch upon the principles on which the calculations of terrestrial longitude are based.

LONGITUDE[1]

Unlike latitude, which as we have seen can be determined independently of almanacs or chronometers, longitude depends upon an arbitrary fiducial point. Both in the heavens and earth there is nothing fixed by which to determine the easterly or westerly positions without either a timetable or a watch or both.

The earth performs a rotation in twenty-four hours of sidereal time (see 'Sidereal Time,' Glossary). A star which crossed the meridian of Greenwich to-day at midnight will cross the 180th meridian, that is over the Fiji Islands, at twelve sidereal hours later, and will recross Greenwich meridian twelve hours later still. Any celestial phenomenon happening, independently of our meridian, to the star, such as an occultation of it by the moon, would, although in itself an instantaneous phenomenon, be seen at different local times in different places, and the difference of local time corresponds exactly with that arc of the earth's rotation which would bring successively the two points of

[1] See Glossary.

observation under the same celestial meridian; in other words, is equal to the difference of terrestrial longitude.

Phenomena such as those just described are very useful in their place, but the chief basis of calculation is the sun, whose transit across the meridian can be observed with the utmost exactness, and having by chronometer the time when he crossed the meridian at some known place, we can at once, without any mathematics, determine the difference of longitude by observing his transit and reducing the difference of time to difference of arc. We use the meridian of Greenwich as our fiducial point, and perhaps the time is not far distant when all the nations will agree upon one common meridian.

Reserving further explanation of the methods available to the surveyor for ascertaining his longitude, we will add to these prefatory remarks an attempt to explain the curious fact which travellers puzzle over when they first cross the Pacific Ocean; viz., that of 'losing' a day in going one way and gaining it when going the other way.

As the sun appears to go round from east to west, a place east of Greenwich, say Paris, will have sunrise a little before Greenwich, or west, as Dublin, a little afterwards. In other words Paris time is ahead of, and Dublin time behind, Greenwich. If we could travel round the world across America as quick as lightning, and left Greenwich at midday of January 1, 1890, when passing New York we should find the early risers at breakfast on New Year's morning. At San Francisco and Honolulu they would still be all in bed, and at Fiji if we kept to the same calendar we should find them at midnight of December 31. Fiji is just half way round, being on the 180th meridian. Instead of its being midnight of New Year's eve, however, it would be styled in that place midnight of January 1, and the reason for this we will explain by supposing that instead of going *viâ* America, we had gone *viâ* Calcutta in our lightning flight. Leaving Greenwich at the same time, viz. midday

of January 1, passing Calcutta we should find them already at their New Year's evening dinner. At Melbourne they would be just going to bed, and at Fiji it would be midnight of January 1. It will be seen therefore that the Fijians might keep their time twelve hours back or twelve hours ahead, according as they chose to consider themselves in the Western or Eastern Hemisphere. Consequently they have the option of one day's date. The writer has been informed by a missionary that the keeping of two Sundays running has been of frequent occurrence in the palmy days of evangelisation upon that group, but they have finally elected to consider themselves in the Eastern Hemisphere in order to be in the same calendar as Australasia.

Now to take the passage of a vessel circumnavigating from Greenwich eastwards ; she would keep putting on her time every day at noon until her time was twelve hours ahead of Greenwich at Fiji, and correct with local time and date there. A vessel circumnavigating westwards from Greenwich would have been putting back her time all the while, until at Fiji she was twelve hours behind Greenwich, and consequently correct with local time there, but one day behind in date.

If they both happened to arrive at Fiji at midnight of January 1, 1890, the eastward navigator would be correct to time and date, but the westward navigator's date would be midnight of December 31, consequently when he passes over into the Eastern Hemisphere he will have to skip a day and call it January 1. The eastward navigator passing into the Western Hemisphere will have to put back a day and call it December 31.

Classification of Methods

There are three different circumstances in which a surveyor is ordinarily placed as regards his astronomical work, modifying the methods which he can use and the degrees of accuracy which he can obtain.

1. When starting from the coast and proceeding with a carefully measured telemetric survey inwards.

2. When starting from the coast and making a route-survey inwards.

3. When carrying on an inland survey and having to locate his starting-point.

The most disadvantageous circumstances, short of having no instruments to all, in which a surveyor can be placed are to be—

1. Far inland and out of telegraphic communication.
2. Possessed of a poor watch.
3. Unable to revisit his points of observation.
4. Compelled to move forward rapidly in an easterly or westerly direction.

As will be explained presently, the surveyor is very dependent upon Greenwich time for his longitude. This is preserved upon important geodetic work with the utmost care. A number of chronometers in padded cases, swinging on patent joints, wound every day at the same time by the same individual with the same number of turns, and constantly compared with one another, enable the observer to obtain his longitude with the same accuracy as his latitude. Very different is the case with the preliminary surveyor. He is rarely able to transport more than one chronometer, he often does not possess even a semi-chronometer watch, and so when out of range of the telegraph has to resort to what are termed 'absolute methods' of determining his longitude. Apart from the great inconvenience of transporting high-class chronometers whilst on rapid transit, their very delicacy renders them less useful under rough usage than semi-chronometer watches, two or more of which in a surveying party are the best instruments available. 'Absolute methods' of finding the longitude are either performed with the sextant by 'lunar distance' or by 'lunar altitudes:' with the transit as in the method termed 'moon-culminating stars'; or with the telescope

alone as by eclipses of Jupiter's satellites or the occultation of a fixed star by the moon. Sextant observations will only be explained briefly, in principle; the surveyor rarely possesses anything of that description larger than a pocket sextant, which is of no use for such purposes.

In the first case, when starting from the coast on a telemetric survey, the daily traverse is reduced to latitude and departure by slide-rule or by a table, such as that in Chambers's mathematical tables. A very close check is thus kept on the astronomical work, and the longitude and latitude by account is used like the dead reckoning at sea for determining the argument of the daily observation. The daily observations should be if possible four.

One for time to correct the watch's rate.

One for longitude by watch.

One for latitude by fixed star.

One for azimuth by sun or star.

If any celestial phenomena are available for 'absolute longitude' they should not be missed.

If only one observation can be taken it should be that for azimuth, and it can be made *en route* from the sun without interrupting the work for more than five or ten minutes. No continuous telemetric traverse should be carried forward into an unknown country without a daily check upon the direction. A slip of one degree which may be set down to change of magnetic deviation may produce an error of a thousand feet in a single day's work.

In the second case, when starting on a route survey from a known point, the observations should be the same but more strictly kept up every day, because the survey covers so much more ground, and the check by latitude and departure is much rougher; otherwise the principle is the same.

In the third case, when the location of the starting-point has to be determined independently, the surveyor must take time to get reliable data to begin with: he should stay

two or three days in the same place to take and reduce a series of observations. The first thing is to determine the meridian, which is best done by equal altitudes of a fixed star. From this, local mean time is obtained by observing the transit of a known star. Latitude is then had from the pole star or a circumpolar star such as one in the Great Bear or Cassiopea. Longitude is determined by the successive transits of the moon and a fixed star close together, or else by an eclipse of Jupiter's satellites or the occultation of a star by the moon. This is supposing he has not got Greenwich time by chronometer to start with.

Three days of favourable weather and careful work will give the surveyor who has nothing more than a six-inch transit to work with, a very fair approximation to his true geographical position.

The starting-point, once fixed, should remain as the basis of the whole survey. It is probable that errors will be found later on by bringing greater precision to bear upon it, but in order to preserve symmetry in the work the starting-point should be assumed as correct for graduating the sheets, so that any error found afterwards will be a constant throughout the work. The local mean time and Greenwich date at starting-point once determined, can be recovered at any time by returning to the spot and observing a star's transit.

The constant check upon the watch by means of sidereal time cannot be too strongly recommended. No mathematics are needed, and no time wasted in waiting. By it the poorest time-keeper can be made valuable, and the best chronometers are none the worse for being checked by it.

Observations for determining the True Meridian.

First and foremost amongst the uses of the stars to the surveyor is the determination of the true north and south line or meridian of the place. He needs it for running his survey line true, and he needs it above all things for his other astronomical calculations.

There are still old-fashioned people who think the magnetic compass is quite near enough for a railway, and star-gazing a superfluous luxury; they are generally of the same sort who think the chain is the only practical way of surveying, and a transit instrument only a means of saving lazy engineers their due amount of manual labour. We will not enter into controversy with them for fear of being hit too hard, but proceed to touch upon the various ways of determining the meridian.

A rough and ready method has been explained in the chapter on route-surveying at p. 40.

The solar compass as described on p. 328 gives the true bearing of any points observed, and the true north and south line.

Some surveyors have found very good results attainable with simple sun-shadows reduced to true north and south line by Davis's tables from the time of day recorded opposite to each shadow.

All such operations, though useful in their place, are very approximate.

A watch keeping apparent time, and held so that the hour hand points towards the sun, will roughly give the meridian, midway between the sun and the XII of the watch.

Meridian by equal Altitudes of a Star

The simplest of the really accurate methods is by observation of a fixed star at the same altitude, east and west of the meridian. The movement being perfectly uniform, the successive bearings are taken from any clearly defined terrestrial point, and the mean bearing corresponds exactly with the meridian.

A good plan is to adjust the transit for collimation and bubble in daylight upon a good solid position, then to adjust the horizontal limb so that the vernier and compass needle both stand at zero. Then, clamping the external

axis and releasing the parallel plates, take the exact bearing on the same vernier of some well-defined terrestrial point at a considerable distance, which will be its magnetic bearing. On *no* account use the compass of the instrument for this bearing.

Cover over the instrument until the stars begin to appear. Choose a bright star as early as possible and about 30° to 45° from the meridian either north or south and about 20° to 40° above the horizon.

If it is a circumpolar star taken to the north, be careful to note from its position relatively to the pole whether it is on its way to upper or lower culmination.

If on its upper path, the first altitude must be observed to the east like the southern stars, but if on its lower path it must first be observed to the westward as shown on Fig. 36.

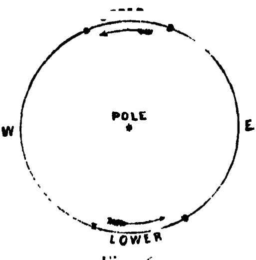


Fig. 36.

This method requires no mathematics whatever. Book the altitude of the star from the vertical limb in its first position, and opposite to it the magnetic bearing from the horizontal limb; *not* from the compass. Unclamp the vertical arc and bring the telescope bubble to the centre of its run, and book any index error which may have crept in since the first adjustment. Set the vertical arc back to the altitude of the star, unclamp the horizontal limb and direct the telescope to the approximate second position of the star, that is to say, about the same angle from the meridian, only on the opposite side. The variation of the compass is known roughly all over the world from a map of equal variations such as that in 'Hints to Travellers' referred to on p. 369, so that the telescope can be placed within a degree or so of its proper place. When the star begins to get near the field, test the vertical arc with the bubble

as before, and if any index error be found, correct it, and clamp the vertical arc again at the same altitude as the first position and free from index error.

Release the horizontal limb, but on no account touch the vertical limb. Bring the telescope right under or over the star, and when it enters the field clamp the horizontal limb with the star on the central vertical hair, and keep it in that position with the tangent screw of the horizontal limb until it reaches the horizontal axial hair.

Book the second bearing from the vernier of the horizontal limb, and the mean of the two bearings is the magnetic bearing of the meridian.

If the observation has been taken to the north, and the bearing of the meridian is greater than zero, or if taken to the south and it is greater than 180°, the variation of the compass is west; and if less than 360° or 180°, as the case may be, the variation is east.

Example. Observed Spica at same altitude to the south.

Mag. bearing in first position . . .	155° 08′ 40″
,, ,, second ,, ...	239° 39′ 40″
	2)394° 48′ 20
Mag. bearing of meridian . . .	197° 24′ 10″
	180° 0′ 0″
Variation of compass W.	17° 24′ 10″

The directrix chosen was a lightning conductor on a large building about 500 feet away, whose magnetic bearing was

	233° 52′ 0″
Applying the variation found . .	17° 24′ 10″
True bearing of directrix –	216° 27′ 50″

Thus when we wish to place the transit in the true meridian we place it in adjustment for collimation and bubble, and clamp the horizontal limb at 216° 27′ 50″. We then release the external axis and adjust the intersection of

the cross hairs to the directrix and *clamp the external axis.* We then release the parallel plate and set the vernier at 0° or 180°, when the telescope will be in the true meridian of the place.

The disadvantage of this method is that it has to be done at night time. In high latitudes there is twilight all through the night during summer, and the stars are hard to see. It takes a long while also, although plotting and other kinds of work may be done during the interval between the two stellar positions.

Meridian by Circumpolar Stars

Another method, which may be applied to rough approximations with a plummet string, but which is a very accurate and convenient observation when performed with a transit, is by watching for two circumpolar stars in the same vertical. It requires the knowledge of the latitude, which can be

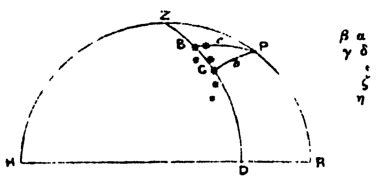


Fig. 37.

α is Dubhé; β is Merak; γ is Phad; δ is Megrez Al Dub; ε is Alioth; ζ is Mizar al Inak; η is Al Kaid.

obtained at the same time from Polaris with the transit (see p. 153), or from two pairs of stars graphically with a plummet as described on p. 154.

If the azimuth is taken with a plummet-string, the point of observation is first fixed by a peg with a nail in it, over which the observer stands with his plummet pointing to the nail. An assistant has another stake in hand consisting of a piece of 9″ × 1′ board pointed at the end. When the stars are very nearly in the same vertical the wide stake

is driven, and at the precise moment the assistant puts in a nail to the directions of the observer; a bullseye lantern is needed to guide the alignment of the nail. The angle of the line connecting the two nails may then be determined by a very simple logarithmic sum, as will be presently described for the accurate method when taken with the transit.

Adjust the transit as already described (p. 125) with the magnetic north as a starting-point, and when the two stars are in the same vertical book their magnetic bearing. This is all that has to be done with the instrument.

The azimuth of the stars or angle from the pole is then determined in the following manner for two stars in Ursa Major.

In Fig. 37 Z represents the zenith, P the pole, HZPR the meridian of the place, and BC the stars; ZBCD is the great circle passing through the zenith and the stars when the latter are in the same vertical. $ZP = 90° - PR$, *i.e.* the co-latitude; hence $\sin ZP = \cos$ lat. Angle Z in triangle PZC is the azimuth; PBC is a spherical triangle which alters its form very slowly because the fixed stars BC only move a few seconds a year. In the spherical triangle ZPC we have[1] Sin Z (azimuth) : $\sin h :: \sin C : \sin ZP$; or in other words sin azim $\times$ cos lat $= \sin h \times \sin C$. This product $\sin h \times \sin C$ is a constant for the whole year within a few seconds of arc. Any two stars can be chosen in the Great Bear, Cassiopea, Draco, or other suitable constellation, and the angle C worked out by the equation given in the Appendix.

The constants for the two well-known stars β and ϵ Ursæ Majoris are given for years 1890 to 1900 at p. 378 of Appendix. This familiar constellation is shown on Fig. 37, B as β or Merak. It is the farther one of the two 'pointers,' and Alioth is the third star from it in the train.

Alioth is useful for knowing the position of the pole

[1] See Appendix. Formula 78 Rem.

star, see 'Alioth,' Glossary. The reader should also study Fig. 122 on p. 395, showing the relative position of all the important circumpolar stars.

Example. In latitude 51° 16′ 45″ observed β and ϵ Ursæ Majoris in the same vertical and west of the meridian, the magnetic bearing being 318° 27′.

Tabular constant for 1890 + 10 . . .	19·72910
Cos latitude	9·79625
Sin azimuth 58° 57′	9·93285
	360°
	58° 57′
True bearing	301° 03′
Magnetic	318° 27′
Variation of compass	17° 24′ W.

The meridian is found from a previously observed terrestrial object as already described, p. 126.

The constants for two other stars, not so well known but easily identified from the sidereal chart on p. 393 are given also for years 1890 to 1900 on p. 378 of Appendix. As there is a considerable time interval between their right ascension and that of the Great Bear, they will serve when the other is not obtainable.

MERIDIAN BY TIME INTERVAL OF CIRCUMPOLAR STARS

A very delicate adjustment of the instrument in the meridian, particularly suitable for a test when it has been previously adjusted as close as possible by the foregoing, is to watch the culmination of two circumpolar stars differing nearly twelve hours in right ascension, such as β Cassiopeæ in R. A. 3 min. 21·77 sec., and γ Ursæ Majoris in R. A. 11 hours 48 min. 05·84 sec.

If the instrument is set to the true meridian and moving in a true vertical plane, these stars will transit at a sidereal interval of 15m. 15·93s. sidereal or 15m. 15·68s. mean time, but if the line of sight be to one side, as shown by the dotted line, the interval will be less or more. In the case shown the error would cause the culmination to be nearly simultaneous. In the tacheometer described on p. 307, the micrometer enables the rate of travel in azimuth to be accurately measured, and by a simple proportion the error is eliminated.

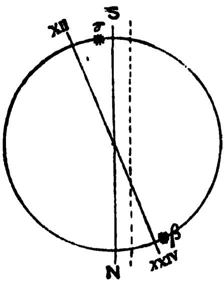


FIG. 38.

MERIDIAN BY POLE STAR

Another and extremely useful method is by an observation of the pole star at elongation. This luminary performs an apparent orbit of an extremely small radius round the pole, but having twenty-four hours to do it in like the rest of the stars. He appears to move straight up and down for a considerable time when at elongation, and to move horizontally when at his upper and lower culmination. He is consequently very suitable for observation at elongation for azimuth, and at culmination for latitude. There is a table given in the N. Alm. for finding the latitude by an observed altitude of Polaris, *out of the meridian*. It requires the local mean time to be known at least with some approach to accuracy; the maximum error produced by an error of a minute of time is about half a minute of latitude. By using this method the latitude and meridian can be determined simultaneously without any mathematics.

The polar distance of the pole star for January 1, 1891, will be 1° 16′ 23·1″ with an annual variation of − 18·9″. Its

azimuth for any latitude when at elongation is given by formula 59, p. 376.

$$\sin \text{ azimuth} = \frac{\sin \text{ polar distance} \times \text{rad}}{\sin \text{ co-latitude}}$$

Example. In latitude 51° 16′ 45″ N. observed Polaris at eastern elongation. Required the azimuth.

Log sin polar dist.	8·34678$5$3
Rad	10·
	18·34678$5$3
Log sin co-latitude	9·7962456
Log sin azim 2° 2′ 9·21″	8·5505397

To find the mean time at place when the elongations will take place is a time problem, which subject we must not forestall. To be done exactly, the hour angle corresponding to the azimuth should be calculated by formula 70, p. 376 :—

Sin hour angle=cos azim ÷ cos polar dist.

The polar distance being so small, the hour angle is within an angular minute or two of the complement of the azimuth. Thus in the preceding example the hour angle would be 87° 57′ 52·9″, which only differs by 2·1″ from the complement of the azimuth.

This is a sidereal interval which reduced to mean time by rule on p. 412 gives the mean time from the culmination. The time of culmination is found from Whitaker or N.A. by rule on p. 412.

The time of elongation ranges according to latitude from 5 hours 49 minutes to 5 hours 54 minutes from the culmination, which is near enough for ordinary purposes.

The following short table of logarithmic values of sin polar distance of Polaris will reduce the calculation to the finding and subtracting the log sin co-latitude.

TABLE XXI.—*Log Sine of Polar Distance of Polaris*

1891 . .	8·34678	1896 . .	8·33763
1892 . .	8·34496	1897 . .	8·33579
1893 . .	8·34313	1898 . .	8·33395
1894 . .	8·34130	1899 . .	8·33211
1895 . .	8·33947	1900 . .	8·33027

Generally speaking the position of the meridian is only required to the nearest minute of azimuth. This can be done in one minute of time by the slide-rule.

Example as above. By slide-rule with the sine-scale in its initial position we find opposite to sin 38° 44′ (the co-latitude) the value ·626. Extend the sine-scale to the right until the sine polar distance 1° 16′ is under the ·626. Then the right hand 1 of the rule will be found over the required angle of 2° 2′.

The approximate time of Polaris culmination and other stars is given in 'Hints to Travellers,' p. 153.

All the foregoing nocturnal observations possess the great advantage of regularity of movement in the celestial bodies observed; it is often necessary, however, to obtain the true meridian *en route*, and the writer has found himself compelled to take observations for azimuth more than once in the day, in order to produce anything like accuracy. The case was one of peculiarly sharp ravines in a densely wooded country, so that the bases were necessarily very short.

The sun is the great stand-by for such operations as these. There is no mistaking him, and if he were only a little more regular in his movements, he would be all that could be desired. Nevertheless, with a little calculation, the

Solar Azimuth

is the handiest and best observation, all things considered, that the surveyor has at his command. The instrumental work is done in a very few minutes: the calculation takes about half an hour. The formula is applicable also to the pole star or any other celestial body.

On Fig. 127 of Glossary the angle at Z in the triangle SPZ is the supplement of the azimuth SZS′ taken from the south point.

Having the three sides, we can by formula 75 at p: 377 of

Appendix (expressed logarithmically, and remembering that $\frac{1}{\sin}$ =cosec) find the angle Z.

Our data are : *First*, the latitude of the place approximately. This is usually computed from the latitude and departure of the day's run as a correction to the observed latitude of the day before (see p. 173) ; or if in possession of previously made maps of sufficient accuracy we can scale it from them.

Secondly, the declination, which we have from the almanack for Greenwich, and which we reduce to local time from the longitude by account.

Thirdly, the altitude of the sun. The three sides of the triangle SPZ, Fig. 127, are all of them complements of these data.

PZ=co-latitude
PS=polar distance
SZ=co-altitude

whence by No. 75, p. 377

$2 \log \cos \frac{1}{2} Z = \log \sin S + \log \sin (S - SP) + \log \operatorname{cosec} ZP + \log \operatorname{cosec} SZ - 20$

Example. At Sevenoaks, April 30, 1890, in latitude by account 51° 16′ 45″ N., observed the altitude of ☉̣ and magnetic bearing of |☉ with transit theodolite. Required the true azimuth, astron. bearing from N. point, and variation of compass, Greenwich time being known by chronometer.

	hrs.	min.	sec.
Times by watch	4	2	20
	4	4	45
	2)8	7	5
	4	3	32·5
Watch slow on Greenwich mean time .	0	2	19
Greenwich time of observation . . .	4	5	51·5

To find the Co-latitude PZ

Latitude by account	$51^\circ\ 16'\ 45''$
	90
PZ.	$38^\circ\ 43'\ 15'$

To find the Co-declination or Polar Distance

Decl. ⊙ at noon by 'N. A.' . . .	$14^\circ\ 50'\ 42''$ N.
Diff. for 4 hours 5 minutes . . .	$+\ 0^\circ\ \ 3'\ \ 5{\cdot}5''$
Decl. at time of observation . .	$14^\circ\ 53'\ 47{\cdot}5''$
	90°
SP.	$75^\circ\ 06'\ 12{\cdot}5''$

Note. If the declination had been south, SP would have been$=90^\circ+$declination.

To find the Co-altitude or Zenith Distance

Altitudes	$\begin{cases} 28^\circ\ 40'\ 20'' \\ 28^\circ\ 18'\ 30'' \end{cases}$
	$2)56^\circ\ 58'\ 50''$
	$28^\circ\ 29'\ 25''$
Refraction (see Glossary)	$-\ 0^\circ\ \ 1'\ 44''$
Semidiameter (N. A.)	$+\ 0^\circ\ 15'\ 55''$
Contraction ,, ...	$-\ 0^\circ\ \ 0'\ \ 1''$
Parallax (see Glossary and N. A. . .	$+\ 0^\circ\ \ 0'\ \ 9''$
True altitude of centre	$28^\circ\ 43'\ 44''$

Whence $SZ=90^\circ-28^\circ\ 43'\ 44''=61^\circ\ 16'\ 16''$

and $S=\dfrac{PZ+SP+SZ}{2}=87^\circ\ 32'\ 51{\cdot}7''$.

Log sin S . . .	$=9{\cdot}9996021$
+ log sin (S − SP) . .	$=9{\cdot}3334351$
+ log cosec ZP . .	$=10{\cdot}2037544$
+ log cosec SZ . .	$=10{\cdot}0570480$
	$39{\cdot}5938396$
	20
	$2)19{\cdot}5938396$
Log cos $\frac{1}{2}$ Z . . .	$=9{\cdot}7969198=51^\circ\ 12'\ 28''$

Whence Z . . .	$= 102° 24' 56''$
	$180°$
Azim. from south point .	$= 77° 35' 04''$
	$180°$
Astron. bearing from north point	$= 257° 35' 04''$

The magnetic bearings of the sun's |⊙ at the two altitudes were taken at the same time, as described on p. 126.

1st observation . . .	$274° 33'$
2nd observation . .	$274° 54'$
	$2)549° 27'$
	$274° 43' 30''$
Semidiameter	$15' 55''$
Mag. bearing ⊕ . .	$274° 59' 25''$
Astron. bearing from N. .	$257° 35' 04''$
Variation of compass .	$17° 24' 21''$ W.

NOTA BENE

When the magnetic bearing is in excess of the true bearing reckoned clear round from the north point, the variation is that much west; and *vice versâ* east.

The meridian can be also obtained by equal altitudes of the sun, similarly to the method by a fixed star. Owing, however, to the sun's position in the heavens having made a sensible change during the period of observation—*i.e.* that which is due in reality to the earth's orbit, not its rotation—a correction termed the equation of equal altitudes has to be calculated at some little length.

Since the methods given are all of them handier and practicable at any time when the solar equal altitude could be taken, this latter will not be gone into for want of space. The rough and ready method described on pp. 39, 40 with the plane-table is practically this problem, without, however, applying the correction.

For nocturnal observations a good directrix is made from a piece of two-inch board with a circular hole three inches diameter in it, and a wire tacked across it up and down. It is convenient for attaching a lantern behind. It should be driven firmly into the ground as far away from the instrument as possible.

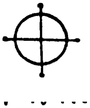


Fig. 39.

Method of determining Local Mean Time and Longitude

The reader should first peruse the following definitions in the Glossary—Apparent time, Astronomical time, Civil time, Mean time (which is the same as mean solar time), Sidereal time, Equation of time, Hour angle, and Longitude.

The subjects of time and longitude are very closely allied.

If we have Greenwich time by chronometer or otherwise, the difference between it and the local mean time is the longitude in time. This will have become plain from the preliminary remarks on p. 118 of this chapter.

The tables for converting angles of the earth's rotation into their time-equivalents are all of course based upon twenty-four hours being equal to 360°, unless the centesimal system of 400° is used, which facilitates calculation, but is not adopted in this work on account of its novelty.

The tables are given in Chambers's 'Mathematical Tables' and Raper.

By the Slide-rule to reduce Arc to Time

	Arc	Time
Multiplying by 4	Degrees give	minutes
	Minutes ,, ...	seconds
	Seconds ,, ...	thirds

Example. Reduce 157° 25′ 32″ to its time equivalent. Write down the angle on one side as follows, and placing

a 1 of the slide over a 4 of the rule write the equivalents down opposite.

Arc		Time
	min.	hrs. min. sec.
157° . . .	= 628 . . .	= 10 28
	seconds	
25′ . . .	= 100 . . .	= 0 1 40
	thirds	
32″ . . .	= 128 . . .	= 0 0 2·13
	Answer . .	10 29 42·13

The hours could have been obtained by dividing the degrees by fifteen. The seconds of arc are generally expressed in seconds and decimals of time, and are therefore also more quickly reduced by dividing by fifteen.

To reduce Time to Arc

	Time	Arc
Multiplying by 15	Hours give . .	degrees
	Minutes ,, . .	minutes
	Seconds ,, . .	seconds
	Thirds ,, . . .	thirds

Example. Reduce 10 hrs. 29 mins. 42·13 sec. to its arc-equivalent.

Time	Arc
10 hours × 15	150°
29 minutes × 15 = 435′ . . . =	7° 15′
40 seconds × 15 = 600″ . . . =	10′
2·133 seconds × 15 = 32″ . . . =	0° 0′ 32″
	157° 25′ 32″

When we have ascertained the true meridian, the simplest method of determining the local mean time is by the

Culmination of a Fixed Star

Any star within the range of the vertical limb of the instrument will do. Stars of small declination, like those in

Orion, appear to pass the field of view much more quickly than those near the pole. When culminating at Greenwich an error of one minute of arc in determining the meridian would correspond with a time error of three to four seconds, whereas a star close to the pole would take twelve to fifteen seconds to make the same change of azimuth ; the former consequently yields the greater precision. An error of four seconds of time corresponds with one angular minute of longitude.

Rule. Find the star's right ascension and add twenty-four hours to it, if necessary. Find the sidereal time at Greenwich mean noon either from Whitaker's Almanack or N. A.; in the Nautical Almanac it is called the sun's mean right ascension at Greenwich mean noon.[1] Correct the sidereal time thus found for the longitude by account, by rule on p. 142. Subtract this result from the star's right ascension. Remainder will be the interval in sidereal time between mean noon and the culmination. Reduce this interval to mean time by rule p. 412. Final result is local mean time of culmination.

Example. On December 31, 1890, in longitude by account 157° 25′ W., find the local mean time of the culmination of Polaris.

	hrs.	min.	sec.
Sid. time M. N. Greenwich, N. Alm. .	18	39	27
Diff. for 1 hr. 9·86″ × lon. in time 10·49 hrs. =	− 0	1	43·4
Sid. time M. N. at place	18	37	43·6
R. A. of Polaris	1	18	52·05
Add	24	0	0
	25	18	52·05
Sid. time M. N. at place	18	37	43·6
Sid. interval from mean noon	6	41	08·05
Correction subtractive, see p. 412 . . .	0	1	5·7
Mean time of culmination	6	40	02·75

[1] See 'Sidereal Time,' Glossary.

Local Mean Time by Solar Transit

The actual culmination of the sun can be observed when not too near the equator by means of the diagonal eyepiece. The meridian having been determined by a solar observation for azimuth on the same day or by the stars the night before, local mean time can be obtained in a similar way to that just described for a star-transit. If the sun souths too near the zenith we have to take an observation for hour angle in apparent time, as will be presently described, but if we can actually observe the culmination we can save mathematics by merely applying the equation of time as explained on p. 399. The equation of time cannot be *perfectly* exact unless we know the longitude in time, which is just what we want to know, because the culmination is only given in the ephemeris for Greenwich noon; but its maximum variation is $1\frac{1}{4}$ seconds per hour and is sometimes a small fraction of a second per hour. If therefore we know our position within one hour, that is 15° of longitude, which is over a thousand miles at the equator, it *could* not make a greater error than $1\frac{1}{4}$ seconds of time.

The equation of time approximately follows the time of year, and the following rough guide may be useful to fix upon the memory the positions of maximum and minimum variation of equation of time.

TABLE XXII.—*Approximate Equation of Time*

		minutes		minutes
January	Sun after clock	$3\frac{1}{2}$	to	$13\frac{1}{2}$
February 1 to 11	,, ,,	$13\frac{1}{2}$	to	$14\frac{1}{2}$
February 11 to 28	,, ,,	$14\frac{1}{2}$	to	$12\frac{1}{2}$
March	,, ,,	$12\frac{1}{2}$	to	4
April 1 to 15	,, ,,	4	to	0
April 15 to 30	Sun before clock	0	to	3
May 1 to 15	,, ,,	3	to	3 50 sec.
May 15 to 31	,, ,,	3 50	to	$2\frac{1}{2}$
June 1 to 15	,, ,,	$2\frac{1}{2}$	to	0
June 15 to 30	Sun after clock	0	to	$3\frac{1}{2}$
July 1 to 26	,, ,,	$3\frac{1}{2}$	to	$6\frac{1}{4}$
July 26 to 31	,, ,,	$6\frac{1}{4}$	to	6

			minutes	minutes
August	Sun before clock	. .	6	to 0
September	,, ,,	. . . .	0	to $10\frac{1}{4}$
October	,, ,,	. . . .	$10\frac{1}{4}$	to $16\frac{1}{4}$
November	,, ,,	. . . .	$16\frac{1}{4}$	to 11
December 1 to 25	,, ,,	. . . .	11	to 0
December 25 to 31	Sun after clock	. .	0	to $3\frac{1}{2}$

The surveyor can hardly be anywhere on land nowadays where he cannot tell his position within 100 miles from some point given on an ordinary atlas.

The maximum error in equation of time at say a latitude of 70° (which is about as far north as he can go) for a distance error of 100 miles would only be ·33 of a second; we may therefore safely say that the surveyor can always obtain, for the purposes of his calculation, the equation of time, and when he observes apparent noon, by applying the equation of time he has the mean time at place.

Example. On December 31, 1890, in longitude by account 157° W., observed the culmination of sun's western limbs at 11h. 58m. 12s. and eastern limb 12h. 0m. 34s.; what was the error of the watch on local mean time?

Longitude in time 10·43 hours behind Greenwich.

	hrs.	min.	sec.
Equation of time at Greenwich, December 31, mean noon	0	3	16
Diff. for 1 hour 1·19 sec. × 10·43 hrs.. .	−0	0	12·4
Equation of time to be deducted from apparent time	0	3	3·6
Apparent noon is	12	0	0
Deduct equation at place	0	3	3·6
Mean time of transit, sun's centre . .	11	56	56·4

	hrs.	min.	sec.
By watch, western limb . .	11	58	12
,, eastern limb . .	12	0	34

	hrs.	min.	sec.
,, transit of centre . . .	11	59	23·0
Watch fast on local mean time . . .	0	2	26·6

When the sun souths too near the zenith, we can obtain the local mean time by a

Solar Hour-angle

This observation may be the same as the one for azimuth. If the reader will refer back to that problem on p. 132 it will save space to use that description up to the finding the three sides of the triangle SPZ.

PZ=co-latitude.

PS=co-declination.

SZ=co-altitude.

Angle SPZ, or for shortness P, is the hour angle because it is the angle of rotation at the pole between the position S where the sun is observed and the position S′ which it will have on the meridian, and by formula 75 at p. 377 of App. we have,

$2 \log \cos \frac{1}{2} P = \log \sin S + \log \sin (S-SZ) + \log \text{cosec } ZP + \log \text{cosec } SP - 20.$

As on p. 138 $SZ = 61° \ 16' \ 16''$; $SP = 75° \ 06' \ 12{\cdot}5''$

$ZP = 38° \ 43' \ 15''$

S, the half sum, $= 87° \ 32' \ 51{\cdot}7''$

and $S - SZ = 26° \ 16' \ 35{\cdot}7''$

Log sin S	9·9996021
Log sin (S − SZ)	9·6461139
Log cosec SP	10·0148468
Log cosec ZP	10·2037544
	39·8643172
	20
	2)19·8643172
Log cos ½ P, 31° 11′ 54·2″	9·9321586
Whence P = 62° 23′ 48·4″	

This hour angle of apparent time is first reduced to its equivalent in apparent time as follows:—

			hrs.	min.	sec.
62°	248 min.		4	8	0
23′	92 sec.		0	1	32
48·4″			0	0	3·1
			4	9	35·1

It is then reduced to mean time by applying the equation of time in N. A. for Greenwich corrected for the assumed longitude. The longitude in time ·0135 hr. is applied here merely to show that an error in assumption of 12 miles would not make an appreciable difference in the result.

	min. sec.	hrs.	min.	sec.
Local apparent time as above		4	9	35·1
Equation of time at Greenwich, mean noon, April 30	−2 54·0			
Long. in time 48·5 sec × ·0135 hr.	0 0·004			
		0	2	54·034
Mean time at place		4	6	41·096
Greenwich mean time by chronometer		4	5	52·8
Difference of time		0	0	48·3
= 12′ 5″ east longitude.				
		hrs.	min.	sec.
Time by watch, see p. 137		4	3	32·5
Local mean time		4	6	41·1
Watch slow on local mean time		0	3	8·6

A sidereal hour-angle is obtained in precisely similar manner from a fixed star, only the arc of rotation, reduced to its time equivalent, is an interval of sidereal time and has to be reduced to its mean time equivalent by rule on p. 412.

The time of apparent noon is also obtained by equal altitudes of the sun by applying the correction of equal altitudes already referred to, or else it may be obtained from the equal altitudes of a fixed star described on p. 124 without any correction.

All the previous methods entail the use of a chronometer keeping Greenwich time, which, if done properly, is

by far the most satisfactory, and indeed the simplest and easiest plan.

The surveyor cannot, however, as a rule place very much dependence upon his semi-chronometer watch. Where he is able to return to a place at which he has previously fixed the direction of the true meridian and the geographical position, he can after any lapse of time recover the true time.

This is a most important matter to bear in mind. It will often pay to take several days' journey out of the course to pick up a former station and so readjust the watch, because with a good timekeeper, although it may suddenly change its rate from rough usage, it is generally a gradual acceleration or retardation, and when a cumulative error has been discovered, it may be distributed proportionally over the unchecked back work with very close results.

The process is simply to set up the transit in the meridian, and finding the time at place from a stellar culmination as described on p. 137 determine the error of the watch over a certain interval of time.

The error of the watch on local mean time can be found anywhere; but that does not give Greenwich time. The object of going back to the known station is to recover Greenwich time.

We now come to observations for longitude by what are termed ABSOLUTE METHODS.

ECLIPSES OF JUPITER'S SATELLITES

These phenomena, visible through a forty-power telescope of one-inch aperture, are timed in the Nautical Almanac for Greenwich mean time. All the observer has to do is to watch both the disappearance and reappearance and reduce the difference of time between the local mean time of its occurrence and its timed occurrence at Greenwich into difference of longitude.

The configuration of Jupiter's satellites is given for every

day in the Nautical Almanac for north latitude, and must be reversed for south latitude. They are also given for an inverting eyepiece. The first satellite is most rapid in his orbit, and is therefore to be preferred.

The eclipse being caused by the shadow of the planet falling upon the satellite, the latter may be at some distance from the planet when it takes place, so the observer should be ready a little beforehand. Raper says of this method that though easy and convenient, it is not very accurate; the eclipse is not instantaneous, and the clearness of the air, and the power of telescope employed, affect considerably the time of the phenomenon. Observers have been found to differ as much as 40 to 50 seconds in the same eclipse.

'The observation can only be considered complete when both immersion and emersion of the same satellite have been observed on the same evening, and as nearly as possible under the same circumstances. Thus if the satellite disappear a little sooner than if the air had been clearer, it will emerge a little later from the same cause, and the mean of the two results may be near the truth.'

Lunar Observations

The moon changes her apparent place in the heavens so rapidly that her R. A. and Decl. are given in the Nautical Almanac ephemeris for every hour at Greenwich. Her spherical distances are also given from the sun, certain well-known fixed stars, and any of the planets to which she may be near, for every three hours at Greenwich. Consequently if we observe some recorded phenomena in connection with the moon, or if we time her culmination relatively to that of a fixed star, or measure her distance from one of the registered stars, we have a ready means of finding Greenwich time. If the ephemeris were absolutely correct, these methods would be more valuable than they are, but, unfortunately, the moon's apparent motion is so eccentric that it

is impossible to register her movements perfectly. In addition to this, the interpolation between the registered times is far from a matter of simple rule of three. To obtain any close results, a lengthy process called Bessel's equation for fourth differences is required, and it is but rarely that the surveyor is able to afford the time for it. There are, however, occasions in which even the approximate computation of the longitude from these lunar phenomena are the only means at the surveyor's disposal.

LONGITUDE BY A LUNAR OCCULTATION [1]

Occultations of fixed stars by the moon are given in the Nautical Almanac with Greenwich mean time of immersion or disappearance and emersion or reappearance. The longitude by account gives a correction for the approximate time of occurrence at place, and the instrument should be set up some time beforehand, as the stars are often of small magnitude and need steady watching.

At the instant of occultation the apparent R. A. of the moon's limb is the same as that of the star. The calculation is somewhat lengthy, to remove the effect of the moon's parallax; which being done the true R. A. is deduced and the G. M. T. found.

If the surveyor only has an ordinary six-inch or five-inch transit-theodolite he will have some difficulty in obtaining a good observation, but with the 'ideal' tacheometer described at p. 307 he will see it very well

Any good three-draw telescope attached to a post will do.

The local mean time has to be first determined by one or other of the methods given on pp. 136–140.

[1] A good illustration of this method is given in Raper's 'Navigation,' p. 305.

THE LUNAR DISTANCE[1]

This observation for longitude has to be taken with the sextant, and will therefore only be explained in principle.

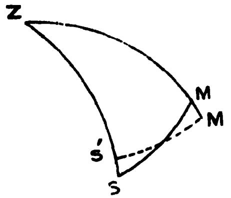


FIG. 40.

Let Z be the zenith and M′S′ be the observed lunar distance from a star, M′Z and S′Z being the zenith distances or complements of the observed altitudes. If M′S′ were the true distance, we could find Greenwich time by interpolating between the registered lunar distances in the Nautical Almanac. But inasmuch as M′ and S′ are not the true places on account of refraction, parallax, and semi-diameter, the distance is really MS. We can make the corrections MM′ and SS′ and then making S equal the sum of the true altitudes H+H′, and s equal the sum of the apparent altitudes $h+h'$, we have by spher. trig. (Chambers's 'Pract. Math.' 751 d),

$$\sin^2 \tfrac{1}{2}\,MS = \cos^2 \tfrac{1}{2}\,S - \frac{\cos H \cdot \cos H' \cdot \cos \tfrac{1}{2}(s + M'S') \cdot \cos \tfrac{1}{2}(s - M'S')}{\cos h \cdot \cos h'}.$$

TERRESTRIAL DIFFERENCE OF LONGITUDE

When two points are correctly determined as to latitude but the longitude is doubtful, their difference of longitude can be computed as follows, providing that they are visible from one another.

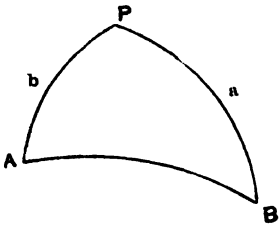


FIG. 41.

Let A, B be the stations, AP, BP their co-latitudes, the angles A and B their reciprocal true azimuths, and APB, or P, the required angular difference of longitude. As-

[1] Examples of this method are given in Raper's 'Navigation,' p. 283, Chambers's Math. p. 449, and a graphic method in 'Hints to Travellers.'

suming the earth to be a sphere we have (by formula 81, p. 378)—

$$\sin \tfrac{1}{2}(a \sim b)$$

LONGITUDE BY MOON-CULMINATING STARS

The Nautical Almanac furnishes the R. A. of the moon's *bright limb* for the lower as well as the upper culmination, marked L. C. and U. C. respectively.

It also gives the variation in R. A. in one hour of longitude, that is the variation in her transit over two meridians equidistant from Greenwich and one hour distant from one another. The figures are calculated from the right ascension of the bright limb and include the effect produced by the change of the semi-diameter.

If we can determine exactly what the right ascension of the moon is at any time and place, we can find, by interpolating between the values for the two nearest hours in the Nautical Almanac, the Greenwich date corresponding to our local time of observation.

The method adopted is to watch the successive culminations of the moon's bright limb and some fixed star close to her whose right ascension is known. The Nautical Almanac gives a list of such stars which are peculiarly suitable from their position for this purpose. They are generally small however, and need some little practice in star-gazing to identify them. They are chosen with as short a time interval as possible and nearly on the same parallel.

Very correct results are obtained by this method when the distance between the meridians is not great.

The best plan is by corresponding observations at two places on the same night.

Such a case is the following example taken from Professor Loomis' 'Astronomy :' Let the right ascension of the moon at the two meridians be A and A′, from which we

know the moon's motion in R. A. during the interval of the two transits A′ – A.

The almanac furnishes the variation of the moon's right ascension corresponding to one hour, which we will represent by V.

We shall therefore have the proportion—

V : A′—A :: 1 hr. : the difference of longitude.

Example 1. The R. A. of the moon's first limb September 6, 1840, was observed at Washington to be 19h. 21m. 29·90s.; and on the same day at Hudson, Ohio, 19h. 22m. 9·72s. Required the difference of longitude of the two places.

Here A′ – A = 39·82 sec.

That value of V must be taken which corresponds to the middle of the interval between the observations, which is found by interpolation to be 135·55 sec.

135·55 sec. : 39·82 sec. :: 1 hr. : 17 m. 37·56 sec.

An accurate method of determining the longitude upon this principle, both for distant as well as near meridians, involving lengthy calculation, may be found in Professor Chauvenet's 'Spherical Astronomy.'

Methods of determining the Latitude

The simplest, most accurate, and most rapid of all the astronomical calculations at the disposal of the surveyor are those for finding the latitude. The observer does not require to know his meridian or his local time, all he needs is a sextant or a transit. In fact, as shown later on, he can even obtain an approximation to the latitude with a plummet-string.

In the prefatory remarks to this chapter it was shown that an altitude of the true pole is equal to the latitude of the place. If the pole star were exactly at the pole, we

should merely have to take its altitude in order to get our latitude. The pole star is not exactly at the celestial pole, so it has to make its own little circle of apparent rotation round the true celestial pole. It will be seen on examining Fig. 127, Glossary, that no star whose polar distance is less than the latitude can set, neither can any star whose polar distance is greater than 90° + co-latitude ever become visible above the horizon.

The pole star crosses the meridian in the Northern Hemisphere twice in the 24 hours like all the rest of the stars. It is in fact a circumpolar star. When at its upper passage, its altitude will be 1° 16′ 42″ greater than, and when on its lower passage as much less than, the latitude. Similarly when we know the declination of any other star we can tell the latitude from a meridional altitude. All we want besides is to know what side of the globe we are on, and in which direction, north or south, the body is culminating.

In north latitude when the polar distance is equal to the complement of the latitude, the body culminates at the zenith (*Z*, Fig. 127) because then the altitude$=$ZP$+$PR.

When the polar distance is greater than the co-latitude, it culminates to the south, and when less to the north ; and when equal to the supplement of the latitude, the body only touches the horizon when making its meridional passage. Its diurnal path is represented by HT (Fig. 127).[1]

When the polar distances are as above in south latitudes the culminations are in the opposite direction.

When the polar distance is 90°, or in other words when the body is on the celestial equator, its meridional altitude EH (Fig. 127) will be the complement of the latitude, because HEPR$=$180° and EP$=$90°, therefore HE$+$PR$=$90°.

The polar distance is either the complement of the declination S′P or 90°+ the declination U′P, and the following rules are deducible from Fig. 127, for obtaining

[1] The paths of the celestial bodies are shown diagrammatically.

the latitude in either hemisphere, when we know the declination and altitude on the meridian.

Rule 1. In the Northern Hemisphere.

(A) When a body culminates to the south of the observer.

(1) When the declination is north. Subtract the declination from the altitude, this will be the co-latitude:

$$HE = S'H - S'E$$ (Fig. 127, Glossary)

(2) When the declination is south. Add the declination to the altitude; this will be the co-latitude:

$$HE = EU' + U'H$$

(B) When a body culminates to the north.

(1) When it culminates above the pole. Subtract the co-declination from the altitude. This will be the latitude:

$$PR = VR - VP$$

(2) When it culminates below the pole. Add the co-declination to the altitude; result is the latitude:

$$PR = V'R + V'P$$

Rule 2. In the Southern Hemisphere.

(A) When the body culminates to the north of the observer.

(1) When the declination is south. Subtract the declination from the altitude; this will be the co-latitude:

$$QR = S''R - QS''$$ (Fig. 128, Glossary)

(2) When the declination is north. Add the declination to the altitude; result will be the co-latitude:

$$QR = U''R + U''Q$$

(B) When the body culminates to the south of the observer.

(1) When it culminates above the south pole. Subtract the co-declin. from the altitude; result will be the latitude:

$$OH = V'H - V'O$$

(2) When it culminates below the south pole. Add the co-declination to the altitude; result will be the latitude:

$$OH = OV + VH$$

At the equator, all bodies having north declination culminate to the north and all bodies having south declination culminate to the south, so that when near 'the line' there can be no mistake as to which rule to adopt. The meridional altitude of a star near either north or south pole, such as α Ursæ Majoris or α Crucis, will at once show what side of the line we are on.

The most accurate method of obtaining the latitude is by a fixed star, but, all things considered, the most useful celestial body is the sun. The moon or one of the planets will all serve the purpose as well as a star; the only difference is that the declinations of the bodies of the solar system vary from day to day and we have to know, approximately at least, what the Greenwich time of our observation is in order to get the correct declination from the almanac. The declination of the fixed stars is only very slightly variable from year to year, so that one of them will serve our purpose just as well if we have not the most remote idea of our longitude. But the great value of the sun arises from its being a daylight observation, and to any one not familiar with the stars an unmistakable object. There are but few cases in which we cannot obtain the longitude by account near enough for determining the declination.

The formidable array of logarithmic figures, which are so associated with astronomical problems, convey the impression of a complexity which does not in fact exist, at least as far as latitude is concerned. There are no trigonometrical calculations at all about obtaining the latitude. The first operation is to free the observed altitude from errors due to dip (if the visible horizon is used), parallax, and refraction, and to correct it for semi-diameter, for all of which terms see explanation in Glossary. The second operation is to reduce the recorded time to Greenwich time from the longitude, as far as it is known by account. The sun's maximum variation of declination is only about an angular minute per hour, and an hour corresponds with 15° longitude, so that

we must be a very long way out in our reckoning for longitude for it to make any serious difference in the calculation.

The third operation is to calculate by a simple rule of three sum the declination at the time of observation as reduced to Greenwich mean time, and finally to apply one of the foregoing rules to the particular case.

Latitude by Circumpolar Stars

An exceedingly exact though lengthy method of obtaining the latitude is by observing the upper and lower culminations of a circumpolar star. This can be done without knowing the meridian, the declination, or the time, because it is sufficient to watch for the maximum and minimum altitudes of the same star. When both observations are corrected for refraction the mean of the two altitudes is the altitude of the celestial pole, that is the latitude of the place: $\frac{VR + V'R}{2} =$ PR (Fig. 127, Gl.).

Generally the time is known and the true meridian can be readily ascertained. In this case the operation is very speedy, for the altitude is measured at either of the culminations and the declination applied as already explained.

Latitude by Meridional Altitude of a Fixed Star

On February 5, 1889, observed the meridional altitude of Sirius in Honolulu, north latitude, star culminating to the south.

Observed altitude	52° 08′ 20″
Refraction	0° 0′ 45
	52° 07′ 35′
S. Declination (N. A.)	16° 33′ 52″
By Rule 1. A. 2, Co-latitude . . .	68° 41′ 27″
	90° 0′ 0″
Latitude	21° 18′ 33″ N.

The latitude by a meridional altitude of the sun, which is the sailor's stand-by, is identical in principle, only subject to correction for parallax, which is insensible in the case of a fixed star.

The following example is taken from Raper's 'Navigation,' illustrating the calculation made on board ship with the sextant. May 3, 1878, longitude 38° W., obs. mer. alt. $\odot$ 56° 10′ to the southward ; ind. corr. + 2′ ; height of eye 20 feet ; required the latitude.

Decl. 3rd Table 60 or N. A.		15° 43′ N.
Corr. for 38° W.		+ 0° 2′
		15° 45′ N.
Obs. alt. $\odot$	56° 10′	
Ind. corr. + 2′ Dip. − 4′	− 0° 2′	
App. alt.	56° 8′	
Refr. − 1′ Semidiam. + 16	+ 0° 15′	
True alt.	56° 23	
Zen. dist.	33° 37′	33° 37′
Latitude		49° 22′ N.

This is a case of Rule 1. A., but in Raper the zenith distance is added to the declination for the latitude. An inspection of Fig. 127, Glossary, will show at once that ZE = PR, because PE = 90°, and ZR = 90°, and ZP is common.

There are a great variety of methods of determining the latitude. The foregoing are the most common, and as they are always available when the heavens are suitable for observation, we will not take up space with more than an allusion to two others.

1st. Latitude by an altitude of the sun out of the meridian. In the triangle PZS (Fig. 127, Glossary), if we know the time, we have the hour angle SPZ. The co-declination PS we also know, and ZS is the co-altitude. From these

data we can obtain the side PZ, which is the co-latitude. The demonstration of this is given in Chambers's 'Mathematics,' art. 887. The bare formula is No. 78 in the Appendix.

2. Latitude by two altitudes of the sun or of a star, and the interval of time between the observations, or the altitudes of two known stars, taken at the same instant, to find the latitude of the place. To those fond of mathematical gymnastics of high order this method is recommended. It involves a maximum amount of labour with a minimum chance of coming out right at the end. Demonstration in Raper, Chambers's 'Mathematics,' &c.

In conclusion, as to observations of meridional altitude, the pole star and circumpolar stars have this great advantage that they are longest at culmination, giving good time for a series of altitudes, and the vital necessity of repeating the observation face right and face left, as explained in tacheometry, cannot be too strongly urged.

The subject of latitude will be concluded with a method of obtaining it without any instrument at all.

Graphic Latitude

The following simple method of obtaining an approximate latitude is described by Mr. Coles of the Geographical Society, and is given here somewhat in his words with a few additional explanations and practical cautions, but the diagram and example are independently worked out.

By observing the times when two pairs of stars are vertically above one another the latitude can be obtained by graphic construction in a very short time. All one requires is a plummet-string, and a table of mean positions of fixed stars, such as is given in Whitaker's Almanack. The operation should be repeated once or twice, and the results meaned. It requires a time interval of from four to eight hours, and a timepiece which is at least accurate to a

few seconds for that interval, although it may be quite wrong on local mean time. The positions of the stars are projected stereographically upon the plane of the celestial equator ; the circle marked primitive being the equator to a radius 1. P, the centre of the circle, is the projection of the pole. The first point of Aries is marked ♈, and the other quadrants of right ascension 6 hrs., 12 hrs., and 18 hrs., indicated by their numbers. The projections of the vertical circles through the stars are obtained as will be explained,

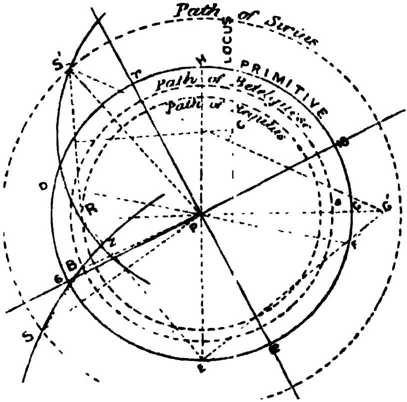


FIG. 42.

and the intersection of these circles being the zenith, the arc ZP, which (Fig. 127, Gl.) is the co-latitude, gives us the latitude of the place.

It will be seen that the circles through the stars are much flatter than the primitive, and it is important to have them cut one another as nearly at right angles as possible, so the first pair of stars should be as nearly east, and the second pair as nearly west, as can be conveniently chosen, with as long a time interval as possible.

One of the stars may belong to both pairs as in the example, but not necessarily so.

We will explain the process direct from the example.

Betelgeuse and Sirius were observed with the plummet-string when bearing about S.E. by S.; and after nearly six hours, when Sirius was nearly setting, it was observed plumb under Regulus, bearing about W.S.W. The four positions of the stars are marked S and B for the first pair, and S′ and R for the second pair. The projections of right ascension are measured by the chords of the angles on the primitive from ♈. The projections of polar distance are measured by the tangents of half those arcs from P upon the radial lines drawn to the positions of the stars in R.A. Thus by slide-rule the chord of R.A. of Betelgeuse = 2 sin ½ angle, and the arc corresponding to 5 hrs. 49m. 10 sec. = 87° 12′ 30″. Placing the extremity of the sine-scale under the 2, we read for 43° 36′, 1·379, which is laid off from ♈. The declination of Betelgeuse is 7° 23′ N., therefore polar distance = 82° 37′. The tangent slide in its initial position gives for 41° 18½′ ·878, and this we lay off from P. The scale of the plot was radius = 1 decimetre, so that the bevelled edge of the slide-rule served for all the scaling. The diagram as printed has been reduced to one-fifth its original size. The first pair of stars being projected, the second pair are treated similarly, except that their right ascension is decreased by the amount of the sidereal time interval between the observations. The projections of the vertical circles are shown on the diagram for both pairs, and their intersection Z, but only the locus C of the second pair is shown, and the auxiliary dotted lines by which it is determined in order to avoid confusion.

Through R draw RPG, and lay off the perpendicular HPE. Draw ERD and DPF. Draw EFG′ cutting RPG produced in G′. Bisect RG′ and draw the locus perpendicularly through its centre. Bisect RS′ and draw the perpendicular

from it to C in the locus, which will be the centre of the projection of the vertical circle RS′.

Draw circle S′R, and similarly, from a fresh locus, the circle SB cutting S′R in Z the zenith. ZP measured by same scale gives the semi-tangent of the co-latitude; being the projection of a great circle passing through the pole, similarly to the circles of polar distance PS′, PR, &c.

At Kukuihaele, Hawaii, in latitude 20° 8′ 9″ by account,

	hrs.	min.	sec.
Observed Betelgeuse and Sirius plumb at .	7	39	15
,, Regulus and Sirius plumb . .	13	26	15
Mean time interval	5	47	0
Correction by slide-rule (see Sid. Time—Glossary)	+ 0	0	57
Sidereal time interval	5	47	57
	hrs.	min.	sec.
R. A. of Betelgeuse	5	49	10
,, ,, Sirius	6	40	15
,, ,, Regulus, less sid. time interval .	4	14	31
,, ,, Sirius, less sid. time interval .	0	52	18

Declination of Betelgeuse 7° 23′ N. and P. D. = 82° 37′
,, ,, Sirius . 16° 34′ S. and P. D. = 106° 34′
,, ,, Regulus 12° 30 34″ N.; P. D. = 77° 29′ 26″

ZP = semi-tangent of co-latitude = by scale of diagram ·698 . . . . . = tan. 34° 55′
2

Co-latitude	69° 50′
	90° 0′
Latitude	20° 10′

CHAPTER V

TACHEOMETRY

TACHEOMETRY, or rapid measurement, is a term which signifies more than one kind of measurement. It may be defined as telescopic surveying, and has for its aim the production of a correct map with the least possible assistance and in the least possible time. It performs at one operation the measurement of distance formerly only possible with the chain, and the measurement of elevation formerly made with the level and staff.

The word is sometimes written tachymetry or takimetry. The French also have a word 'takitechnie' or 'rapid art,' which includes the use of the slide-rule. The principal difference in the methods of tacheometry is in the mode of mapping. If the plane-table and stadia are used, the mapping is done in the field, but if the transit and stadia are used, the mapping is done at home from field notes.

The instruments used are very numerous, some of them highly complicated, and many of them exceedingly ingenious. Their names often affect an originality of principle or an extent of accomplishment which does not really belong to them, whilst they fail to classify them according to their organic type.

The measurement of distance performed optically is the essential principle of the stadia, and places it in the front rank of all tacheometers.

There should be a means of distinguishing those instruments which measure angles for the calculation of distance

—such as the Hadley sextant when used in hydrography; those which demand the measurement of a base by tape, chain, or pacing, like telemeters and range-finders; or those which require the use of a rule of three sum or reference to a table, like Eckhold's 'omnimeter,' or the micrometer telescope—from those in which the distance is practically *seen*, or at least *read at a glance* from a graduated rod. This latter achievement, though not by any means new in principle, deserves a special and typical designation in its modern form.

The word metroscope, forming a good third with telescope and microscope, from *metron* a measurement and *skopeo* I behold, would express more satisfactorily the stadia principle.

Mr. B. H. Brough, in his paper in the 'Min. Proc. Inst. C. E.' vol. xci., attributes the invention of the stadia to Mr. William Green in 1778. It was used with a simple tube having in its field three horizontal wires, and was used upon the principle that since rays of light travel in straight lines except as diverted by refraction, the distance of an object is sensibly measured by the extent of the optic retina which is covered by it.

Thus, supposing we see a man and a boy standing together 400 yards away from us, and the image of the boy upon the eye occupies three-fourths of that of the man; then, if the boy comes forward so as to be 300 yards from us, he will appear to be exactly the same height as the man.

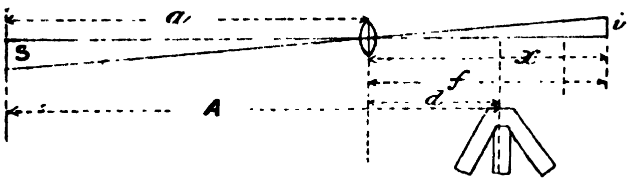
If we arrange the two extreme horizontal wires in a plain tube so that 20 feet away from us a graduated staff will show 2 feet subtended by the wires, then at a distance of 40 feet they will subtend 4 feet and so on.

If we term the distance of the hairs from the eye 'hair distance,' and the distance of the staff 'staff distance,' also the distance apart of the hairs 'hair height' and the space subtended by them on the staff 'staff height,' we have by simple proportion—

Staff distance : *staff height* :: *hair distance* : *hair height*

from which we can calculate the distance of the staff when we know the other three factors. If, however, we graduate the staff so that one subdivision represents 10 or 100 of horizontal measurement, and if we then direct the lower hair to an even figure on the staff, we can simply *read* the distance from the staff.

When telescopes are used (and the lens-power is the backbone of stadia measurements) the factor of 'hair distance' varies with the focus, so that we have no longer a single ratio as a multiplier for determining the distance, and to avoid making a calculation every time, we have to reduce the varying ratio to a single ratio plus a constant, or else an



optical device is resorted to in the instrument itself to produce the effect described on p. 302.

Fig. 43 applies to a horizontal sight when the staff is vertical, or to any sight when the staff is held at right angles to the line of sight.

Let s be the intercepted height on the staff: i, the height of its image; a, the distance of the staff from the object-glass: x, the distance of the image from the object-glass; f, the focal length; d, the distance of the axis of the instrument in rear of the object-lens; A, the distance of the staff from the axis of the instrument.

Then by simple proportion, as before, we have

$$\frac{a}{x}=\frac{s}{i}$$

But the general formula for the foci of lenses is

$$\frac{1}{f}=\frac{1}{x}+\frac{1}{a}$$

This multiplied by a, becomes

$$\frac{a}{x}+1=\frac{a}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and substituting for $\frac{a}{x}$ on the one side of the equation its value from (1) we have

$$a=f\frac{s}{i}+f$$

or,

$$A=a+d=f\frac{s}{i}+f+d \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$f\frac{s}{i}$ is a simple ratio, and $f+d$ is a constant, which ranges in different instruments from 12 to 24 inches. $\frac{f}{i}$ is generally constructed $=100$, that is to say for a fourteen-inch telescope the hairs are placed one seven-hundredths of an inch above and the other seven-hundredths below the axial hair. An amateur will find the best way to do this, to mark the lines on the brass diaphragm with a needle point, as explained on p. 312, then to fix one hair permanently with shellac but to fasten only the extremities of the other hair at the outer edges of the diaphragm ; this will permit of a slight adjustment of the hair to fix it finally by drawing it up or down at the inner edges of the diaphragm with a quill tipped with shellac. If the measurement of the spaces is carefully made, it will only require to be moved by about its own thickness, if at all. This may not leave the hairs perfectly equidistant from the axial hair, but that is of small importance when the two distances are known. Most surveyors who put in their stadia hairs put them in at haphazard, and find their value by experiment, reducing

the distance by slide-rule, but by so doing they sacrifice for the sake of a very little extra trouble the most valuable feature of stadia measurement, and open to themselves a fertile source of error. The diaphragm will hardly ever have to be taken out more than once to finally fix the hairs.

The constant of $f+d$ with a fourteen-inch telescope in which the vertical axis was eight inches from the object-glass would be 22 inches or 1·8 feet; but inasmuch as the readings are only taken to even feet, the constant to be added would be 2 feet, or with a ten-inch telescope hung concentrically it would be 1·25 feet; but unless at very close distances, only 1 foot would be deducted. This requires no entry in the book: it is usually added systematically down the whole row of figures as they are entered by the recorder.

The principle of all telemetry or tacheometry is triangulation, but it may be otherwise styled the determination of parallax. This is a great word amongst astronomers, for by it the whole solar system is plotted to scale, and it serves as a complete illustration of the value of tacheometry. Parallax is the angle at a distant object which subtends a given base. The greater the base, and the more exact the angular measurement, the more correct will the measurement of the distance be. It is possible to *obtain a greater degree of accuracy with a small base* and high powers of angular measurement, than with a long base but inaccurate angles.

For instance, with a range-finder, a base is run out as explained on p. 338; it may be 100 or 200 feet, but if the angle is taken with a low power and the base roughly measured, it will not produce results nearly as correct as those of a stadia telescope of high power when forming upon a distant graduated staff a base of only a few feet.

The base which the astronomers have for determining the distance of the sun is only the diameter of the earth, and

that is to the distance itself something like a hair's breadth to a distance of six feet.

Another analogy that should be borne in mind is that exactness is always proportionate to magnitude. It is supposed that the error in the sun's distance does not exceed 125,000 miles; this in itself is a large amount, but in 93,000,000 it only represents $\frac{1}{760}$, which, considering the size of the base, is marvellously accurate. Porro, who in 1823 introduced the anallatic stadia telescope of high power, claimed never to have exceeded an error of $\frac{1}{2000}$ up to 660 feet, or $\frac{1}{1000}$ in distances up to 1,320 feet. This would mean that at a distance of 660 feet, he could read to one-third of a hundredth of a foot on a staff graduated in our way. His telescope must have been an uncommonly good one.

If a preliminary survey is correct to the limits of the scale, it is all that can be demanded of it, and 100 feet to the inch is about the largest scale used for that purpose, at which less than 1 foot would be practically unscaleable. Sights should not be taken at distances too great for the instrument to read the graduation distinctly, and this limit varies from 300 feet in small to 1,000 feet in large instruments, beyond which only check sights, or those upon the accuracy of which the survey does not depend, should be taken.

Even with large telescopes, 300 feet is about the best working limit for the length of sights. In the first place, the reliability of the levels rapidly diminishes beyond that distance just as it does in ordinary levelling, and in the second place, detail is sure to be missed.

The value of long shots, either within the range of the stadia or beyond it, with the micrometer is very great, both as a check, and also for connecting the survey with useful outlying points, of which the detail is not required, and which cannot be got by intersections.

Methods of Taking the Sights

In level country it is convenient to set the telescope horizontal like an ordinary level and read off all three hairs, booking the stadia hairs in their column and the axial hair in the column for foresight or intermediate ; there will then be no entry under the vertical limb column, and the level will be reduced by the collimation method. This is only possible where the staff does not travel outside the field.

When it becomes necessary to use the vertical arc, the lower hair should preferably be directed to an even foot on the staff, and that one which will bring the axial hair as near as possible to the same height above ground at the staff as the line of sight is above ground at the instrument. For instance, if the telescope is 5 feet above ground and the staff is 350 feet away, the stadia being arranged at 1 in 100, the lower hair should be at 3·00, then the upper will be at 6·50, or, if there is an instrumental constant of 1 foot at 6·49, the axial hair being at 4·745.

There are two methods in vogue for holding the staff. In Germany the practice is to hold it at right angles to the line of sight, in America to hold it vertically. It can be easily shown that in both figures, 44 and 45, angle x=angle X ; and calling the difference of the two stadia hair readings S and S′, and the reading of the axial hair H, we have for Fig. 44,

or for the case of an instrument whose stadia value is 1 per 100, and constant 1·5 feet,

$$B = A \times \sin X$$
$$C = A.\cos X + b = A.\cos X + H.\sin X$$

In Fig. 45, $S = S'.\cos X$

$$\therefore A = S'.\cos X \times$$

Or for similar case,

$$A = 100\,S'.\cos X + 1{\cdot}5$$
$$B = A.\sin X = 100\,S'.\cos X.\sin X$$
$$C = A.\cos X = 100\,S'.\cos^2 X + 1{\cdot}5$$

The writer prefers the first method for the following reasons :

1. When the staff is held vertically, the accuracy is dependent upon the staff-holder, who has to watch a circular

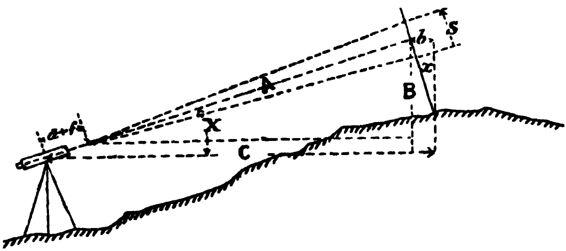


FIG. 44.

bubble or plumb-bob, but the observer has no means of telling whether the staff is correctly held or not. When the staff is held at right angles, the staff can be furnished with a

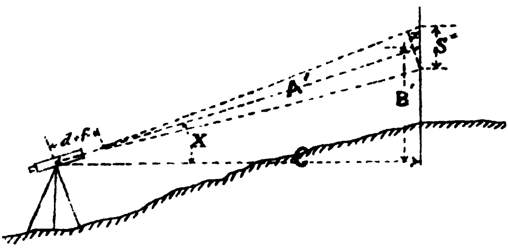


FIG. 45.

piece of wood whose two faces are each three or four inches long, one painted white and the other black ; this serves the twofold purpose of enabling the staff-holder to sight square, and as a tell-tale to the observer, who by a wave of the signal-flag makes the staff-holder adjust it with perfect correctness.

2. In the first method, the only correction for steep angles in the method of reduction is to add to C the distance *l*, which, as will be seen, is seldom a scaleable quantity, and from the short table on p. 167 the surveyor will be able to calculate mentally whether the correction will be appreciable or not. There is, strictly speaking, a second correction for B, due to H being shorter in Fig. 44 than Fig. 45 by H—H cos X; but this rarely enters into the calculations.

In the second method the use of $\cos^2 X$ is required which necessitates a specially made slide-rule, or failing a slide-rule a double calculation. For B also a double multiplier is introduced in the shape of $\sin X \times \cos X$. These formulæ have to be applied to all vertical angles.

The metallic slide-rule made by Mr. J. Kern, of Aarau, Switzerland, is a topographical speciality, which will greatly assist those who prefer to have the staff held vertically, as it gives the horizontal and vertical components for that method. The values of $\cos^2$ are given upon a short slide and $\sin \times \cos$ upon the usual long slide. The metallic division is not more accurate than that of either the boxwood or celluloid rules, and the fit of the slide upon the one examined by the writer was not equal to them. It is more trying to the eye to read, and it is of course much more expensive. It has, on the other hand, the advantage of being more durable and less susceptible to humidity.

3. If the staff is intended to be kept always in a vertical position, an error on the part of the assistant is much more fatal to accuracy than when it is kept at right angles to the line of sight.

For instance, let the stadia read one per cent.; let the line of sight be at an elevation of 45°, and let the direct distance be 300 feet. An error of 3° from the vertical position of the staff will produce an error of 10·3 feet in horizontal distance: whereas the same error from a normal position of the staff will only produce an error of 0·3 foot in horizontal distance.

TABLE XXIII.—*Functions of Angles in per centage of perpendicular to base.*

Angle in degrees	Angle in degrees and minutes	Percentage perpendicular to base	Angle in degrees	Angle in degrees and minutes	Percentage perpendicular to base	Angle in degrees	Angle in degrees and minutes	Percentage perpendicular to base
1·00	1° 0′	1·7	6·00	6° 0′	10·5	10·77	10° 46′	19·0
1·15	1° 9′	2·0	6·28	6° 17′	11·0	11·00	11° 0′	19·4
1·73	1° 44′	3·0	6·85	6° 51′	12·0	11·32	11° 19′	20·0
2·00	2° 0′	3·5	7·00	7° 0′	12·3	11·87	11° 52′	21·0
2·30	2° 18′	4·0	7·42	7° 25′	13·0	12·00	12° 0′	21·2
2·87	2° 52′	5·0	7·97	7° 58′	14·0	12·42	12° 25′	22·0
3·00	3° 0′	5·2	8·00	8° 0′	14·05	12·95	12° 57′	23·0
3·43	3° 26′	6·0	8·53	8° 32′	15·0	13·00	13° 0′	23·1
4·00	4° 0′	7·0	9·00	9° 0′	15·8	13·50	13° 30′	24·0
4·[illegible]	4° 35′	8·0	9·10	9° 6′	16·0	14·00	14° 0′	24·9
5·00	5° 0′	8·7	9·65	9° 39′	17·0	14·03	14° 2′	25·0
5·15	5° 9′	9·0	10·00	10° 0′	17·6	14·58	14° 35′	26·0
5·72	5° 43′	10·0	10·20	10° 12′	18·0	15·00	15° 0′	26·8

Example 1. Required by first method, B and C. $\frac{f}{i}$ being 100; $f+d=1·5$; $H=5·0$; $S=3·00$; and $X=5·72°$. This angle corresponds with a slope of 1 in 10, which is a steeper ascent than is ordinarily met with in roads.

$$A=100\ S+1·5=301·5$$
$$B=A\times\sin X=301·5\times·0996=30·03$$
$$l=10 \text{ per cent. of } 5 \text{ feet}=·50 \text{ feet}$$
$$C=A\times\cos X+l=301·5\times0·995+·5$$
$$=299·99 \text{ feet}+·5$$
$$=300·49 \text{ feet}$$

Note. B′ is the height that is actually needed for the reduction of the levels, so that B would be, strictly speaking, subject to an addition of $H-H.\cos X=0·01$ feet, making $B'=30·04$.

Example 2. Required A′, B′, and C′ in Fig. 45; the data being the same, except that H will become 5·02 nearly and $S'=3·02$ nearly. At that distance only the nearest hundredth can be read to, so the results cannot be made exactly to correspond with example 1; the difference of 0·1 in S′ makes a difference of ·1 ft. in B′.

$$A' = 100\ S'.\cos X + 1{\cdot}5$$
$$= 100 \times 0{\cdot}995 \times 3{\cdot}02 + 1{\cdot}5 = 302{\cdot}00$$
$$\text{(which is practically } A + l\text{)}$$
$$B' = (100\ S'.\cos X + 1{\cdot}5) \times \sin X$$
$$= 302 \times 0{\cdot}995 \times 0{\cdot}0996$$
$$= 30{\cdot}08$$
$$C = 100\ S'.\cos^2 X + 1{\cdot}5$$
$$= 302 \times 0{\cdot}995 \times 0{\cdot}995 + 1{\cdot}5$$
$$= 300{\cdot}5$$

It is true that by a specially constructed slide-rule, the multiplication by $\cos^2$ and by $\sin \times \cos$ is as simple as that by cos and sin; but beyond the limits of 300 feet with angles as steep as that in the example, the slide-rule will not give the result to a tenth of a foot in elevation, and therefore most of the turning sights and many others also where special accuracy is needed have to be reduced numerically, and then the extra labour of the second method is felt.

Usually a limit of accuracy is required of about one foot of elevation and ten feet of distance. This upon a scale of 4 miles horizontal and 100 feet vertical per inch would not be scaleable, whilst on scales of 400 feet horizontal and 40 feet vertical per inch they would represent ·025 inch, which is not a large amount, and *this accuracy can be attained by beginners.*

The field work performed with the tacheometer alone, apart from the auxiliary work of contouring and plane-tabling detail, consists of surveying and levelling, and though they are performed simultaneously we will consider them separately; and, first—

Surveying

The greater part of the measurements are independent rays whose bearings are read from the horizontal limb. Great care is needed to avoid misreadings, as there is but little opportunity for checking them. Some useful checks are practicable which will be presently described, but the chief

means of keeping the general direction true is the observation for azimuth described on p. 136, and a frequent comparison with the compass-needle.

When the distance of any point is measured by the stadia and its bearing also taken it can be plotted either by a protractor or by latitude and departure. Before commencing a survey the true meridian is acertained and some convenient reference-mark or 'directrix' chosen. Then the staff is carried to all the points in succession which are to be shown on the plan, and finally to the new instrument station, the observations to which are taken with extra care, and a check-sight taken on the directrix before shifting the instrument. It is advisable, if the instrument is fitted with two verniers to the horizontal limb, to remove one of the microscope 'readers' during all the observations so as to avoid getting on to the wrong vernier.

When adjusted at the new station, the horizontal limb is clamped to the same bearing, entered in the book from the first to the second station, and the telescope is directed by means of the external axis to the staff held at the first station. The reader is then transferred to the opposite vernier so as to give the bearing $\pm$ 180° and so replace the zero of the horizontal limb in correspondence with the meridian.

It may be mentioned that in some countries the south point is called 360° and the north point 180°.

If the bases are short it will be necessary to align the telescope from station to station by a plummet, because the staff being three or four inches wide will give a margin of error.

At the new station the staff is read a second time by the stadia as a backsight, so that all the primary lines of the traverse are measured twice over. *It must be borne in mind that the instrument cannot be carried forward as in levelling ahead of the staff to take a backsight.* At every fresh instrumental station the staff and instrument must change places. The station must be clearly marked. A good stout peg should

be driven down within an inch of the ground and a reference stake about three feet long driven a couple of yards away, having the number or letter of the station chalked on it. Where the intermediate staff stations are at defined points, such as corners of fences, it will be sufficient to describe them in the column for remark, but if they are undefined they should be marked by smaller stakes or builder's laths. A convenient notation on a continuous traverse is to name the instrument stations by the capital letters of the alphabet to the end, and then the small letters, after which it will be safe to begin again with the capitals. The staff stations can then be marked with the letter of the station from which their position has been first determined, and their number as A_1, B_{26}, and so on, a separate column being given to each. When a new instrument station is observed from an old one, or a backsight from a new one upon an old one, the reading of the staff is booked thus :

Inst. station	Staff station	Bearing
A	B	175°
B	A	355°

THE STADIA HAIRS

As has been already stated, when the vertical arc is used the lower stadia hair should be directed to an even foot so that the eye will not have the effort of endeavouring to read simultaneously three heights upon the staff. There is generally a slight movement of the staff which makes it very difficult to read unless the telescope is adjusted in this manner ; but by so doing the stadia are read at a glance, and for the intermediate stations it is not necessary to read the axial hair at all ; since the stadia hairs are equidistant from it, the mean of the two readings will be the reading of the axial hair. It is advisable, however, at the turning points to read the axial hair as a check.

Some writers advise directing the axial hair to the same

height on the staff as the height of the instrument above the ground or peg; the value of B or B′ will then be the difference of level of the two stations. This plan loses more time in observation than it saves in calculation.

Auxiliary Work

Especially with beginners there is a liability to make a misreading of the horizontal limb with the intermediate sights, and where the work is very particular it is advisable to let an assistant tape from point to point. Mistakes are generally in even degrees, which would at once be shown by the check lines. Taping also comes in very handily where instrumental sights are only practicable at long intervals.

In a topographical survey of a portion of Windward Hawaii, Sandwich Islands, the author adopted the following plan for the ravines. These great gorges, locally termed gulches, were sometimes a quarter of a mile wide, and the side slopes varied in height from 100 to 400 feet and in angle from a precipice to 20°. The growth of running shrubs, matted ferns, and ironwood was such that in one place it took two hours to obtain a single sight. Half an hour was a very frequent delay.

The instrument was set up on one side of the ravine and the staff-party remained on the other. They consisted of a staff-holder, clinometer man, and two axemen. All four had axes, and the first thing was to clear for a sight. When the first point was fixed, the slope was measured and a subsidiary tape-traverse was run just wherever the jungle was penetrable. The lengths varied from 10 to 50 feet and the total tape traverse sometimes extended to several hundred feet. Each tape measurement was aligned by the prismatic compass, levelled by the Abney level, and the slope taken up and down with the clinometer. As soon as the staff-party reached a place practicable for a clearing, all four

would fall to with the axes again and a fresh point would be thus determined from the instrument. The established points were first plotted, and then the tape traverse plotted on tracing paper and squared in with them ; the coincidence was generally very close, except where the magnetic deviation was great.

When the staff-party reached the limit of instrumental range, a fresh station was selected on the same side of the gulch by means of a backsight with a second staff taken to the station just left, and also check sights to any of the stations on the opposite side where the clearing had been sufficient to put in a flagpole visible from the new station. There was in this way a subsidiary triangulation by which to eliminate error, and great accuracy was obtained.

When one side of the gulch had been surveyed, the staff and instrument changed sides and the latter occupied the stations in the clearings, the staff party repeating their *modus operandi* on the other side of the gulch.

When the surveyor is in good practice with the tacheometer, he will very rarely make a misreading, but it *is always a very useful check to let the staff-holder carry a passometer (not a pedometer) and book it at every station ; this he can do while the sight is being taken without impeding the progress.*

Reduction of the Traverse

The method of latitude and departure has this advantage that protractor work is replaced by square scaling, and the position can be determined in the field at any time whether for the purpose of a check measurement to some known object or for the assistance of the observation for azimuth. When no tables or slide-rule are at hand it becomes a very tedious process and is not often resorted to.

Chambers's Mathematical Tables give difference of latitude and departure for every single degree and for each foot of distance up to 300. The actual distances of a traverse

rarely exceed 600 feet, so that the latitude and departure can be very readily booked from these tables, if close accuracy is not required, especially as it is generally possible to arrange the base lines so that they come for the most part on even degrees, by waving the staff-holder to the right or left until he comes to the nearest degree.

With the slide-rule it is done in the following manner :

First. Reduce the bearing to its equivalent azimuth by Table XVII. (which should also be pasted on to the slide-rule).

Second. Find the difference of the angle thus found from 90°, *i.e.* the complement.

Third. Place the initial point of the sine-scale under the distance on the rule, and read off the latitude opposite to the complementary angle and the departure opposite to the angle. With small angles it will give the nearest tenth, but with larger angles the nearest foot.

Example. Find latitude and departure of base line A △ in fieldbook, p. 182. Bearing 347·03°, dist. 258. (The distance must of course be taken as reduced to the horizontal, which is equal to dist. × cos vert. angle or else by Table XXIV.)

The azimuth will be 13·97° N.W. and its complement 76·03°. Place the left hand 1 of the sine-scale under the 258 of the rule, and opposite to angle 13·97° will be found 62·4 feet and opposite to 76°, 250 feet.

To reduce the difference of longitude and latitude thus found to true longitude and latitude from Greenwich by Tables XXV. and XXVI.

This is useful for astronomical observations and check points.

Example

Started on January 1 in long. 157·582° W.
„ „ lat. 21·45° N.
Total northings for the day's run 33,425 feet.
„ westings „ „ 17,295 „

Northings in miles by slide-rule 6·33, multiplied by $\frac{1}{1\cdot146}$ by slide-rule$=$5·52′ of latitude N.

Westings in miles by slide-rule$=$3·27; multiplied by $\frac{1}{1\cdot072}$ $=$3·01′ of longitude W.

Position on January 2:

Long. 157° 31·3′ + 3·01′ = 157° 34·31′ W.
Lat. 21° 27′ + 5·52′ = 21° 32·52′ N.

As the maritime and geographical positions are given in degrees, minutes, and seconds it is more convenient to retain the old notation for that purpose.

TABLE XXIV.[1] —*Difference between hypotenuse and base in feet per hundred feet of hypotenuse, for sloping ground.* (*Note.*—This should be marked on the vertical limb of the transit; on Y theodolites it usually is.)

Slope in degrees	Deduct. feet	Slope in degrees	Deduct. feet	Slope in degrees	Deduct. feet	Slope in degrees	Deduct. feet
¼	·001	¼	·420		1·596	¼	3·521
½	·004	½	·460		1·675	½	3·637
¾	·009	¾	·503		1·755	¾	3·754
	·015	6	·548	11	1·837	16	3·874
	·024	¼	·594		1·921	¼	3·995
	·034	½	·643		2·008	½	4·118
	·047	¾	·693		2·095	¾	4·243
	·061	7	·745	12	2·185	17	4·370
¼	·077	¼	·800	¼	2·277	¼	4·498
	·095	½	·856	½	2·370	½	4·628
	·115	¾	·913	¾	2·466	¾	4·760
	·137	8	·973	13	2·563	18	4·894
	·161	¼	1·035	¼	2·662	¼	5·030
½	·187	½	1·098	½	2·763	½	5·168
¾	·214	¾	1·164	¾	2·866	¾	5·307
4	·244	9	1·231	14	2·970	19	5·448
¼	·275	¼	1·300	¼	3·077		
½	·308	½	1·371	½	3·185		
¾	·343	¾	1·444	¾	3·295		
5	·381	10	1·519	15	3·407	20	6·031

[1] Copied from Trautwine's 'Pocket Book.'

TABLE XXV.—*Length of a minute of longitude in different latitudes at the level of the sea in statute miles of 5,280 feet.*

Degrees of latitude	1 minute longitude in miles	Degrees of latitude	1 minute longitude in miles	Degrees of latitude	1 minute longitude in miles	Degrees of latitude	1 minute longitude in miles
0	1·153	18	1·097	36	0·933	54	0·679
2	1·152	20	1·084	38	0·909	56	0·646
4	1·150	22	1·069	40	0·886	58	0·612
6	1·146	24	1·053	42	0·858	60	0·578
8	1·142	26	1·037	44	0·831	62	0·542
10	1·135	28	1·019	46	0·802	64	0·507
12	1·128	30	0·999	48	0·773	66	0·470
14	1·119	32	0·979	50	0·743	68	0·433
16	1·108	34	0·957	52	0·711	70	0·395

TABLE XXVI.—*Length of a minute of latitude in different latitudes at the level of the sea in statute miles of 5,280 feet.*

Degrees of latitude	1 minute latitude in miles	Degrees of latitude	1 minute latitude in miles	Degrees of latitude	1 minute latitude in miles
0	1·145	30	1·148	60	1·152
10	1·145	40	1·150	70	1·154
20	1·146	50	1·151	80	1·155

LEVELLING

The elevations of the staff-stations are obtained by first determining the elevation of the optical axis of the instrument from some bench-mark ; in other words the height of the centre of the trunnion of the telescope above any known or assumed datum. From this elevation the elevations of all succeeding intermediate staff-stations and the next instrument station are determined by calculating the vertical component of the direct distance, and adding to or deducting from it the height of the staff from the ground to where it is intercepted by the axial hair of the telescope.

Whether at the commencement or at a turning-point, the elevation of the optical axis is obtained by a backsight, and all other sights are foresights. The elevation of optical axis is henceforth termed O. A. to distinguish it from the

height of the instrument itself above the peg over which it stands, which will be called H. I.

This latter needs a separate column because it is an independent check. The elevation of instrumental station will be signified by E.I.S., and that of the staff station by E.S.S. The direct distance will be marked D., the horizontal component H. C., and the vertical component V. C.[1]

The Backsight

The elevation of optical axis, O. A., is obtained by the following formulæ :

a. When the vertical angle is one of elevation, which is termed *plus*, O.A.=E.S.S.+B.S.−V.C.

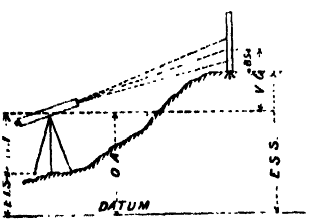


Fig. 46.

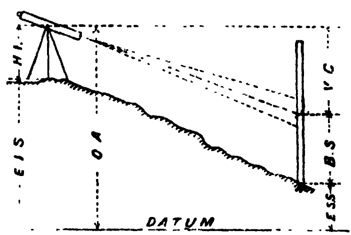


Fig. 47.

b. When the vertical angle is *minus*, O.A.=E.S.S.+B.S. +V.C.

The elevation of instrumental station E.I.S. is obtained in either case by deducting H.I. from O.A. It is needed as a check from the next station.

The Foresight

The elevation of any intermediate staff-station, or of the next instrumental station, is obtained by the following formulæ :

[1] See also remarks on adjustment of axial hair to same height as that of instrument, p. 174 at foot.

(*a*) When the vertical angle is plus, E.S.S.=O.A.+V.C. - F.S.

(*b*) When the vertical angle is minus, E.S.S.=O.A.−(V.C.+F.S.).

In these figures the staff is shown vertical, and the instrument not anallatic, but the rules apply to whatever method is adopted of obtaining V.C.

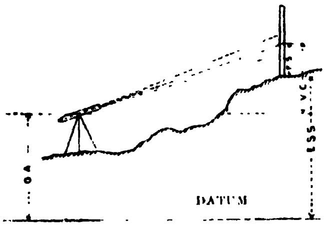


FIG. 48.

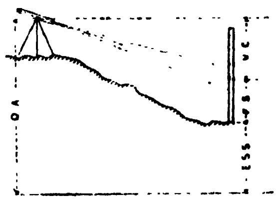


FIG. 49.

The first operation backsight A to ⁄⁀ is reduced thus

E. S. S. = 122·12
+ B.S. 5·28

127·40
− V. C. 22·34

O. A. = 105·06

The second intermediate sight A to A_1 thus :

O. A. = 105·06 1·63 V. C.
− (V. C. + F. S.) 4·39 2·76 + F. S.

E. S. S. 100·67

The foresight A to B, O. A. 105·06 11·96 V. C.
− (V.C. + F.S.) 17·48 5·52 + F. S.

E.S.S. 87·58

It should be remembered that there may be a fall with a plus angle when the staff-reading is greater than the V.C., but never a rise with a minus angle.

The last backsight here agrees exactly with the foresight,

Form of Fieldbook

$\frac{f}{i} = 100$; $f + d = 2$ *ft.*

I. S.	S. S.	Hor. limb	Vert. limb	Stadia	D.	H. C.	V. C.	E. I. S.	E. S. S.	H. I.	B. S.	F. S.	O. A.	Remarks
A	∧	347° 02′	4° 57′	6·57 4·00	259	258	22·34	100·00	122·12	5·06	5·28		105·06	M. S. 4½ from London, 122·12 feet above Ord. datum
	A_1	114° 28′	−0° 22′	3·52 1·00	254	254	1·63		100·67			2·76		
	A_2	66° 30′	+0° 5′	6·42 3·00	344	344	0·50		100·85			4·71		
	A_3	21° 32′	−0° 15′	4·11 1·00	313	313	1·37		101·14			2·55		
	B	163° 21′	−2 14′	7·05 4·00	307	307	11·96		87·58			5·52		
B	A	343° 21′	+2° 12⅓′	6·05 3·00	307	307	11·81	87·58	100·00	5·13	4·52		92·71	

but if there is a discrepancy exceeding the working limit of accuracy the sights should be repeated ; if they only differ by a small amount, such as one or two hundredths, for close levelling the mean should be taken for the O.A. of the new station. The accuracy of the levelling never can equal that of an ordinary levelling instrument, inasmuch as there is always a liability to error in the measurement of the distance, otherwise the accuracy would be the same. Error in the vertical angle is more or less present with the intermediate sights ; but, in the back- and foresights, it is eliminated by the precaution of making each observation in duplicate, face right and face left, and taking the mean angle. This is done by releasing the parallel plate, and rotating the telescope 180° ; then, revolving it in its trunnions, the measurement of the vertical angle in the reversed position will reverse the index error if any. This should be very little in a day's use of the instrument, if it has been properly adjusted in the morning as explained on p. 313.

The importance of this operation for eliminating index error cannot be overstated. It should never be neglected, however rapid the rate of march.

Thus, if the vertical angle from A to B was 2° 13′ 20″ when the vertical circle faced to the right, and in the reversed position 2° 14′ 40″, the true vertical angle would be 2° 14′.

The value of the tacheometer for long sights and great differences of level is a cardinal point, but it must be remembered that the accuracy is in opposite ratio both to the distance and the height. The writer has obtained a difference of elevation exceeding a hundred feet in one shot *exactly*, the oblique distance being between 300 and 400 feet, but that cannot be relied upon with a tacheometer any more than long shots with the level. The surveyor should therefore keep to short sights for the centre-line of his survey if running a railway traverse, but the outlying points, such as fences, &c., can be put in with longer sights.

In working out intermediate sights by slide-rule, or in checking the back- and foresights, the following short table will serve to show where the decimal point will come in the result; for instance, in the backsight A ⟋⟍ the angle is nearly 5°, and the distance nearly 250, so it will be at once seen that the result on the slide-rule is 22·34, and not 2·23 or 223·4. The list of multipliers and other factors on the back of the Mannheim rule is not of much use to surveyors, so it is best to paste them over and write a table such as this one; the rules for back- and foresights on pp. 180, 181; the table of square-roots of small decimals given on p. 250: and some others in this book, in the place of them.

TABLE XXVII.—*Sines multiplied by various distances.*

Degrees	Degrees, minutes, and seconds	Sine angle multipled by 50	Sine angle multiplied by 250	Sine angle multiplied by 750
·01	0° 0′ 36″	·0087	·043	·13
·05	0° 3′ 0″	·0436	·22	·65
·10	0° 6′ 0″	·0872	·44	1·31
·20	0° 12′ 0″	·174	·87	2·62
·30	0° 18′ 0″	·262	1·31	3·93
·50	0° 30′ 0″	·436	2·18	6·55
·75	0° 45′ 0″	·65	3·27	9·82
1·00	1° 0′ 0″	·87	4·36	13·09
3·00	3° 0′ 0″	2·61	13·07	39·22
5·00	5° 0′ 0″	4·36	21·80	65·37
7·00	7° 0′ 0″	6·09	30·47	91·40
10·00	10° 0′ 0″	8·68	43·41	130·23
15·00	15° 0′ 0″	12·94	64·71	194·13
20·00	20° 0′ 0″	17·10	85·50	256·51
25·00	25° 0′ 0″	21·13	105·65	—
30·00	30° 0′ 0″	25·00	125·00	—
35·00	35° 0′ 0′	28·68	143·39	—
40·00	40° 0′ 0″	32·14	160·69	—
45·00	45° 0′ 0″	35·36	176·78	
50·00	50° 0′ 0″	38·28	191·41	
55·00	55° 0′ 0″	40·96	204·79	—
60·00	60° 0′ 0″	43·30	216·50	—

Referring back to the two first sights in fieldbook, p. 182, to multiply sin 4° 57′ by 250. Place a 1 of the sine-scale

under 259 of the rule, then opposite to 4° 57′ will be found 223, which by reference to Table XXVII. is seen to mean 22·3. The slide-rule will only give the nearest tenth, but as it is a backsight it ought to be worked out from a table of sines and tangents. If in the office, Crelle's tables, which give at a single inspection the products of three figures by three figures will be found of great assistance, and this is as close as it is necessary to go. For intermediate sight AA_1, to multiply sin 22′ by 254. Place the fiducial mark for minutes on the number scale of the slide, under the 22 of the rule. Then opposite to 254 on the slide will be found 163, which by inspection of the table will be at once identified as 1·63 feet.

If the elevations are required in metres, the staff must be graduated also in metres, but the operation is the same.

Contouring

The elevations of suitable points having been determined in the foregoing manner, the intervening ground may be topographically represented by dotted lines of equal elevation called contours, in precisely the same manner as described on p. 61. It is well to distinguish contours which are drawn between two fixed points from those which only depend upon a slope taken from one point, the former being naturally more reliable when the distances are not great.

Profile

The profile, as it is called in America, or section, as termed in England, is produced from the contours, which are drawn sufficiently close to enable the elevation of each 100 feet station to be judged to the nearest foot by the eye. The gradients are first fixed approximately. In Fig. 50 for instance, a suitable place was found about a mile up the gulch for a horseshoe curve. Crossing the river at a suitable height would leave a gradient of about 1 per cent.

falling on both sides, so as to meet the rising bottom of the gulch. The next operation was then to mark off at say every 500 feet the position where the gradient intersects the surface. Then with railway sweeps a trial line is drawn to pass as nearly as possible through these points, having due regard to curvature. In a case like this, where the line is located on the side of a hill, the whole of the line must be thrown into cutting, except where the slope is flat enough to admit of embankment.

It is nearly always cheaper to take out a cutting, even in rock, than to build up a retaining wall. Where this latter course becomes necessary, it is usual to bench the hillside and build up cross-logging, jointed at the ends with axe-cuts, termed in America cribwork. In other places it is found cheaper and otherwise preferable to build a wall of dry-rubble, but, apart from the question of expense, these erections are liable to destruction from wash-outs and decay, and a little extra expense in cutting is more economical in the long run.

When the location is on tolerably even country, the trial line should aim at equalising the cuttings and embankments, making them all as short as possible. When the line is on level ground subject to floods, it should be placed entirely in embankment, so as to keep it above highest known flood-level, with frequent openings for letting off the flood waters.

The details of curvature should all be written up on the plan, for which see p. 217.

In plotting the profile, one man should take the plan, with contours and trial line drawn on, and another man the ruled profile-paper, which dispenses with all scaling. The first then reads out the hundred-foot stations, and if necessary, intermediate points with their elevations, and the other plots them. Thus: station 200, elevation 354; station 201, elevation 353; station 202, elevation 350½; station 202 + 10, 350, and so on.

Fig. 50 is a sketch from memory of one of the Hawaiian

gulches, and is merely intended as an illustration of the method. The scales are sufficiently given by the centre line of the location and the profile. The sharp curve of 40°

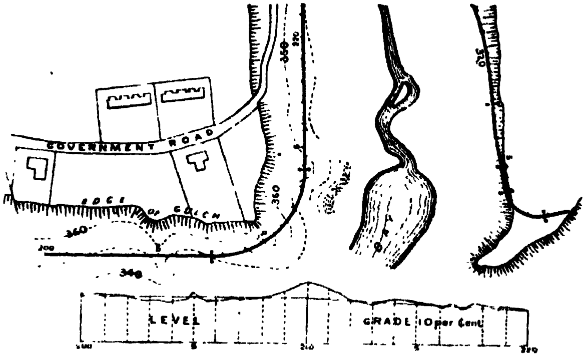


FIG. 50.

(146·2′ rad.) on the right of the figure was frequently resorted to either to turn the head of a gulch, or to cut through a rock-spur as shown.

TACHEOMETRIC CURVE-RANGING

Curve-ranging is performed by tangential angles with the tacheometer on the same principle as by the transit and

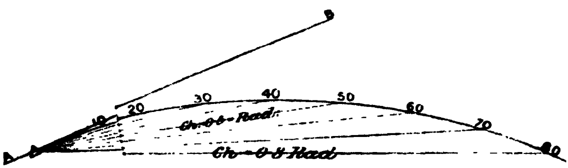


FIG. 51.

chain, with the exception. Measurements by the chain are necessarily repetitions of a short chord round the arc of a circle, whereas measurements with the tacheometer are each of them an independent chord, as shown on Fig. 51, where

A is the B.C. (beginning of curve) and AB the tangent produced; A_{10} A_{20} A_{30}, &c., are each of them rays from the instrument, which may be either chords forming equidistant points upon the curve, in which case every radius demands a different series of tangential angles, or else they may be in decimal parts of the radius, which is the method shown on Fig. 51. Whatever the curve may be, points upon it can be at once determined from the following table.

TABLE XXVIII.—*Tangential angles for chords with radius* = 100 *For tacheometric curve-ranging.*

Chord	Tangential angle in degrees	Chord	Tangential angle in degrees	Chord	Tangential angle in degrees	Chord	Tangential angle in degrees	Chord	Tangential angle in degrees
1	0·28	21	6·03	41	11·83	61	17·76	81	[illegible]·89
2	0·57	22	6·32	42	12·12	62	18·06	82	[illegible]4·21
3	0·86	23	6·60	43	12·42	63	18·36	83	[illegible]4·52
4	1·15	24	6·89	44	12·71	64	18·67	84	[illegible]4·83
5	1·35	25	7·18	45	13·00	65	18·97	85	[illegible]5·15
6	1·72	26	7·47	46	13·30	66	19·27	86	25·47
7	2·01	27	7·76	47	13·59	67	19·57	87	25·78
8	2·29	28	8·05	48	13·89	68	19·87	88	26·13
9	2·58	29	8·33	49	14·28	69	20·18	89	26·42
10	2·87	30	8·62	50	14·48	70	20·49	90	26·74
11	3·15	31	8·92	51	14·77	71	20·79	91	27·07
12	3·44	32	9·21	52	15·07	72	21·10	92	27·39
13	3·73	33	9·50	53	15·37	73	21·41	9[illegible]	27·71
14	4·02	34	9·78	54	15·67	74	21·72	9[illegible]	28·03
15	4·30	35	10·07	55	15·96	75	22·02	9[illegible]	28·36
16	4·59	36	10·37	56	16·26	76	22·33	[illegible]	28·68
17	4·88	37	10·66	57	16·56	77	22·64	[illegible]	29·02
18	5·17	38	10·95	58	16·86	78	22·96	[illegible]	29·33
19	5·45	39	11·24	59	17·16	79	23·27	[illegible]	29·67
20	5·74	40	11·53	60	17·46	80	23·58	[illegible]	30·00

Let us suppose that the bearing of AB is 360 and we wish to set out a curve of 5°, that is of 1,146ft. rad in chords of about 100ft. If 114·6ft. is near enough, will take the tens and begin with 2°·87, but if it be necessary to come closer than that, we should take 9, 18, 27, & and begin with 2·58°, which would give us for a first chord 103·14ft. The one is as simple as the other by the slide rule.

Let us suppose that we take the 9. Place the right hand 1 of the slide under the 1,146 of the rule, and read off the first dist. 103ft. under the 9, and set the instrument at 2°·58. Put the staff-holder in line, and make him take 37 ordinary steps forward. Keeping him in line wave him back or forward by the Morse 'B' or 'F' signal until his staff records the distance by the stadia-hairs, and then put a stake in. Clamp the instrument at 5°·17, and read off the distance opposite to 18 on the slide-rule=206ft. The sub-chords A–10, 10–20, 20–30, 30–40, &c., lengthen as the curve is ranged, but that is quite immaterial for a preliminary survey. If A–10 were 103, 70–80 would be 109ft., and 60–70 would be 107½.

Any labourer of average sense will learn to step each distance within a yard, and the time occupied in ranging him in line and moving him back or forward is no longer than that of the ordinary curve-ranging.

This method is quite suitable for ranging the permanent centre-line of a railway where great precision is not required.

By leaving a mark at A the instrument can be shifted to any one of the points on the curve, and directed to the tangent. For instance, if it were desired to set up the instrument on the tangent at point 80, of which the tangential angle is 23°·58, we should clamp the parallel plates at 23°·58, and, reversing the telescope, direct it by means of the external axis on A. Then, reversing it back again with the external axis clamped, release the parallel plates, and set the vernier at 47°·16, which is twice the tangential angle, or in other words the angle of deflection, and the instrument would then be on the tangent.

It is even more advantageous to keep the curve-ranging on true astronomical bearings when on preliminary survey than when on construction, but the manner of doing so is considered in the chapter on curve-ranging.

Method of Ranging a Curve to Finish at a given Alignment without Shifting the Instrument [1]

It frequently happens that the chaining of a trial line in order to obtain the topography sufficiently to locate the best curve would involve a great deal of labour, and would run a very small chance of being the final one. It is possible without shifting the instrument to locate a curve upon the ground which will pass through any given points and terminate on a given alignment without chaining. Let us suppose that a tangent has ended upon the edge of a ravine, and it is desired to lay out a horse-shoe curve which will follow the slope of the hill; turn the head of the valley with a trestle, and, skirting the opposite hillside, terminate in a parallel to a boundary of some property which could not be interfered with except at considerable cost. Any other circumstances may fix the direction of the final tangent. If there are no obstructions such as the one in the figure, the tangent will be chosen to suit the ground, and in the nearest direction practicable to the objective point of the railway.

The surveyor-in-chief will leave an assistant at the transit, and proceed to the neighbourhood of B. He will then fix upon a suitable direction for the final tangent BY, and measure its magnetic bearing with the pocket altazimuth described on p. 321, or by an ordinary prismatic compass. Knowing the variation of the compass by previous observations, he then reduces the bearing to an astronomical bearing, and telegraphs it to the transit man by means of a flag. See Flag-signals (p. 93). The transit-man then works out the deflection angle D, and tangential angle T, which is equal to the angle XAB, and setting his instrument to AB, gives the surveyor-in-chief the E.C. point B. It is

[1] Before perusing the following, the student should read the chapter on curve-ranging.

possible that B may fall considerably more to one side than was intended, so that it would be necessary to prolong or shorten the tangent at A. It is possible also that, after locating D, it may not be the best place for the trestle

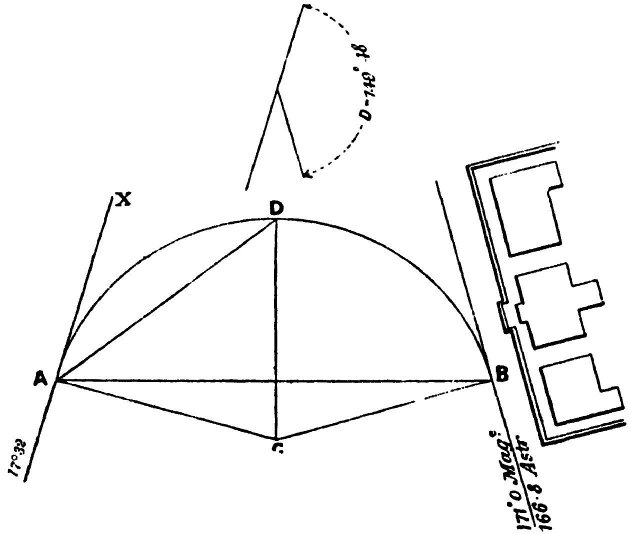


FIG. 52.

Preliminary Calculations and Formulæ.

$$AB = 700 \text{ feet.}$$
$$D = 149{\cdot}48 = ACB.$$
$$T = 74{\cdot}74 = BAX.$$
$$\text{Rad} = AC = \frac{AB}{2 \sin T} = 362{\cdot}8 \text{ feet}$$
$$\frac{T}{2} = XAD = 37{\cdot}37^\circ$$
$$AD = \frac{AB}{2 \cos \frac{T}{2}} = 440{\cdot}4 \text{ feet}$$
$$AB = 2R \sin T$$

Neither of these eventualities need necessitate a change in the direction of BY, but merely in the position of A and B, and the radius of the curve. Assuming that B is a suitable point for the E.C., the staff is held there, and the

transit-man measures the length of AB by the stadia, and telegraphs it. He then works out the radius and AD by above formulæ and telegraphs them. The surveyor-in-chief then considers whether the radius is a suitable one, or whether, by moving his final tangent a little, he can obtain either a longer radius, or one which corresponds to a more simple degree of curve, or one which is on better ground, and, if necessary, telegraphs back the radius which he desires. In the illustration, the radius being 362·8 feet, and a 16° curve having a radius of 359·3 feet, he would telegraph back simply 16, and place his staff for fresh alignment about 7 feet nearer to the transit. The transit-man will then work out AB afresh $= 2 \times 359{\cdot}3 \times \sin T =$ 693·3 feet, and AD $= 436{\cdot}2$. The chief surveyor will then first drive a stake at the E.C., and then, proceeding to D, will get his alignment and distance from the transit-man, and, if suitable, drive another stake at D marked apex.

It may be advisable to know the difference of level between A and D, to determine the height of the trestle before finally selecting D. This the transit-man will do by a foresight, as described on p. 176, and if necessary the chief surveyor will take one or more trial points above or below D. Since he is possessed of all the data of the curve, he can with the slide-rule plot the curve on a sketchboard and alternative curves above or below D, if he pleases.

When the E.C. and apex points have been satisfactorily fixed, and stakes driven, the chief surveyor returns to the transit, during which time the transit man has worked out the data of the curve, and is ready to range it in the method described on p. 183.

The particulars of this fieldbook will be better understood after reading the chapter on curve-ranging. The column 'chainage of,' is so marked for simplicity, although the chain is not actually used. The columns of subtangent and apex distance are not filled in because, the curve not being run to intersection, it is needless to calculate them.

Description of Curve.

Total angle of deflection		Astronomical bearing				Degree	Radius	Subtangent	Length	Apex distance	Chainage of		
Right	Left	Old tangent	Apex point	Chord	New tangent						B. C.	Apex	E. C.
deg.	deg.	deg.	deg.	deg.	deg.		feet	feet	feet	feet			
–	149·48	17·32	54·69	92·06	166·80	16	359·3	—	934·3	—	113 +24	117 +91·1	122 +58·3

But little is said in Chapter IX. about the various types of tacheometer which are before the public, many of them well worth description, and but little has been said here about the method of using the stadia-rod vertically. The formulæ for $\cos^2$ and $\cos \times \sin$ are only tabulated for the centesimal degree in a somewhat bulky form, published by Cuartero of Madrid; they can be obtained through Messrs. Troughton & Sims. The method given in these pages requires nothing more than an ordinary table of sines and cosines, and the writer has proved its accuracy by considerable use of it. Upon the Hawaiian survey, a section of seventy miles of railway included a little over one hundred ravines of sizes varying from a hundred feet wide by fifty deep, to a quarter of a mile wide by four hundred feet deep. The intermediate survey on level ground between the ravines was made with the transit, chain, and level, by a separate party in the usual manner. The large ravines were all surveyed with the tacheometer, and frequently where two ravines were close together the intervening ground was also surveyed and afterwards checked with the results obtained by the chain party. In many cases the one party used the stakes of the other as reference marks both for distance and elevation, and the accuracy of the stadia work was on the whole decidedly greater than that of the chaining. The space of this work does not admit of dwelling exhaustively upon many different

methods of doing the same thing; the one given here is the simplest.

The following are the conclusions of Mr. Lyman, in his paper, read before the Franklin Institute in 1868, on the experience gained by him from the Schuylkill topographical survey, United States of America.

1. That the additional lenses and complications adopted by Porro, to cause the centre of anallatism to fall in the exact vertical axis of the instrument, were needless, as the inconvenience from adding to every distance observed a constant quantity equal to the sum of the focal distance of the object-glass, and the distance of that glass in front of the vertical axis, rarely exceeding 1 foot, is comparatively trifling.

2. That three horizontal hairs or wires are sufficient for all purposes, as the reading of the middle wire affords sufficient check on the other two; that fixed are preferable to movable wires with protected adjusting screws; and that the fixed wires should be so set that the visible height on the staff intercepted between the middle and either outer wire should bear some exact ratio to the distance such as 1 foot to 100, thus avoiding calculation.

3. That the staff should be graduated to hundredths of a foot.

4. That in combination with the above arrangement, a telescope magnifying only twenty times and reading to the 200th of a foot at a distance of 660 feet, will produce results as correct as those of Porro's larger and more complicated instrument.

5. That the errors arising from spherical aberration may under these circumstances be neglected in angles of less than 10° on either side of the focal axis.

The only point in which the writer's views differ from those of Mr. Lyman is in the power of the telescope necessary to produce the degree of accuracy named: $\frac{1}{200}$ of a foot at 660 feet distance. That he maintains is only possible with double the power specified by Mr. Lyman.

THE PLANE-TABLE AND STADIA

The plane-table itself is described at p. 319 *et seq.*, but when used in combination with a distance-measuring telescope it becomes an instrument of much greater value. The simple sight-rule is developed into a heavy brass ruler termed an alidade, to which is attached a telescope usually of 10 inches focal length, furnished with stadia hairs. Instead of having to fill in detail with taping, pacing, and sketching, it is all done with the stadia telescope. The main triangulation still forms the basis of the work, and occasionally the stadia work itself is checked by supplementary triangulation.

The levelling is done in the same way as with the tacheometer. The essential difference between the two methods consists in the absence of a graduated horizontal limb in the alidade, so that when traversing, the accuracy depends upon the back and forward ray with the alidade, which are transferred directly to the map. The transit and stadia are superseding the plane-table and stadia in America. The reasons for the writer's preference for the plane-table as an auxiliary and not as a universal instrument have been stated on p. 35, but it may be added here—

1. The plane-table is not equal to the tacheometer in wooded country, because triangulation is often impracticable.
2. It is not so suitable where only short bases are possible.
3. It is not suitable to rainy country.
4. It is not suitable for putting in railway curves.
5. It is more awkward to handle.
6. It is not so well adapted to astronomical observations.

An engineer does not want to over-burden himself with instruments, but seeks a maximum of *efficiency*, *portability*, and *economy*. The tacheometer will do all that the plane-

table used as a universal instrument will do; it will do most things better, and the peculiar cases in which the plane-table is superior are not of sufficiently frequent occurrence to a man in general practice to make him choose the instrument.

RATE OF EXECUTION

The Hawaiian survey previously alluded to was an example of as rough a piece of country as could be found anywhere. The average rate of progress was under half a mile per diem by two parties, one with the tacheometer and the other with transit and chain. The work included a belt of from 500 to 1,500 feet wide, a hundred ravines of more or less size, and seventeen plantation villages; the whole being mapped to a scale of 100 feet per inch in order to show the contours at every five feet on the steep side-hill. The ravines were densely wooded, and most of the cultivated land was covered with sugar-cane standing eight to ten feet high.

The greatest amount of work done in one day with the tacheometer alone was two miles. In that case the country was cleared, the cane was short, there were only two short ravines and one small settlement of about a dozen cottages.

The little survey of Greatness Mill is intended to show the convenient manner of doing an awkward piece of work by the tacheometer. Everything except building detail was put in from the three stations A, B, and C, and in a few hours. The buildings were afterwards plotted on a tracing with the sketchboard plane-table. Every check-line taken tallied exactly. The profile of the road was obtained from the sights taken to survey it. A photograph was taken at the same time somewhat in the direction of AB. In the case of a railway being made through such a spot as that, the information obtained would show—

1. From the survey a depression of ground calling for a crossing by bridge or viaduct.

2. The narrowest part of the water for a crossing.

3. By the photograph the nature of the property to be interfered with.

Mr. William Bell Dawson surveyed with the tacheometer and one assistant an area of 180 square miles, including one hundred lakes from seven miles long downwards, in five months. The object was to produce a map of the gold

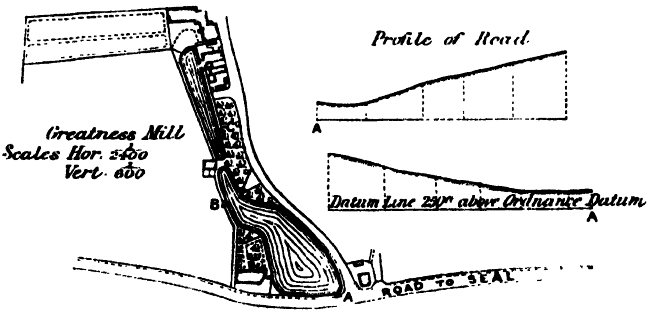


FIG. 53.

fields on the Atlantic coast of Nova Scotia. The traverses generally closed within an area of 20 to 30 feet radius, but closing errors were further eliminated by independent checks. The map was on a scale of two miles to the inch. He used a 6-inch theodolite with stadia hairs and a Rochon micrometer. The total cost was about 3*l.* 11*s.* 6*d.* per square mile.

A level should always be furnished with stadia hairs, and if it has a compass also contouring and small surveys can be often done with it of sufficient accuracy for the purpose, and so save bringing out the larger instrument.

CHAPTER VI

CHAIN-SURVEYING

THIS department of survey practice will be only treated in a condensed form to serve as a reference for the preliminary surveyor. In exceptional cases it becomes expedient to make a survey, measure an acreage, or range a curve with the chain or tape alone, or at most with the assistance of an optical square or cross-staff. The most frequent of such cases is where the time of the tacheometer party cannot be profitably employed on a solitary piece of detail, and one or two hands are told off to fill it in with the chain only.

A short description will also be given of the methods in vogue in America for running trial lines for railway location with the transit and chain, together with a few notes on curve-ranging with the chain only, reserving for the next chapter the subject of curve-ranging with the theodolite.

CHAINING

If chaining could be carried out with rigorous exactness, it would be possible to perform the whole survey with no instruments whatever, although it would be a lengthy affair, especially when the afterwork of levelling over the staked survey is considered. But there are also many essential difficulties which tend to produce error, and render chain surveys unsatisfactory.

It is just because chaining seems so very simple that it

is often relegated to inferior assistants, who do not study a systematic method of performing their work, and errors frequently arise from the following causes:

First. The liability of a poor chainman to 'drop a chain,' that is, to make a mis-count with the arrows.

Secondly. The tendency to measure on sloping ground without making the proper deductions.

Thirdly. The liability of the chain to being stretched by tension, or to expansion and contraction by variations of temperature, all of which require regular attention, not always devoted to them.

Fourthly. From twists in the map, due to incorrect tie-lines at the angles of the base lines.

Fifthly. From confusion in the fieldbook.

On a level piece of ground, where the plot is drawn to a large scale, such as 20 or 30 feet to the inch, the survey should be made with the transit and chain, as the maximum error should not exceed three inches. The tacheometer will not give this exactitude, and the chain alone is almost sure to develop twists. A survey of this kind, such as a town-district or a railway terminal depôt, is out of our province to describe. The main distinguishing features about it are: First, a network of triangulation wherever it is possible to chain bases, or, combined with triangulation, a closed traverse. Second, a system of ordinates or offsets from the bases thus fixed to all the points required to be plotted.

The same operation can be performed with the surveyor's compass, but the theodolite is much more precise in the angular measurement.

There are two or three methods of counting the chainage and it is not of much consequence which system is used, but it is most important that the same system should be rigidly adhered to. The following method is recommended. The leader carries a ranging rod, and the ten arrows, with the forward end of the chain. The follower carries the hinder end of the chain and a clinometer; he also has a tape

attached to his button-hole. The ranging rod has a handkerchief or piece of paper attached to it at the height of the follower's eye.

The leader drags forward the chain to the follower's direction, and hauls it taut.

The follower shakes it up to get rid of kinks and curvature.

The leader holds up the ranging rod, to get the final direction, and when fixed, puts the iron point through the handle of the chain to give the last stretch, and then puts in an arrow, and goes ahead. When he has put in his last arrow he runs out the next chain without an arrow, aligns it by the rod, and, leaving the iron point through the handle of the chain, signals to the follower to come forward with the ten arrows.

The follower first ties a knot in his tape to mark the ten chains, and then comes on, leaving the chain to be dragged forward by the leader when he has received his arrows. The leader then puts in one of the ten arrows at the rod, and proceeds as before.

When the slope exceeds a degree (or, if the greatest accuracy is required, at every chain) the follower books the slope, and at the end of the line sums up the corrections and deducts the result from the chainage. He stands erect and directs the clinometer to the mark on the ranging rod.

Measuring by short lengths on a side-slope is very unsatisfactory; it is difficult to keep the chain level, and still more difficult to plumb down exactly from the high end of the chain when rapidity is an object.

A table is given at p. 174 from which the deductions from the chainage on sloping ground can be readily made for any given angle.

Setting out a Square

Let it be intended to lay off BD perpendicular to AC with the chain alone. Measure off AB = 30 feet. Take the

10 mark on the chain to A and the opposite end of the chain to B. Put an arrow through the 40 mark nearest to B. Draw out the chain and shake it taut, so that the BD line

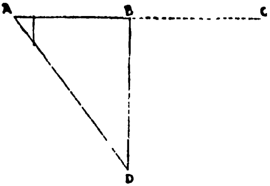


FIG. 54.

which is equal to 40 feet, and the AD which is 50 feet, shall each be perfectly straight; then because $3^2+4^2=5^2$ ABD is a right angle. Links of a Gunter's chain will of course do just as well, and any multiples of 3, 4, and 5, such as, 6, 8, and 10; 9, 12, and 15: or 45, 60, and 75.

To Measure the Deflection Distance of a New Base Line

This is a problem of curve-ranging, but is needed also in all cases where closed triangles cannot be formed.

Let it be required to find the deflection distance of BD from BC. Measure as long a base B*b* as possible, align *b* correctly with AB, set off *bd* square and measure it. If possible choose 100 feet or 200 feet for B*b*, put in a peg

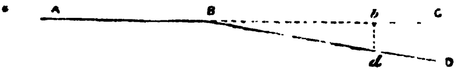


FIG. 55.

with a nail at *b*, and measure *bd* with a steel tape. BD can then be plotted direct from these data, or else the angle can be calculated thus: $\tan \text{CBD} = \dfrac{db}{\text{B}b}$. The entry in the

fieldbook is marked R or L, according as the new base line turns right or left.

ANOTHER METHOD

When the angle of deflection is great, it is better to make the tie-line by the chord of the angle. Measure B*b* as long as possible and also B*d* equal to it. The fieldbook should

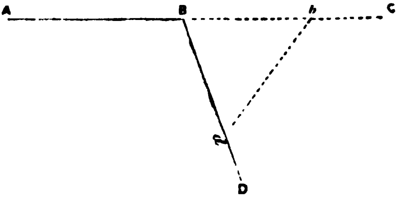


FIG. 56.

show by a sketch how the tie-line has been taken. The distance *bd* should be measured with a steel tape, and the base can be plotted from the data, or else

$$\sin \tfrac{1}{2}\text{CBD} = \frac{bd}{2\text{B}b}$$

In making a triangulation survey the aim should be to get as long bases as possible, and the triangles 'well-con-

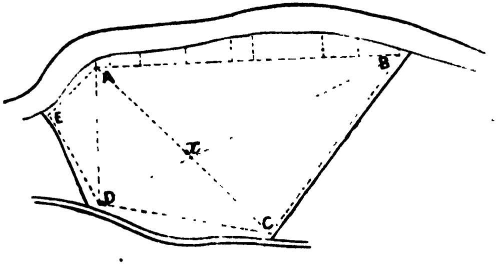


FIG. 57.

ditioned,' that is approximating to equilaterals. When the interior of the survey is partly inaccessible this is impossible,

and recourse has to be had to external ties like the above. When the triangulation can be made internally, a cross-staff is of great assistance. The handiest form is a brass octagon fitted with hair sights in slits. It stands on a short pole with an iron shoe, and only costs from 5*s.* to 10*s.* The

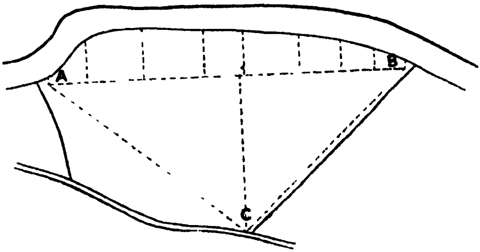


FIG. 58.

advantage of the cross-staff will be seen by comparing Figs. 57 and 58. It enables the offsets to be made up to 100 feet with the tape, and so diminishes the base lines.

The fieldbook is arranged with a central column for distances measured along the base lines, and on the right and left are the offsets opposite to the distances at which they are taken, with remarks to identify them. The field-book is commenced at the bottom of the page.

Left offsets		Base	Right offsets
		B	
	18	716	
	49	655	
	68	587	
	80	505	
	98	392	
		382	330 to C.
	98	312	
	105	190	
	85	80	
To river	17 From	A	

Acreage

The area is divided up into triangles and trapezoids. The area of the triangles is obtained by either of the two following formulæ

Rule 1 To find the area of a triangle from the lengths of the three sides, a useful method when no perpendicular can be measured across the triangle

Let a, b, c be the sides, and s=their half sum, A the

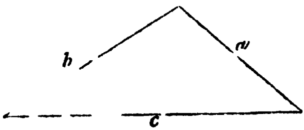


Fig 59

area in square measure of the unit adopted, such as square feet area for lineal feet measurement of the sides

$$A = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}$$

or logarithmically

$$A = \tfrac{1}{2}[\log s + \log(s-a) + \log(s-b) + (\log s-c)]$$

Rule 2 To find the area of a triangle when any one of the sides and the perpendicular from it to the opposite angle are given

Let B=base and P=perpendicular area$=\frac{B \times P}{2}$

The area of a trapezoid is obtained as follows

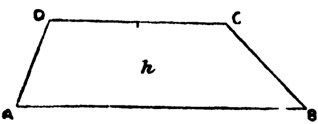


Fig 60

Rule 3 Multiply the sum of the parallel sides by the

perpendicular distance between them, and half the product is the area.

$$A=\frac{(AB+CD)\times h}{2}$$

or logarithmically

$$\text{Log } 2\,A=\log(AB+CD)+\log h$$

To reduce areas in Gunter's chains to acres.

10 square chains = 1 acre
100,000 square links = 1 acre

To reduce square feet to acreage.

43,560 square feet = 1 acre
1 acre = 4 roods
1 rood = 40 perches

PRELIMINARY WORK WITH TRANSIT AND CHAIN

In running trial lines with the transit and chain, the curves, unless exceptionally sharp, are not put in. The base lines are measured, and staked from intersection to intersection; the angles measured, and the stakes afterwards levelled over. The vertical arc of the transit is not used unless when on a steep gradient it is desired to follow the required inclination, in which case the vertical arc is clamped to the corresponding angle, and the line run as nearly as possible to it.

In America the levelling staff is called a rod, and the ranging rod is called a picket. In running a line there are generally two picket men, sometimes only one. The transit-man sends out a picket-man in the direction which he intends to take, aligns the picket, and takes a reference sight to some good fiducial point.

There are two chain-men, leader and follower, and one or more axe-men, who prepare stakes about three feet long, pointed at one end, and shaped on the other for a chalk mark.

The transit-man aligns the chaining by the leader's picket, and the axe-men put in a stake at every hundred feet, or at shorter distances where necessary.

The chief engineer generally accompanies the chaining party in order to select the ground.

When the end of the base is reached, the picket-man, leaving a peg at the end of the base, returns to the instrument, and changes places with the transit-man to enable the latter to range out a new base.

There are three methods in use for registering the deflection of the one base from the other.

The first is the most common—see Fig. 61. It takes no

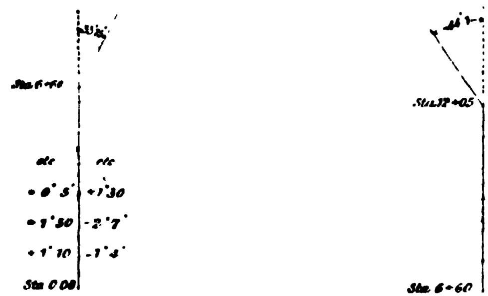


FIG. 61.

account of the astronomical meridian and only uses the compass-needle as an occasional check.

The field notes will explain themselves with scarcely any description. The instrument is set back to zero every time; the slopes of the ground, right or left, are either taken with the vertical arc of the transit or by the clinometer.

Method 2—see Fig. 62. The second method is by working out the magnetic or astronomical azimuths. If the former, it is sufficient to commence by making the zero of the horizontal limb coincide with the position of the needle when at rest. If the latter, the meridian must first be determined by one of the methods mentioned in Chapter IV.

Almost all transits are graduated from 0 to 360, so that this method entails the reduction of the angle as explained on p. 59; it is, however, very well suited to plotting by latitude and departure.

Method 3. By bearings from 0 to 360—see Fig. 63. These are now also commonly termed azimuths, though not, strictly speaking, azimuths unless taken from south as well as north point. Some distinctive term is necessary for these bearings. 'Northerly bearing' would express the first idea, but then it needs to be defined as to whether it is astronomical or

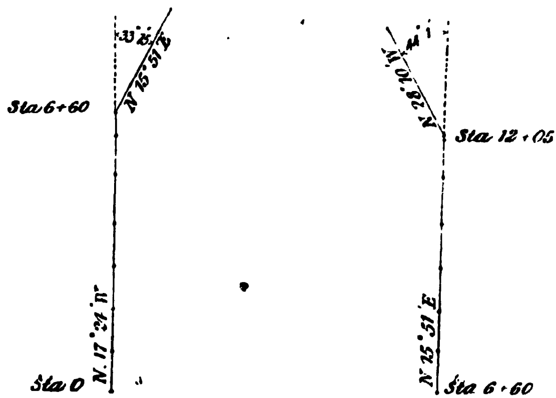


Fig. 62.

magnetic bearing, which would be expressed by N. Ast. Bearing, or N. Mag. Bearing.

If we are furthermore to be visited with a centesimal graduation in competition with the old graduation, we should require another letter or two of the alphabet to define our *modus operandi*. If the term 'course' could by universal consent be relegated to the magnetic, and 'bearing' reserved for the astronomical direction, both being kept solely for graduation from 0 to 360, whilst the term 'azimuth' was confined to what it strictly means, it would

simplify nomenclature ; but it is to be feared that the sailors would not fall in with the proposal.

It is not necessary either in Methods 2 or 3 to measure the angles of deflection separately. They are only inserted here in order to compare one method with the other. The transit is fitted with two verniers, and when making a deviation the instrument is reversed and directed to the previous station, one of the verniers being set at the bearing of the line just run. This is called the backsight. The instrument is then again reversed so as to point forwards, and, unclamp-

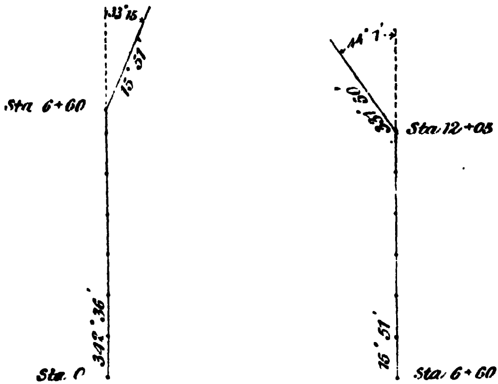


FIG. 63.

ing the parallel plate, directed along the new base-line to the picket fixed at the end of it. The bearing is then read by the other vernier, and booked.

When two 'readers' are provided, it is advisable to use only one, and at each change of direction shift it over to the other vernier. Before commencing work, it is of course necessary to see that the line of sight is in true correspondence with the zero of the horizontal limb as explained in Chapter IX. It is also necessary to have an instrument in which the graduation is correct, so that the

readings of the two verniers differ by 180°, or in the centesimal instrument by 200. When this is so, using the two verniers forms a check upon the readings.

The slopes are put in by all three methods as shown on Fig. 61.

CIRCULAR CURVE-RANGING WITH THE CHAIN

Four methods will be briefly dwelt upon.

Method 1. Kröhnke's tangential system.

This is a scheme for obtaining equidistant points upon a curve by laying out abscissæ BB′, BB″, &c., along the

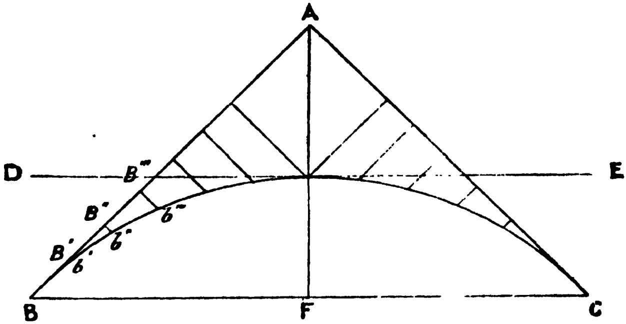


FIG. 64.

subtangent of the curve and setting off ordinates B′ b′, B″ b″, &c., to the required points. When the offsets become long the curve is bisected, each section treated in a similar manner. Kröhnke's tables of forty-seven pages are published for setting out curves of any radius by this method.

Method 2. Jackson's six-point equidistant system. As in Kröhnke's method, he obtains equidistant points from unequal abscissæ, but by reducing the number of points to six he obtains a manageable curve and much shorter tables. They are given in his 'Aids to Survey Practice,' Crosby Lockwood & Co.

Method 3. The method given in Kennedy and Hackwood's 'Curve-tables' (E. & F. N. Spon) of equal abscissæ and unequal distances on the curve. This is much more simple, and although, for purposes of construction, it is a disadvantage to have points that are not equidistant, for preliminary survey it makes no difference. The deflection distances are given for Gunter's chains in the end column of each table.

It may be here as well to explain the principle of laying off deflection distances and to show how points may be set off from a tangent to any required equidistant points upon a circular curve.

One of the fundamental properties of the circle is that equal chords subtend equal angles at the centre. It requires

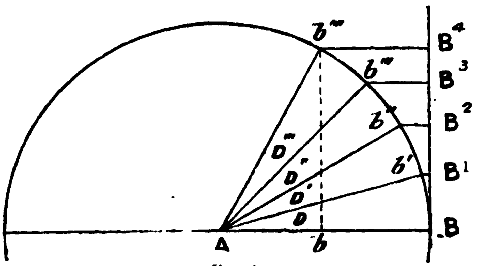


FIG. 65

no demonstration that in Fig. 63, BB^1 is $= \sin D \times AB$, or $BB^2 = \sin (D + D') \times AB$, or that $B^1b' = AB - AB.\cos D$, or $B^4b'''' = AB - AB.\cos (D \text{ to } D''')$.

It will be presently shown in the chapter on curve-ranging that the angles D, D + D', &c., are each of them double the corresponding tangential angle given in the tables.

Thus, supposing the radius 40 chains, we have a tangential angle for 1 chain of $0° 42' 58'' =$ a deflection or central angle of $1° 25' 56''$.

If we multiply the radius, 2,640 feet, by sin $1° 25' 56''$, we shall get 65·985 feet as the co-ordinate corresponding to

a chord of 1 Gunter's chain, and by cos 1° 25′ 56″ we shall get 2,639·17 feet, which deducted from the radius gives us the offset ·83 feet or 9·96 inches. The table enables us to find the tangential angle due to length of curve of any radius, and from this we can obtain the deflection distance in the manner just described. This will be better understood after reading the next chapter. In order to throw off a new tangent when the offsets become too long, we have only to go back two, three, or four stations and measure off from it the deflection distance due to its number from the last peg. For instance, if we wished to lay off a tangent at b'''' we should go back say to b''' and lay off the deflection distance for 1, or to b'', and lay off the deflection distance for 2 chains. We should also lay off from b'''' the abscissæ = BB^1 or BB^2 as the case may be, and this would be a true tangent at b'''', which we could produce and treat in the same manner as BB^4.

Example. The total curvature Δ' of the crowning curve on p. 230 is 63°·6, and the radius 36·93 feet; it is required to put in equidistant points b', b'', b''' upon the curve. Subdivide Δ' into a suitable number of parts, say 10, then D (Fig. 65) = 6°·36; (D + D′) = 12°·72 &c. $BB^1 = 36{\cdot}93 \times \sin 6^\circ{\cdot}36 = 4{\cdot}09$ feet. $BB^2 = 36{\cdot}93 \times \sin 12^\circ{\cdot}72 = 8{\cdot}13$ feet. $B^1 b' = 36{\cdot}93\,(1 - \cos 6^\circ{\cdot}36) = 0{\cdot}23$ feet. $B^2 b'' = 36{\cdot}93\,(1 - \cos 12^\circ{\cdot}72) = 0{\cdot}91$ feet, &c.

Method 4. The oldest and simplest plan for setting out curves with the chain is the only one which can be done without tables (see Fig. 66).

The first point in the curve is found similarly to the last method, but the succeeding points are fixed by ranging lines Bb^1a^2; $b^1b^2a^3$, and so on, from which the offsets a^2b^2, a^3b^3 will be each of them double the first offset a^1b^1. The first offset a^1b^1 is termed the tangential distance, because it is the chord subtending the tangential angle to a radius of 1 chain. The succeeding offsets $a^2 b^2$, $a^3 b^3$, &c., are termed the deflection distances, because they are equal to the chord

subtending the angle made by the intersection of two tangents to the curve at the ends of a chain and to a radius of 1 chain (see more on nomenclature of curve-ranging at p. 214).

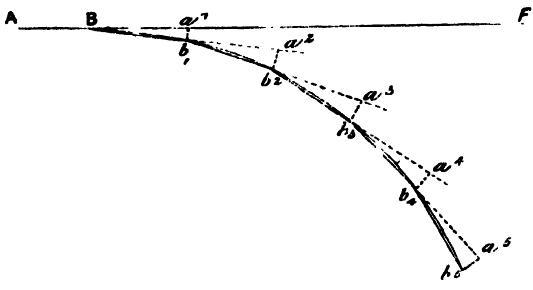


FIG. 66.

The formula used in the field from which to find the offsets by this method is

for tangential dist. offset $= \dfrac{\text{chord}^2}{2R}$.

„ deflection dist. offset $= \dfrac{\text{chord}^2}{R}$

The chord may be 1 Gunter's chain in links, in which case the radius will be expressed also in links and the offset will be in links, or it may be in feet or metres or any other measure.

This formula is not quite exact for small radii.

On a curve of 100 feet radius, the tangential distance for 100 feet chord would be 51·76 feet against 50 feet by the formula. At 500 the formula would be but ·013 feet too little. So that for curves of 10 chains and upwards, set out with 1 chain chords, the formula is practically correct.

The subject of curve-ranging with transit and chain is reserved for another chapter.

CHAPTER VII

CURVE-RANGING WITH TRANSIT AND CHAIN

THE subject of curve-ranging has been briefly touched upon in the previous chapter as far as it can be done with the chain only, but it was deemed appropriate to devote a whole chapter to the subject of curve-ranging with the instrument and chain as being *par excellence* curve-ranging.

There are still those who prefer the chain only to any instrument for the purpose of curve-ranging. In the hands of experts, very correct results may be obtained in level country, but it would be interesting to know how the chain-curve-ranger would put in track centres on a 40° curve on the top of an embankment 30 feet high, the total deflection angle being 180°.

To take it all round, curve-ranging with the chain only is, as compared with work by better methods, poor, fudgy, and muddling.

NOMENCLATURE

The principles of curve-ranging are best understood by keeping in one's mind the idea of a ship's course at sea.

In the previous chapter, it will have probably been noticed how the methods of running base lines with the transit and chain, especially Method 2, resemble the determination of a ship's course and position by dead reckoning.

The angular changes of direction 33° 15′ and 44° 1′ in Fig. 62 are the *angles of deflection*, and this is the very root-idea of curve-ranging. The internal angle between the tan-

gents of a curve, commonly termed I in English textbooks, forms no part whatever of the theory to be described here. It conveys no idea of the curve beyond the mere fact that it is the supplement of the angle of deflection.

This angle of deflection may be either represented by its angular value or else by the difference of bearing of the base-lines, as in navigation.

Curve-ranging may be defined as *deflecting by degrees.* It is for this reason that American surveyors have adopted the curve-nomenclature by which the quickness or slowness of a curve is expressed by the number of degrees of a circle which the *curve, not the chord,* deflects at each chain length.

Thus a curve in which a 100-foot chord subtends 1° of curvature is termed a 1-degree curve.

The reason for this is obvious. When the total number of degrees contained in the deflection angle have been turned, the new base line or tangent will have been reached, and the number of chain lengths contained in the curve will be given by simply dividing the total angle of deflection by the 'degree of curve.'

For instance, if the total angle of deflection be 17°22′ and it is desired to put in a 2° curve, that is to say, a curve in which, if a chain length is measured along it, the tangents to the curve at either end of the chain length will deflect from one another by 2°, the total length of the curve will be $\frac{17 \cdot 22}{2} \times 100 = 861$ feet if a 100 feet chain is used, or 861 links if a Gunter's chain.

Contrast this simple computation with that of English curve-ranging, where it is expressed as follows :

Let x be half the angle of intersection, and R the radius.

Length of the curve $= \cdot 000582$ R $(5400 - x)$. *Note :* x must be expressed in *minutes.*

The writer has not taken the trouble to ascertain whether this formula gives the true circular measure of the curve, or the measure in chains and parts of a chain. If, as is pro-

bable, it is the former, it has not even the advantage of being correct, because curves are not set out by true circular segments but by short chords of chains or parts of a chain, and this is the principle of the American formula.

The English formula is adapted to logarithmic calculation, whereas the American is performed either by slide-rule or even in the head.

The ordinary graduation of instruments into degrees and minutes necessitates the reduction of the deflection angle to decimals of a degree, as for instance where the angle is 29° 47′ 30″ and the degree of curve is 6° 35′ it requires some

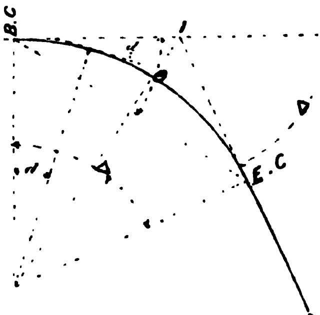


FIG. 67.

little calculation; but by the author's system of decimal graduation of the ordinary degree, it is done by the slide-rule instantaneously.

In Fig. 67, it will be observed that there are two angles marked Δ, and two marked *d*. The upper Δ is the angle of deflection of the curve and is equal to the angle at the centre subtending the curve, which is therefore also marked Δ. Similarly the angle of deflection of a portion of the curve of which the chord is 100 is $=d$, and is equal to the central angle also marked *d*. There has been, and is still, considerable confusion of nomenclature in both England and America, from the fact that some writers insist on term-

ing the tangential angle Θ the angle of deflection because it is the angle by which the chord deflects from the tangent. This has been termed the tangential angle, at least from the first of Rankine's books on the subject, and is still so termed by the best authorities. There might have been some room for choice in naming this angle, but it could not with propriety be termed the angle of deflection.

The only apology for spending so much time in this demonstration is that, unfortunately, writers of high standing have taken up this perplexing nomenclature. To put it concisely, the deflection of a curve *is its actual change of direction, or curvature*, and those who adopt the wrong designation would have to describe a curve which had turned round a quarter of a circle as having deflected forty-five degrees, because the angle between the chord and the tangent (the tangential angle) would be 45°. The term 'tangential angle' is used either for the angle for 1 chain of 66 feet or of 100 feet or for the total angle, which is vague. The term 'total tangential angle' should be given to that for the whole curve, and 'single tangential angle' for that for the unit of measurement. The term 'total deflection angle,' and 'single deflection angle,' will similarly apply to those angles, the latter being the 'degree' of curve.

One English writer, imperfectly acquainted with American curve-ranging principles, describes their nomenclature as 'a confusion between the angle at the centre and the angle of deflection,' from which it would appear that the writer was not himself aware that these angles were equal to one another.

The tangential angle T, or Θ, is equal to half the angle of deflection in circular curves.

In America the straight is termed the tangent, and the prolongation from the springing to the intersection is termed the subtangent. The point of intersection is so named, or else the vertex,' and the midway point of the curve is called the 'apex,' or 'crowning point,' or 'summit.' The curve is

marked B.C. at beginning of curve, and E.C. at end of curve. Wherever a change of position occurs with the transit, a 'hub,' or short stump, is driven in with a nail in it, and a reference stake about three feet long close to it. All the other points are marked by long stakes having their distance chalked on them in stations of 100 feet and their excess. Thus 1,143 feet would be marked 11+43.

Only short curves are run to intersection point. The method of keeping the fieldbook herein described enables the surveyor to range a curve of any radius, alter it, compound it, reverse it, and calculate his position with reference to his starting point without running the curve to intersection. The only difference in this system from the ordinary practice in America is that the lines are kept on their astronomical bearing. It is a little more trouble when using the graduation of minutes and seconds, but no more trouble with the decimal subdivision, and has the advantage of always affording a ready means of defining the position and checking the work.

General Properties of Circular Curves

The following demonstrations are amongst the most useful problems occurring in practice (see Fig. 68).

Let the deflection or total curvature Δ be represented by DIE, the radius BO or OE, IB the subtangent, and IA the apex distance.

1. Prove that DIE=BOE.

In the quadrilateral IBOE angles IBO and IEO are right angles.

But the internal angles of any quadrilateral are together equal to four right angles.

Therefore angle BIE+angle BOE are equal to two right angles.

But angles DIE+BIE are also equal to two right angles; hence, equating and eliminating BIE, angle DIE =angle BOE.

2. Prove that angle $IBE = \frac{DIE}{2}$ or $= \frac{BOE}{2}$ or $= BOI$.

BOI = EOI because A is at the middle of the curve. IBE = IEB because IBE is an isosceles triangle.

The three angles of any triangle being equal to two right angles, and it having been already shown that BIE + BOE = two right angles, therefore in the triangle BIE angles BIE, IBE, BEI are equal to the angles BIE + BOE. Equating, eliminating BIE, and setting IBE + BEI = 2IBE; 2 IBE = BOE, or, which is the same thing, $IBE = \frac{1}{2}DIE$.

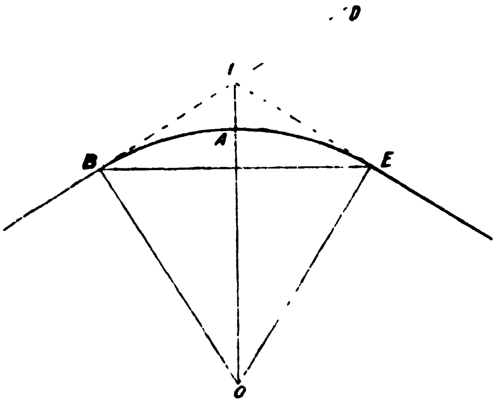


Fig. 68.

3. Prove that the subtangent BI = the radius BO multiplied by the tab. tan. of the tangential angle IBE. IB is evidently proportional to the tangent of angle BOI, which angle we have shown to be equal to IBE.

4. Prove that the apex distance

$$IA = \frac{\text{radius}}{\cos IBE}.$$

IA = IO − AO, but by No. 5, p. 373:

$$\therefore \text{IO} = \frac{\text{BO}}{\cos \text{BOI}} \text{ and BOI} = \text{IBE}$$

$$\text{whence IA} = \frac{\text{radius}}{\cos \text{IBE}} - \text{radius}$$

General Formulæ

Let Δ be the total deflection$=2\Theta$, the tangential angle; $d=$ the degree of curve or curvature in 100 feet chord; R $=$the radius; C $=$ the chord subtending the whole curve; S.T. $=$the subtangent; AI $=$ the apex distance; L $=$ length of curve in feet.

$$R = \frac{C}{2 \sin \frac{\Delta}{2}} = \frac{C}{2 \sin \Theta} = \frac{50}{\sin \frac{d}{2}}$$

$$\sin \frac{d}{2} = \frac{50}{R} = \frac{100 \sin \frac{\Delta}{2}}{C} = \frac{100 \sin \Theta}{C}$$

$$\text{S.T.} = \text{R.} \tan \Theta \text{ ; AI} = \frac{R}{\cos \Theta} - R$$

$$L = \frac{100\,\Delta}{d} \text{ ; } \Delta = \frac{Ld}{100}$$

We have now sufficient data for ranging a curve.

Example 1. The intersection point occurred at station 753 + 34.

The angle of deflection was 37°·916. It is intended to put in a 5° curve. Required, the length of the subtangent, the apex distance, the length of the curve, and the chainage of the B. C. and E. C. points and the apex.

The radius of the curve is found from the table p. 379 $=$1,146 feet.

$$\text{Subtan} = 1{,}146 \times \tan \frac{37^\circ{\cdot}916}{2} = 393{\cdot}7$$

$$\text{Apex dist.} = \frac{1146}{\cos \frac{37^\circ{\cdot}916}{2}} - 1{,}146 = 65{\cdot}7 \text{ feet}$$

$$\text{Length of curve} = \frac{37^\circ{\cdot}916}{5} \times 100 = 758{\cdot}3 \text{ feet}$$

B. C. point $= 753 + 34 - 393{\cdot}7$ feet $= 749 + 40{\cdot}3$
E. C. point $= 749 + 40{\cdot}3 + 758{\cdot}3$ feet $= 756 + 98{\cdot}6$

Apex point at $749 + 40{\cdot}3 + \frac{758{\cdot}3}{2}$ feet $= 753 + 19{\cdot}9$

Lay off angle EIA Fig. 68 $= \frac{180^\circ - D}{2}$ and measure off IA $= 65{\cdot}7$ feet, and put a stake in marked 'Apex, $753 + 19{\cdot}9$.'

Measure the subtangent both ways, and put in pegs with nails, aligning them from the transit at the intersection point. Put in reference stakes marked B.C. $749 + 40{\cdot}3$ and E.C. $756 + 98{\cdot}6$.

Shift the transit to the B. C. point and commence to range the curve. The first odd distance will be $59{\cdot}7$ feet; the tangential angle for this will be $\frac{5}{2} \times \frac{59{\cdot}7}{100} = 1^\circ{\cdot}49$. The instrument is set to read this angle and a stake put in marked 750. The next angle will be $2^\circ{\cdot}5$ $(2^\circ\ 30') + 1{\cdot}49 = 3^\circ{\cdot}99$, which is set off and a stake aligned and driven at another hundred feet and marked 751, and so on. After stake 753 is driven, at a tangential angle of $8^\circ{\cdot}98$ the odd distance, $19{\cdot}9$ to the apex point, is set off, the angle being $2{\cdot}5 \times \frac{19{\cdot}9}{100} = 0^\circ{\cdot}50$, or angle from the beginning $8^\circ{\cdot}98 + {\cdot}50 = 9^\circ{\cdot}48$. This will be one-half the total tangential angle, or one-quarter of the angle of deflection. Both the odd distances to the apex and to the E.C. point are given to the chain-men, and if they do not coincide with the stakes already driven the chain-men either signal the transit-man the amount of divergence when small, or, if large, they return and report. If, for instance, they signal, E.C. point six-tenths to the right, the transit-man will signal back to them to come back three stations or four stations, and he will distribute the error over them. If he is constructing an iron trestle or a brick viaduct he will, of course, repeat

his work until there is no sensible error, but it would be a waste of time for an embankment or cutting, when the error is only a few inches. With ordinary care on the part of the transit-man the error, if any, is nearly always in the chainage.

The following fieldbook is arranged for astronomical bearings. If the curve is run from zero point, the only difference will be that the entries in the bearing column will be dispensed with. The operation at a turning-point is this. Taking as an example the first shift of the transit at Station

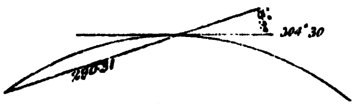


Fig. 69.

755. In the fieldbook on next page a picket-man is sent to the B.C. The instrument is set up at Station 755 over a peg or 'hub' with a nail in it, fixed by the transit from the B.C. With one of the verniers clamped at 290·31 and the telescope upside down, a backsight is taken by means of the external axis upon the B.C.

Description of Curve.

Total angle of deflection		Astronomical bearing				Degree	Radius	Subtangent	Length	Apex distance	Chainage of		
Right	Left	Old tangent	Apex point	Chord	New tangent						B. C.	Apex	E. C.
deg.	deg.	deg.	deg.	deg.	deg.		feet	feet	feet	feet			
37·92	--	276·32	285·81	295·28	314·24	5	1,146	393·7	758·3	65·7	749+ 40·3	753+ 19·9	756+ 98·6

The external axis is then clamped, the telescope is reversed right side up, the parallel plate released, and the instrument directed to the tangent of the curve at Sta. 755 by reading with the same vernier the angle 304°·30. Comparing Fig. 69 with the fieldbook we shall at once see that as the

Fieldbook.

Chainage	Bearing	Length of chord	t	T	Remarks
749 + 40·3	276°·32	0	0	0	B. C.
750	277·81	59·7	1·49	1·49	
751	280·31	100·0	2·5	3·99	
752	282·81	100·0	2·5	6·49	
753	285·31	100·0	2·5	8·99	
753 + 19·9	285·81	19·9	0·5	9·49	Apex
754	287·81	80·1	2·0	11·49	
755	290·31	100·0	2·5	13·99	Turning point
	13·99				
755	304·30	0	0	0	On tangent at 755
756	306·80	100·0	2·5	2·5	
756 + 98·6	309·27	98·6	2·47	4·97	E. C.
	4·97				
	314·24			13·99	
				4·97	
				18·96	On new tangent
				2	
				37·92	Check

tangential angle from the B.C. to 755 is 13°·99, if we add to this an equal amount we obtain the angle of deflection, which laid off from Sta. 755 throws us on to the tangent.

The following fieldbook is for the same curve, only turned to the left instead of to the right.

Description of Curve.

Total angle of deflection		Astronomical bearing				Degree	Radius	Subtangent	Length	Apex distance	Chainage of		
Right	Left	Old tangent	Apex point	Chord	New tangent						B. C.	Apex	E. C.
—	37·92	276·32	266·84	257·36	238·40	5	1,146	393·7	758·3	65·7	749+ 40·3	753+ 19·9	756+ 98·6

Fieldbook.

Chainage	Bearing	Length of chord	t	T	Remarks
749+40·3	276·32	0	0	0	B. C.
750	274·83	59·7	1·49	1·49	
751	272·33	100·0	2·5	3·99	
752	269·83	100·0	2·5	6·49	
753	267·33	100·0	2·5	8·99	
753+19·9	266·83	19·9	0·5	9·49	Apex
754	264·83	80·1	2·0	11·49	
755	262·33	100·0	2·5	13·99	Turning point
	13·99				
755	248·34	0	0	0	On tangent at 755
756	245·84	100·0	2·5	2·5	
756+98·6	243·37	98·6	2·47	4·97	E. C.
	4·97				
	238·40				On new tangent

In practice some of these entries are dispensed with, especially when in a hurry. *It is however very bad policy to keep a fieldbook that nobody else can understand.* If the columns are headed beforehand, or printed, very little time is taken up in making all the entries, and a great deal of time is saved to the reviser when the curve has to be picked up again at any point.

In England, curve-ranging is not done with a fieldbook. Tables like those of Kennedy and Hackwood supply the tangential angles for curves of radii in Gunter's chains. The B.C. and E.C. points are marked usually with two reference pegs, one on either side of the centre peg and close to it. Some surveyors also drive two pegs at every ten chains. Long stakes like those driven in America, with chainage chalked on them, would be welcomed for firewood by the villagers in England.

The American system of curve-nomenclature by degrees of deflection angle could be also used for Gunter's chains. The chord being expressed as 100 links, and the radius also

in links, the degrees could be used just as they are in the tables, *i.e.* a curve of 1° deflection angle per chord of 100 links has a radius of 57·30 chains. Compare Table LIV.

Curve-ranging is nearly always performed with a single theodolite. There is another method with two theodolites which dispenses with chaining ; it is rarely resorted to, and will not be further described than by the remark that the location of points upon the curve is determined by the intersection of two tangential angles from the two instruments. Thus if the two instruments were set up one at the B.C. and the other at the E.C. point in the fieldbook on p. 218 the first point at 750 would be formed by the intersection of an angle of 1°·49 from the B.C. with one of 17°·47 from the E.C., &c. The points are fixed by a picket held by an assistant, first in line with one theodolite and then retreating and advancing along that line until in the line of the other theodolite. If the chain is not used at all, the curves had best be put in telemetrically as described on p. 183; but if it is used at all there is no time saved by dispensing with it on the curves, unless in the case of a trestle across a steep ravine on a very sharp curve, especially if the measurement is impeded by undergrowth. As such cases are exceptional it is not advisable to have the time of two transit-men taken up at the one spot on that account.

An ingenious instrument has been lately invented by Mr. Dalrymple-Hay for ranging curves of radii expressed in Gunter's chains, but it could be easily modified for curves of any other radii. The principle is the adoption of an open and clear graduation in terms of tangential angles, by means of an extended horizontal arc. The index of the limb for a curve of say 20 chains is set at 1 for the tangential angle of 1 chain, at 2 for 2 chains, &c., and thereby simplicity of reading is obtained.

This device can be fitted to any transit theodolite for eight to twelve guineas.

The objections to it are :

t, that in ordinary practice a transit is wanted alternately for running tangents and curves, and when on the tangent the curve apparatus is only an awkward appendage.

Secondly, it tends to make curve-ranging more mechanical. This cannot be called an advantage, because there is hardly anything that calls for more intelligent skill in surveying than this branch of it, and a system which enables a man to do work without knowing why he does it will not be likely to produce a good workman.

Reverse Curves

It is always preferable, having regard to the running gear of the rolling stock, to put in a piece of tangent between the ends of a reverse curve, and when this is done the problem is in nothing different from ranging two distinct curves, one to the right and the other to the left, in the manner just described.

When the main tangents are parallel it is impossible to unite them by any other than a reverse curve; but when they are inclined to one another the only reason for preferring a reverse to a plain connecting curve is to obtain a more rapid transit, in which case the reverse curve will either intersect or lie wholly on the further side of one of the main tangents. A sub-tangent at the point of contrary flexure can be chosen, which will make any desired angle with the main tangents; and an endless number of cases can be formed with a pair of curves of equal deflection and unequal radii, or unequal deflection and equal radii. If curves are wanted which will meet without any intervening tangent, the lengths of sub-tangents are the fixed data, and from the formula on p. 215 transposed, we have $R = S.T. \cot \Theta$, from which we can obtain the radius and other elements.

The following problem is the commonest amongst true reverse curves, and suitable to turn-outs and cross-over roads between parallel tangents.

Whatever the radii, the deflections Δ of the two curves will each of them be equal to twice the inclination Θ of the common tangent to either of the main tangents. When the radii are equal the point of contrary flexure I bisects the common tangent. When unequal they will be proportional to the abscissæ of the common tangent BI and IE.

Reverse curve of unequal radii between parallel tangents.

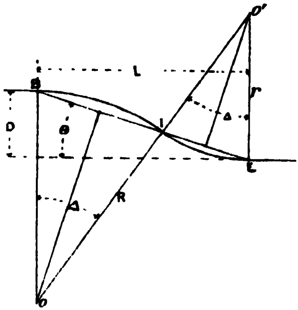


FIG. 70.

If the distance D on the square between the tangents be given, and either L the straight reach, or BE the common tangent be given, the other elements can be found from the following expressions:

$$\tan \Theta = \frac{D}{L}\ ; \ \cos \Theta = \frac{BE}{L}.$$

$$R = \frac{BI}{2 \sin \Theta}\ ; \ r = \frac{IE}{2 \sin \Theta}$$

$$L = D \cot \Theta\ ; \ D = L \tan \Theta$$

or if one of the radii R be given, and the distances L and D or BE be given, to find the other radius.

$$\frac{\text{[illegible]}}{2 \sin \Theta} = \frac{\text{[illegible]}}{2D}$$

Turn-outs are now generally curved from the commencement of the switch, when the switch is of the split type, such as is always used on passenger railways in England.

In America, the old-fashioned stub-switch, having its heel at the point of curvature, is still largely used. The switch, being common to both tracks, is straight, and the curvature to which it is a tangent commences from its further end.

To lay out a turn-out for a stub-switch, instead of the main tangent, take the line of direction of the switch when directed towards the curve and which forms an angle i with the main tangent of which

$$\sin i = \frac{\text{throw of switch}}{\text{length of switch}}$$

Treat the end of the switch as the point of curvature, and calculate from it as if it were B. The two tangents will then not be quite parallel, but the curve can either be run out to a parallel with the switch, by foregoing formula, and produced to a parallel with the main tangent, or else a separate calculation can be made, which want of space precludes our giving here.

To Divert a Curve to a Parallel Tangent

Very frequently the end of a curve comes too near some fixed point, and it is desired to throw it to one side

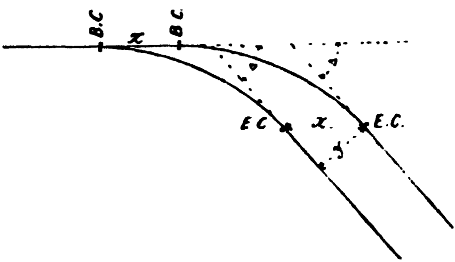


Fig. 71.

or the other by advancing or receding the B.C. point upon the old tangent.

Let the distance between the two parallel tangents be y,

and the length of advance or retreat of the B.C. point be x, the deflection angle being Δ.

Then $x = \dfrac{y}{\sin \Delta}$ or $y = x \times \sin \Delta$

To find the Linear Advance of the Inner Rail on Circular Curves

In consequence of the outer rail being longer than the inner, the joints of the latter gradually forge ahead unless a special rail is put in occasionally, or the rails cut. Let

LA = difference of length between outer and inner rails.

Δ = deflection angle or total curvature.

G = gauge.

General Rule

$$LA = G \times \Delta \times \cdot 01745 \text{ (log } 8\cdot 24188)$$ [1]

Or if LA be in inches and G in feet—

$$LA = G \times \Delta \times \cdot 20944 \text{ (log } 9\cdot 32106)$$

Rule for Standard Gauge of 4 feet 8½ inches

$$LA \text{ in inches} = \Delta \times \cdot 98611 \text{ (log } 9\cdot 99392)$$

Rule for 3 feet 6 inch Gauge

$$LA \text{ in inches} = \Delta \times \cdot 73304 \text{ (log } 9\cdot 86513)$$

Rule for 3 feet Gauge

$$LA \text{ in inches} = \Delta \times \cdot 62832 \text{ (log } 9\cdot 79818)$$

Rule for Metre Gauge

$$LA \text{ in millimetres} = \Delta \times 17\cdot 4533 \text{ (log } 1\cdot 2418774)$$

Generally a consignment of rails includes some 'specials,' six inches or twelve inches shorter than the usual length; in which case we can find the extent of curvature needed for putting in a 'special' by substituting for

[1] Ten added to index No. of unity as with log. sines.

L A the decrement l, and inverting the equation the general rule becomes $\Delta = \frac{l}{G} \times 57{\cdot}296$, and Δ has following values in the several cases:

	Δ
Standard gauge, 6 inch decrement	$6^\circ{\cdot}084$
3 feet 6 inch gauge ,, ,, ...	$8{\cdot}185$
3 feet gauge ,, ,, ...	$9{\cdot}549$
Metre gauge ; decrement of 2 decimetres . .	$11{\cdot}459$

If the radius and length of the curve are given, find from Table LIV. p. 379 the degree of curve, and $\Delta = \frac{\mathrm{L}d}{100}$. Thus if the curve were 1,146 feet radius, and 2,762 feet long, from Table LIV. $d = 5^\circ{\cdot}00$, and $\Delta = \frac{2{,}762 \times 5}{100} = 138^\circ{\cdot}1$.

Transition Curves

The transition or parabolic curve is a means of toning down the abrupt passage from a tangent into a circular curve. It is called the 'Uebergang' by the Germans and constructed as a true parabola.

The objection to it in this form is that if a curve becomes deformed or requires renewal the tracklayers are apt to put in what one of them termed to a friend of mine 'somethink of a paraboler.' As the man literally knew as much as the proverbial cow about conic sections, the curve was unique.

Mr. Searles in his little book entitled 'The Railroad Spiral' has placed the matter on a much more practical basis, using a combination of short circular curves, forming a close approximation to the parabola, in which, with equal chords, the curvature increases by an equal amount at the end of each chord. He adopts ten angular minutes as the basis, and uses chords of from ten to one hundred feet. Whatever its length, the curvature of the first chord is 10, the second 20, the third 30, the fourth 40 minutes, and so on.

'Mr. Searles gives exhaustive and lengthy tables and formulæ for putting in any kind of spiral or for substituting a spiral for an existing circular curve whilst retaining as nearly as possible the same ground and the same length of line.

The object of the parabolic curve is not to dispense entirely with the circular arc, but to attain gradually any suitable radius with which to form a circular connecting or 'crowning curve' and leave it again by the same gradual transition to join the further tangent.

The advantages are chiefly :

First. Less wear upon the running-gear.

Second. Less discomfort to the passengers.

Sharp curves and high speed are the factors that make the demand for the transition-curve. The explanations herein will deal firstly with a case suitable to a tramway or workshop siding of about 40 feet radius, because a better illustration can be given where the whole curve is shown. Spirals for street-railways are becoming every year more important. The radii cannot be increased, but the speed is constantly being accelerated up to the utmost limit in response to the public demand for rapid transit. Motors of one kind and another have been invented which meet the requirements of speed combined with safety. The company protects its cars from derailment upon sharp curves by means of guard rails, but cannot prevent its passengers from the unpleasant swing and jolt when the cars turn a square corner, nor can it avoid the wear and tear to the car wheels and axles from the sudden cross-strain. 'C'est le premier pas qui coûte.'

As pointed out at p. 17 in Chapter I., the wear of locomotive tyres proves itself to depend largely upon the degree of shock which is imparted. Once the running gear has accommodated itself by its structural flexibility to a sharp curve, the extra wear due to pressure against the outer rail is comparatively small. The super-elevation of the outer rail relieves it, and by the transition curve this elevation is

commenced and gradually increased with the curve, so that the unpleasant sensation of being tilted up in one's carriage whilst still on the tangent is avoided.

The spiral, even on the sharpest curves, can be made to follow very closely the same ground as a circular arc. It will be seen from Fig. 72 how very slightly the two curves differ, and yet the spiral commences with a radius of 573 feet instead of one of 40 feet.

The spiral adopted in the following pages differs from that of Mr. Searles in having for its base ·2 of a degree instead of ten minutes. For the sharper spirals the radii are given in feet, for the others the degree of curvature per 100 feet chord. Sharp short curves are best put in by offsets as shown in Fig. 73; flatter and longer ones by tangential angles as explained at p. 215 &c.

Table LV. of Appendix gives the general elements of the decimal spiral which are common to all the other tables. The first column, n, gives the number of the chord from the commencement. The second, $n.\ c.$, gives the curvature in that chord, which is the same whatever its length may be. The length of the short chord c in feet defines the spiral: thus No. 2 spiral is one in which c is 2 feet.

The third column, s, gives the total curvature of the spiral from the commencement, in other words the angle of deflection formed by a tangent at any point n with the main tangent.[1] The fourth column, k, is the inclination of any chord to the main tangent, in other words the total curvature opposite the middle point of said chord; it is the basis of the computation of the ordinates x and y.

The fifth column, i, is the tangential angle formed by the long chord C at the point of spiral S with the main tangent. It is needed for setting out the curve by tangential angles similarly to a circular curve. In the other tables the column r is the radius of curvature; d the degree or deflection of 100 feet chord. The column x is the ordinate to the main tan-

[1] It is called by Mr. Searles the spiral angle.

gent from any point n, and y is the corresponding abscissa. The column L gives the length of the spiral. The letter R denoting the radius of the crowning curve does not occur in the tables. C is a long chord ; c the short chord.

General formulæ of the spiral:

$$s = \frac{n \times (n+1)}{10}\,;\ k = \frac{n^2}{10}\,;\ \tan i = \frac{x}{y}$$

$$x = \Sigma^*.\ \text{chord} \times \sin k\,;\ y = \Sigma.\ \text{chord} \times \cos k$$

$$C = \frac{x}{\sin i} = \frac{y}{\cos i}\,;\ r = \frac{c}{2\ \sin \frac{n.c.}{2}}$$

$$\left.\begin{array}{l}\text{sub-tangent along}\\ \text{main - tangent for}\\ \text{any value of } n\end{array}\right\} = y - x.\ \cot s$$

$$\text{corresponding sub-tangent to } n = \frac{x}{\sin s}$$

It will be seen, on comparing these columns with the corresponding ones in Mr. Searles' tables, how much simpler they become by the use of the decimal degree.

The angles λ and μ indicated in Fig. 72, are formed by the semichord of the crowning curve with a normal to the main tangent and the central radius of the curve respectively.

They are found as follows (see Fig. 73):

$$\lambda = 90° - \tfrac{1}{2}\,(\Theta + s)\,;\ \mu = 90° - \tfrac{1}{2}\,(\Theta - s)$$

and as a check $\lambda + \mu + \Theta = 180°$.

The point to be aimed at with spirals, as with ordinary curve-ranging, is to obtain the easiest curve possible within the limits prescribed by the situation. With tramway curves, and in many other cases, it is often the crown of the curve or apex which fixes its other elements. The formulæ for putting in a spiral to conform to a pre-determined apex distance is somewhat longer than the other, and requires the finding of angles λ and μ. A piece of spiral is first fixed

* The symbol Σ is used for the summation up to any point of the product of each chord by the corresponding value of k.

upon, and the distance of the point of spiral S from the point of intersection I, and the radius R of the crowning curve, are found by the following formulæ:

$$R=\frac{AI.\cos\Theta-x}{2\cos\lambda.\cos\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and $IS=y+\tan\lambda\left[(AI.\cos\Theta)-x\right]+AI.\sin\Theta$ . . (2)

As will be presently shown, these formulæ are very useful for putting in a spiral upon a tramway; but it is not always practicable to fix the apex distance in this way, neither is it always important to do so. The data for a crowning curve of any required radius can be found by the following formulæ for a spiral of any chord-length and any curvature.

Let R be the radius of the crowning curve.

Let Θ be the half total curvature.

Let s be the total curvature of one spiral.

Let x be the ordinate to end of spiral.

Let y be the abscissa to end of spiral.

Let IS be the distance of point of spiral from point of intersection.

Let AI be the apex distance.

Then $IS=y+\tan\Theta\,(R.\cos s+x)-R.\sin s$ . . (3)

and $AI=\frac{R.\cos s+x}{\cos\Theta}-R$ . . . . (4)

The radius of an approximately corresponding circular arc, or trial curve, should be first ascertained by rule on p. 215 for the whole curvature, remembering that the spiral will always somewhat sharpen the rate of curvature at the crown. The next operation is to select from one of the tables a portion of spiral which will lead in a convenient manner into a crowning curve of not much less radius than the trial curve just found. The third step is to work out the data, either from a fixed apex distance by (1) and (2), or from the fixed radius of crowning curve by (3) and (4).

If not limited by a fixed apex distance, a radius of crowning curve will naturally be chosen which forms a harmonious transition from the spiral, but any radius may be put in to join any spiral, provided s is less than Θ. When $s=\Theta$, the two spirals meet, and form a continuous parabola, or nearly so.

It is also possible to have two spirals of different length and different curvature at the ends of a crowning curve, but the discussion of such peculiar cases as that would be out of place in this work.

TRAMWAY SPIRAL

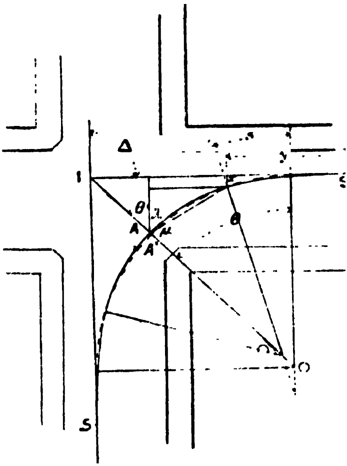
(Nos. 2 and 5 Spiral) Tables LVI. and LVII.

Fig. 72 represents the very common case of a square street-turning, the streets being only 30 feet wide, with 5 feet footpaths, and the centre-line is placed somewhat to one side in order to give equal clearance at the corner, and on the further side of the street. This, of course, is only a secondary matter, and has nothing to do with the principle of the spiral; but the figure will serve to show how, when the limit of apex distance has been fixed by a trial curve, a spiral can be put in *which will differ very slightly from it, and have more clearance at the tight corner.*

The data are as follows.

$\Delta=90°$; $\Theta=45°$; from which a 40 feet radius is selected for a trial curve, and the subtangent ID, and apex distance A′I found by rule on p. 215. It is presumed that the limit of curve-radius is 35 feet. A′I is found to be equal to 16·57 feet. In order that the spiral may approach coincidence with the trial curve, the crowning curve must lie outside, and the spiral inside of it. It is found that an approximately constant relation exists between the ordinate x of the end of the spiral, and the distance AA′ between the trial curve and the crowning curve at the crown, and for curves of this character it will do to diminish A′I by one-tenth of x.

To select a spiral, on examination of Table LVI. it will be seen that any of the points 11 to 14 are of a curvature greater than 40 feet, so that in using them there will be no danger of the crowning curve flattening the spiral. Whichever point we choose, we first reduce the apex distance by one-tenth of x. Supposing we use point 11. Our data are,



$r=52{\cdot}09$ feet : $s=13^{\circ}{\cdot}2$; $x=1{\cdot}76$ feet ; $y=21{\cdot}88$ feet. Deducting from A'I one-tenth of x we have AI $=16{\cdot}40$ feet.

and by (1) $R=\dfrac{16{\cdot}40\times\cos 45^{\circ}-1{\cdot}76}{2\cos 60^{\circ}{\cdot}9\times\cos 74^{\circ}{\cdot}1}=36{\cdot}93$ feet

and by (2) $IS=21{\cdot}88$ feet $+\,16{\cdot}40\times\sin 45^{\circ}+\tan 60^{\circ}{\cdot}9$
$\times(16{\cdot}40\cos 45^{\circ}-1{\cdot}76)=51{\cdot}16$ feet

The transition at end of spiral will be from $r=52{\cdot}09$, to $R=39{\cdot}93$, or a change of $15{\cdot}16$ feet.

If we were to end the spiral at point 13, we should have

AI$=16\cdot57-\cdot28=16\cdot29$, and adapting λ and μ to the altered data, R would become $35\cdot74$ feet, and IS$=51\cdot36$ feet. The transition would then be from $r=44\cdot08$ to R$=35\cdot74=8\cdot34$ feet. The change would be more gradual, but the crowning curve would be $1\cdot19$ feet sharper. In either case the travelling public would be none the wiser, the rolling stock would not be affected, and it would be difficult to say which of the points of 11, 12, and 13, would be preferable.

If it were not necessary to fix the apex distance absolutely and it was desired to have a crowning curve with a perfectly harmonious transition, we should choose for a value of R that of r next in order to the point we selected as the end of spiral. Thus with $n=13$ we should take R$=40\cdot93$ feet: which is the value of r when $n=14$. We do not then need λ or μ; and by (3)—

$$\mathrm{IS}=25\cdot73+\tan\ 45^\circ\ (40\cdot93\times\cos\ 18^\circ\cdot2+2\cdot84) \\ -40\cdot93\ \sin\ 18^\circ\cdot2$$

and by (4)—

$$\mathrm{AI}=\frac{40\cdot93\ \cos\ 18^\circ\cdot2+2\cdot84}{\cos\ 45^\circ}-40\cdot93$$

or IS $=54\cdot67$ feet and AI$=18\cdot07$ feet.

The point of spiral would not be at an inconvenient length, but the apex would be $1\cdot67$ feet nearer to the footpath.

When finally selected and calculated the curve should be tabulated for reference and a working drawing made to a large scale in the form of Fig. 73. *Any practical track-layer can then put it in or replace it with nothing more than a chalk-line, a set-square, and a steel tape, graduated to feet and hundredths.*

When the survey has been made previous to construction, as it always should be if possible, the data of each curve should be worked out and the rails bent at the rolling mills to the required spirals and crowning curves, painted and stamped so as to identify them. When there is no survey

and where no facilities for accurate bending are available at the job, a number of spiral rails should be included in the shipment, from 12 to 30 feet long, having holes for fish-bolts at every two feet in all lengths exceeding 12 feet, so that they will only need cutting at the most suitable value of *n* for each particular curve.

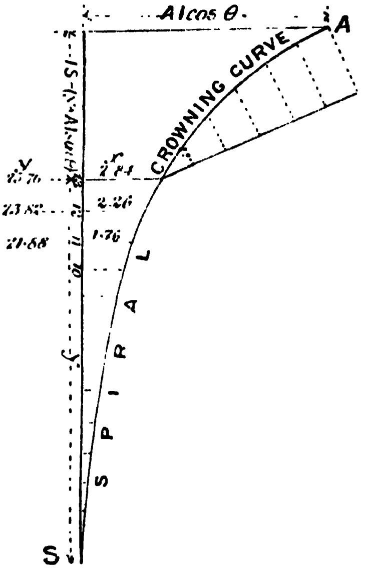


FIG. 73.

A template will be made for the centre line of the spiral track, and each of the rails adjusted to it; they will need a little humouring with the 'Jim Crow' to bring them to gauge, not being perfectly concentric circular arcs.

Another and more exact method is to calculate the spiral for the outer rail, and make a second working drawing from it by laying off ordinates at every two feet equal to the

gauge. The ordinates for the crowning curve can be put in from the tangent by formula on p. 207, or from the chord as shown on Fig. 73 by formula in Trautwine's 'Pocket Book.'

The Horse-Shoe Spiral

(Nos. 5, 10, 15, and 25) Tables LVII. to LX.

When the total curvature approaches a semicircle (see Fig. 74) it is impossible to run the tangents to an intersection. Long before so great a curvature is reached, as shown in the example, it becomes very inconvenient to do so, and recourse should be had to the apex tangent HAK The deflection Δ of the main tangents can be measured by

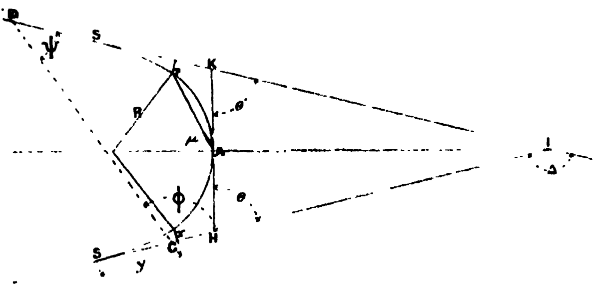


Fig. 74.

running an auxiliary base-line DG from one tangent to the other. The sum of the interior angles IDG, IGD is equal to Δ, but if the instrument is kept to the astronomical bearing as explained on p. 203, the difference of bearing between the two tangents will give their deflection, even though two or three auxiliary base-lines have been run between them.

The point A is generally fixed by inspection of the ground and measured; HAK being set out with angle IHK=Θ.

In exceptional cases, such as when A falls in the middle of a torrent, H and K being on the sides of a precipitous

ravine, it is inconvenient to measure HK with the chain, and telemetric measurement may be also difficult on account of heavy brushwood; we can then measure the distance HG from the end of the auxiliary base to the point opposite the intended apex, and calculate AH by the following formula:

$$AH = \frac{DG.\sin\phi}{2\sin\Theta} - HG.\cos\Theta - DG.\sin(\Theta - \psi).\cot\Theta \quad . . (5)$$

Although lengthy, the only angles involved are Θ and the two interior angles IDG, IGD, so that the formula is less tedious than it looks.

When the distance AH has been obtained, the radius of a circular curve is calculated which will join the tangents and pass through A by the formula

$$R' = AH.\cot\frac{\Theta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

R′ should be from 5 to 10 per cent. greater than the curve-limit of the survey, in order to allow for the diminution due to the spiral. If R′ is too small, a fresh point A′ must be chosen somewhat further from the intersection point. It is generally possible, even in very rough country, to obtain a sufficient approximation to AH by stepping or even guessing in order to be within the limit of radius. For instance, supposing Θ to be 76·25°, and the curve-limit 250 feet. AH must not be *less* than $250 \times \tan\frac{\Theta}{2} =$ 198·8 feet, therefore before selecting A, HK is roughly measured to be sure that it is over 400 feet. If the object be the staking out of the curve for actual construction, it will not do to put in A with the tacheometer; it must be fixed to a tenth of a foot by the chain, but for the first approximation to HK it can be measured by the tacheometer, by pacing, or by the aperture of the two-foot rule, explained on p. 72. Flags are fixed at two points on the first main tangent, such as D and S, and the assistant, keeping himself

in line with them, moves forward along the tangent until he is in the line of HK, the direction of which is given by the surveyor at H with either the theodolite or prismatic compass.

When a point A has been found which appears to suit the ground best and also to be within the curve-limit, it is finally determined either by the exact measurement of AH (=HK) or by calculation with formula (5).

To select a spiral, examine any of the tables suitable to the class of curve intended and find one which attains a radius $r=$ or somewhat $> R'$ within a suitable length of spiral. For instance, although it is possible to apply No. 2 spiral to a 1° curve on a trunk railroad, it would be no use whatever, because it attains a radius of that degree at a distance of eight feet.

No. 50 would attain that radius in a length of 200 feet, or No. 100 in 400 feet, so that one of the last three spirals would be chosen.

At the transition point T or end of spiral, s should be somewhere between one-fourth and one-half of Θ; one-third is best.

When the spiral is chosen, angles λ and μ are calculated (see p. 228) and the data of point of spiral KS and radius of crowning curve R are given by the following formulæ:

$$R = \frac{AH.\ \sin\Theta - x}{2\cos\lambda.\ \cos\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$KS = y - x.\ \cot\Theta + \frac{2R}{\sin\Theta}\cos^2\mu \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Example (see Fig. 74). Let the curve limit be 250 feet. $\Theta=76°{\cdot}25$. AH is measured$=217$ feet.

Selecting No. 15 spiral at point 15, we have further data, $x=32'{\cdot}06$; $y=220'{\cdot}96$; $s=24°{\cdot}00$. From these we obtain $\lambda=39°{\cdot}875$, $\mu=63°{\cdot}875$, $R=217\times\sin 76°{\cdot}25-32'{\cdot}06\div 2\cos 39°{\cdot}875.\cos 63°{\cdot}875=264'{\cdot}43$; $KS=220'{\cdot}96-32'{\cdot}06.$
$\cot 76°{\cdot}25+\dfrac{2\times 264'{\cdot}43}{\sin 76°{\cdot}25}\cos^2 63°{\cdot}875=318'{\cdot}69.$

This will give a perfect transition ; r being 286·5 at the transition point. The length of spiral, 225 feet, will be ample for a curve of this character, being as long as many narrow-gauge trains.

If we had chosen No. 10 spiral at point 10, we should have had a crowning curve of 274'·32 radius, the last value of r being 286'·52. The transition would be equally good, but the length of spiral, 100 feet, would not be sufficient to be thoroughly effective.

Spirals up to No. 15 inclusive are more suitable for setting out by the ordinates of x and y, although values of i, the tangential angle, are given from point 10 onwards in No. 15 spiral. Supposing a horseshoe of total curvature 170 degrees were required with a curve limit of 200 feet radius, it would be advisable to adopt No. 15 spiral at point 21, rather than No. 10 spiral at point 13, and the ordinate x of 82·74 would be inconveniently large for tape measurement.

THE MOUNTAIN SPIRAL

Tables LX. and LXI.

This term has been chosen to distinguish spirals Nos. 25 and 50 as suitable for sharp curves in a standard-gauge line or ordinary curves in a narrow-gauge line.

The ordinates of x and y will not be used in the field, as they become inconveniently large. The curve will be ranged by the tangential angle i, similarly to the ranging of circular curves.

Example. Having a total curve deflection of 30°, it is desired to put in a 5° crowning curve with a uniform transition from a spiral.

Choosing a No. 25 spiral we find the transition point is at $n=6$ and the data are :

$\Theta=15·0$; $s=4°·2$; $x=3·97$; $y=149·91$.
$r=1,193·6$; R$=1,146$ feet.

$$\text{By (3) IS} = 149{\cdot}91 + \tan 15^\circ{\cdot}0\,(1{,}146 \cos 4^\circ{\cdot}2 + 3{\cdot}97) - 1{,}146 \sin 4^\circ{\cdot}2 = 373{\cdot}28 \text{ feet}$$

$$\text{By (4) AI} = \frac{1{,}146 \cos 4^\circ{\cdot}2 + 3{\cdot}97}{\cos 15^\circ{\cdot}0} - 1{,}146 = 41{\cdot}33 \text{ feet}$$

The apex distance will here only differ by 0·9 foot from that of a 5° curve throughout.

The Trunk Line Spiral

Tables LXI. to LXIII.

This term has been chosen to distinguish spirals 75 and 100, but No. 50 is also applicable in many cases.

Example. Having a total curve deflection of 45° between the two main tangents, it is desired to put in a 1° crowning curve, so as to form a uniform transition from the spiral.

Choosing a No. 100 spiral, we find the transition point for a 1° curve to be where $n=4$, and the data are :

$\Theta=22^\circ{\cdot}5$; $s=2^\circ$; $x=5{\cdot}23$ feet : $y=399{\cdot}94$; $R=5{,}730$ feet

$$\text{By (3) IS} = 399{\cdot}94 + \tan 22^\circ{\cdot}5\,(5{,}730 \cos 2^\circ + 5{\cdot}23) - 5{,}730 + \sin 2^\circ = 2{,}574{\cdot}19 \text{ feet.}$$

$$\text{By (4) AI} = \frac{5{,}730 \cos 2^\circ + 5{\cdot}23}{\cos 22{\cdot}5} - 5{,}730 = 474{\cdot}00 \text{ feet}$$

The apex distance will here only differ by two feet from that of a 1° curve throughout. For practice the reader might take the apex distance, 474 feet, as the fixed quantity and find the radius and distance IS by formulæ (1) and (2). The results will agree with the foregoing assumptions within a small fraction of a foot, but inasmuch as AI is small as compared with R, in order to get R correct to two places of decimals, AI should be given to three or four places, which is not necessary for practical curve-ranging.

Ranging the Spiral from an Intermediate Point

Hitherto the spiral has been supposed to be visible from end to end. Such is generally the case, because when the first trial lines are run, the bush is cleared, and it is only when unusual obstructions, such as sharp rock-points, interfere with the view that the spiral cannot be ranged continuously. When, however, from any cause, a break or turning point has to be made, the operation is analogous to that described in circular curves on p. 217, except that the tangential angle i is not half the deflection or spiral angle s.

Supposing a No. 25 spiral is being ranged from a main tangent whose bearing is $33^\circ{\cdot}34$, and it is desired to make a turning point, where $n=16$, the curve being to the right. Point 16 will be ranged from s, with a tangential angle $i=$ $9{\cdot}33$; consequently on a bearing of $33{\cdot}34+9{\cdot}33=42{\cdot}67$, on shifting the instrument to n, we require for the tangent at that point a bearing of $33^\circ{\cdot}34+s$, that is $60^\circ{\cdot}54$; we therefore clamp the vernier at $42^\circ{\cdot}67$ for the back sight, clamp the external axis, reverse the telescope to the forward position, and set the vernier to $60^\circ{\cdot}54$, which will bring the line of sight on to the tangent.

In ranging the remainder of the curve either by ordinates or tangential angles, a separate set of values of k', x', y', and i' have to be used as given in Table LXIV. If the curve is ranged by ordinates, the values of x' and y' have to be calculated by simple proportion from those of No. 100 spiral given in the table; for instance a No. 25 spiral will have values of x and $y=25$ per cent. of those in the table. They are found by the same equation as those for the primary tables, viz. $x'=\Sigma$. chord. sin k', $y'=\Sigma$. chord. cos k'. If the curve is ranged by tangential angles i', no calculation is required, as i' is the same for all spirals.

By keeping the instrument on the astronomical or any continuous bearing, the transition point or end of spiral can

be very conveniently checked, however many turning points may have been taken. For instance, supposing a curve to the right, and calling the bearing of the main tangent B, the bearing of the transition point when viewed from S will be $B+i$. With one turning point it will be $B+i+i'$ and so on. When the transition point is reached, the direction of the tangent to the crowning curve is found by taking a back sight to the last turning point with the bearing $B+i+i'$ &c., and reversing the telescope, with the external axis clamped, and the vernier released, the line of sight is then set to the bearing $B+s$. It can easily be seen on the ground whether this is a tangent to the last chord of the spiral, and so check the calculation.

It is also useful as an independent check to lay off and measure the subtangent from the transition point, or if necessary from any turning point, to the main tangent, which is done by the formula on p. 228.

Wyes and Loops

When a substitute for a turn-table is needed, it is usual in America to put in a pair of curves turning to the right and left on the same side of the main track, and terminating in a common tangent at right angles to the main track ; the engine runs round one curve into the common tangent, and backshunts on to the main track through the other curve, so coming out end for end. As a substitute for a cross-over road on a double track a loop is sometimes made, by which the train, after describing a complete circle, occupies the other track, but in a reverse position.

CHAPTER VIII

GRAPHIC CALCULATION FOR PRELIMINARY ESTIMATES

THE surveyor is compelled to form his first estimates of cost without detailed measurements with chain, tape, and rule, but as little as possible by guess-work.

If the estimate is based upon a walk-over survey and sketches, he must rely upon his experience of similarly constructed works, and he will judge the cost mile by mile according to its general character.

When sufficient time is allowed to produce a topographical map of more or less accuracy, the operations of the preliminary surveyor, although more rapid, are analogous in principle to those of the executive engineer, who succeeds him with more time at his command.

The earthwork is measured on a profile plotted from the contours; the trestles scaled for distance and heights from the same profile, as also the bridges and culverts. The quantities in all cases are usually taken from tables.

The use of quantity diagrams combined with the slide-rule is much more suitable to this class of work than long tables of figures and elaborate formulæ.

CALCULATION BY SLIDE-RULE

Before entering upon the subject of estimates, it will be necessary to describe somewhat fully the use of the slide-rule.

The instrument itself is described on p. 361, but it is

desired to give a detailed description of its uses as applied to preliminary survey.

The printed explanations sold with the slide-rule contain directions for its use, but it is preferred here to arrange them in a different form.

The easiest way to become familiar with the instrument is to look upon it as a—

1. *Geometrical proportion or rule of three sum.* Place a 1 of the slide opposite to a 9 of the rule, and give the figures on the right-hand side of the rule ten times their indicated value. Then, beginning with the central 1 of the slide, it will be found under 9 of the rule, 2 under 18, 3 under 27, 4 under 36, 1·1 under 9·9, 2·3 under 20·7, 3·7 under 33·3, and so on, every value of the rule corresponding with the slide $\times 9$. We may put it in the form of multiplication as $9 \times 3 = 27 \times 1$, or in the form of division as $\frac{9}{1} = \frac{27}{3}$, or in the ordinary rule of three form 9 : 1 :: 27 : 3. In any case, the figures on the rule are opposite to their proportionals on the slide.

We may give the figures on the slide ten times their indicated value, and those on the rule 100 times, but the proportion remains the same. Thus, in the above illustration 3·7 would become 37 on the slide, and it would be under 333 on the rule.

One of the most useful cases of proportion by the slide-rule is the finding of the angular value of odd distances in railway curves by tangential angles, or in getting the logarithms of intermediate numbers or angles by interpolation.

Example. What is the cosecant of 35° 11′ 13″·3?

Log. cosec. 35° 11′ by table . . . =	10·2394308
Tab. difference for 60″ = − 1,791 ;	
$\frac{13 \cdot 3}{60} \times 1{,}791$ by slide-rule . . =	−397
Log. cosec. 35° 11′ 13″·3 . . =	10·2393911

This is done by placing the 1,791 of the slide opposite to the 60 on the rule, and looking for the value on the slide

corresponding with 13·3 on the rule. It is more exactly done by the lower scales of the rule, although it involves two operations. If the upper scales be used we can only read 1,790 instead of 1,791, but there will still be no mistake about the 397 to an eye which has had a little practice in estimating the value of the subdivisions. We will, however, take the lower scales.

Giving the slide 1,000 times its indicated value, we place the 1,791 opposite the 6 of the rule, which we call 60, and we see that the 13·3 on the rule has been overshot altogether, the left hand 1 of the slide being opposite to the 3·35 on the rule. We slide back until the right hand 1 of the slide corresponds with 3·35 of the rule, and we shall be still in adjustment for giving the proportion of $\frac{1791}{60}$ as before. Opposite to the 1·33 of the rule (which we give the value of 13·3) we find the exact figure of 397 on the slide.

This operation is again much simplified by decimal graduation.

The whole of the succeeding examples of scale reduction and plotting, weights and measures and coinage, are based upon this principle of proportion or geometrical ratio.

2. *Multiplication.* When, as described on last page, we put the 1 of the slide opposite the 9 of the rule, we multiply it by 9, and any other figure on the slide is likewise multiplied by 9 on the rule opposite to it; therefore, if we want to multiply by any number, we place a 1 of the slide opposite that number, and the slide and rule will be in adjustment to read like a column of a multiplication table headed by that number. Thus, if we wish to multiply 57 × 35, we place a 1 of the lower slide-scale over the latter number. Every figure on the slide will then be opposite to 35 times its value on the rule, and 57 will be found opposite to 1,995 exactly.

3. *Division.* This is the converse of multiplication contained in the same principle of proportion.

Thus, in the preceding example, to divide 1,995 by 57 we adjust the instrument in that ratio by placing the 57 of the slide over the 1,995 of the rule, or *vice versa*, and read off the quotient 35 under the corresponding 1.

4. *Involution and Evolution.* This is done by simple inspection without using the slide. Any of the figures on the lower scale of the rule are the square roots of those on the upper scale, and *vice versa*. They are made to 'teach' with one another by the brass marker. Thus the 3 on the lower scale is under the 9 of the upper.

Example a. Find the square root of 718. Direct the brass index to that figure on the upper scale of the rule, and on the lower scale will be found 26·8.

Example b. Find the square of 718. The answer will obviously have six figures. Placing the marker at 718 on the lower scale, the upper scale is, as near as one can read, 516—that is to say, 516,000. The exact answer is 515,524.

If we wanted it exactly we should have to make a double inspection with the aid of the lower slide.

Thus, 718 × 700	= 502,600
718 × 18	= 12,924
	515,524

and in so doing we should scarcely save any time, because directly we begin to have to put down figures we might as well work it out.

The Slide-Rule as a Universal Decimal Scale

Nothing can compare with the slide-rule for plotting in the field. In the Mannheim rule one of the edges is bevelled so as to be used on the plot, and is graduated to millimètres. If we set the slide to read with the rule the proportion of millimètres to the given scale of feet, chains, miles, or what not, we can then apply the millimètre scale directly to the paper without any further calculation, and this

with the most awkward scales imaginable. *Example* (a). Let it be desired to plot with the slide-rule from a scale which was intended to be 6 inches to the mile, but which by contraction of the paper has shrunk to 5·95 inches to the mile. On the back of the rule in the table of useful memoranda will be found under *mesures anglaises*, pied (foot)=M. 0·3048, *i.e.* 12 inches=304·8 millimètres. If we then make 304·8 on the slide correspond with 12 on the rule, we shall find opposite to 5·95 on the rule 151 on the slide. This being the scale value of a mile in millimètres, we can make 151 correspond with 5,280, to get the number of millimètres corresponding with feet, or 151 with 8,000, to find the value for links. Thus, adjusting the 151 of the rule opposite the 80 of the slide, we have for, say, 23·11 chains on slide 43·62 on the rule. The second place of decimals, that is, single links, can scarcely be relied upon at this scale with a small Mannheim rule, but as it cannot be estimated by the eye on the millimètre scale, *the rule will give the measurement as closely as it can be scaled.* To use the lower scale we must first reduce the proportion of $\frac{151}{8000}$ to its equivalent of $\frac{1}{53}$ in order to keep within the range of the rule. See p. 243.

The way in which odd scales of paces can be figured off at a glance in feet or links is explained on p. 57 of chapter on Route Surveying, and they can be plotted on the plan without any further reduction.

If the scale of miles, chains, or feet, to which it is intended to plot with the slide-rule, be given on the plan, the first process of determining the value of a mile, or other English measure of distance, in millimètres to scale, is dispensed with by merely applying the millimètre scale to the paper, and then adjusting the slide-rule to the proportion.

Railway Gradients

The nomenclature of gradients on English parliamentary maps for roads or railways is the ratio of perpendicular to base, and is expressed as inclination 1 in —

The nomenclature of railway promoters abroad and ordinary business men is the same ratio expressed in feet per mile.

Engineers have adopted percentage, such as 1 per cent. instead of 1 in 100, $2\frac{1}{2}$ per cent. instead of 1 in 40 &c., as being the most convenient, both for levelling and contouring. Side slopes are named by the ratio of base to perpendicular, as 2 to 1, &c., or sometimes in degrees of slope from the horizontal.

To Reduce Percentage to Feet per Mile by the Slide-rule

As 1 per cent. = 52·8 feet per mile, place a 1 of the lower scale of the slide over the 52·8 of the lower scale of the rule, then 5 of the slide will be over 26·4, that is to say, ·5 per cent. is equal to 26·4 feet per mile, or 5 per cent. equal to 264 feet per mile.

Example 1. What percentage will be a grade of 324 feet per mile? Opposite the 324 on the rule we find 6·13 on the slide.

The left hand 1 of the slide gives grades of 1 to 1·9 per cent., or 10 to 19 per cent.

The right hand 1 gives grades of 1·9 to 10 per cent., and 19 to 100 per cent.

To Reduce the Inclination, such as 1 in 20, 1 in 30, &c., to Feet per Mile

The result is obviously 5,280 divided by the ratio. Place a 1 of the lower slide-scale over the ratio on the rule, and read off the feet per mile on the slide opposite 5,280 on the rule.

Example 2. How many feet per mile are there in a grade of 1 in 18·1?

Place the left hand 1 of the slide opposite the 18·1 on the rule, and opposite 5,280 on the rule will be found 292·1 on the slide.

To Reduce the Inclination as above to the Percentage

Place the slide as above, but read the percentage on the slide opposite the right hand of the rule. Thus, in the preceding example the 5·52 is found opposite to the right hand 1 of the rule.

To Find the Angle of Slope corresponding to any Gradient

1. Reduce the grade in whatever form it is given to its equivalent percentage as just explained.

2. Find the angle from the line of tangents by placing it in its initial position, and reading off the angle on the slide opposite to the percentage on the rule.

Example 3. What is the angle corresponding to 270 feet per mile?

Placing the 270 on the slide above the 5,280 on the rule, we have opposite to the 1 of the rule 5·11.

Reversing the slide to line of tangents, we have opposite to 5·11 the angle 2° 55′, or 2°·92.

Example 4. What is the angle corresponding to 27 feet per mile?

Similarly to the preceding example we find the percentage=·51, and this is less than the line of tangents will give. Without reversing the slide we bring the mark for single minutes situated at 3·44 on the slide, and indicated by a single stroke 1, opposite to the 1 on the rule, then the slide will give tangents or sines of small angles which are alike proportional to those angles. The following table shows that ·51 per cent. lies somewhere between 6′ 30″ and 34′ 18″ of angular value, and we find the exact angle 17·5′ under the 51 of the rule.

Example 5. Suitable for flow of rivers. What is the angle of slope corresponding to a fall of six inches to the

mile? We see from Table XXIX. that the percentage is somewhere between ·002 and ·01, and the angle between 3″ and 20″. Placing 5 over 5,280 we find the percentage ·00948, and by the double stroke mark for seconds the angle 19·5″.

Example 6. What is the angle of a 1 in 7·32 slope? With a 1 of the slide over 7·32 we find the percentage to be 13·66, and from the tangent scale in its initial position angle =7° 46′.

TABLE XXIX.—*Of leading values of slopes in percentage and angle for checking the slide-rule.*

Feet per mile	S = tan of slope	Percentage	Angle (sexagesimal)	Angle (decimal)
0·1	·0000189	·00189	0° 0′ 3·9″	·0011
0·528	·0001000	·01000	0° 0′ 20·6″	·0057
1·0	·0001894	·01894	0° 0′ 39·0″	·0108
5·28	·001000	·10000	0° 3′ 25·8″	·0571
10·0	·001894	·1894	0° 6′ 30·0″	·1083
52·8	·010000	1·0000	0° 34′ 18·0″	·5717
100·0	·01894	1·894	1° 5′ 14·0″	1·0833
528·0	·10000	10·000	5° 42′ 38·0″	5·7106
1000·0	·18940	18·940	10° 43′ 45·0″	10·7292

All the values of S are also equal to the sines except the last three, which are somewhat larger.

To interpolate between the values in this table.

Example 7. Find all particulars as in the table for a grade of 7 feet per mile. S=7 × ·000189=·00134. Percentage=7 × ·0189=·134. Angle=7 × 39″=4′ 33″.

SQUARES AND SQUARE-ROOTS OF SMALL DECIMALS

Table XXX. will facilitate the use of the slide-rule in the involution and evolution of small decimal numbers, specially of those used in Kutter's formula.

Example 8. Find the square of ·0029. From the table we see that this must be between ·00000625 and ·00001, and by inspection of the rule find it to be ·00000841.

TABLE XXX.—*Of squares and cubes for checking the slide-rule.*

Number	Square	Cube	Number	Square	Cube
·0025	·00000625		1·414	2·00	2·818
·00316	·00001		1·50	2·25	3·375
·005	·000025		1·732	3·00	4·196
·0075	·00005625		2·0	4·00	5·359
·01	·0001	·0000010	2·1544	4·6416	10·0
·025	·000625	·0000156	3·1623	10·0000	31·6228
·0316	·001	·0000316	4·6416	21·5443	100·0
·05	·0025	·000125	10·0	100·0	1000·0
·075	·005625	·000422	21·5443	464·1587	10000·0
·1	·01	·001	31·6228	1000·0	31622·8
·25	·0625	·0156	46·416	2154·4	100000·0
·316	·1	·0316	100·0	10000·0	1000000·0
·5	·25	·125	215·4435	46415·87	10000000·0
·75	·5625	·422	316·227	100000·0	31622777·0
1·0	1·0	1·0	464·1587	215443·5	100000000·0
1·25	1·563	1·953	1000·0	1000000·0	1000000000·0

Example 9. Find the square root of ·00095. From the table we see that this must be between ·025 and ·0316. From inspection of the rule it is determined as ·0308.

If the student will take the trouble to work out one or two of these sums in the ordinary way, and then endeavour to obtain one or two decimal square roots by the slide-rule alone without the table, he will at once see what an assistance it is.

CUBES AND CUBE-ROOTS OF NUMBERS

Invert the slide, keeping the numerical scale upwards. Adjust it so that what was the left-hand upper scale of the slide becomes the right-hand lower scale, with the figures upside down. Now place the number to be cubed on the slide over the same number on the rule, and read off the cube on the slide opposite to the left hand 1 of the rule.

Example 10. Find the cube of ·373. From the table we see it will be somewhere between ·0316 and ·125. Placing the 373 of the slide over the 373 of the rule, we find above the left hand 1 of the rule 519, which we read ·0519.

To extract the cube-root with the rule as above, place the number on the slide over the left hand 1 of the rule, and search for a number on the slide which is opposite to the same number on the rule, that is the cube root.

Example 11. Extract the cube root of 1,575. We see from the table that it will be somewhere between 21 and 10. Place the left hand 1 of the rule under the 1,575 of the slide, and it will be found that the coincident number is 11·63.

Example 12. Extract the cube root of ·0954. From the table we see that it will be between ·5 and ·316, and placing the left hand 1 of the rule under the 954 of the slide, we find the coincident number to be 457, which we write as ·457.

RAILWAY SLEEPERS

Method of obtaining by a single adjustment of the rule, when the dimensions and pitch are given, the number, quantity of cubic feet, quantity of cubic yards,[1] weight in tons of 2,240 lbs., and price in sterling or dollars. Place the pitch in feet and decimals upon the slide opposite 5,280 on the rule.

Read the number per mile on the rule in thousands opposite the 1 of the slide.

Read the quantity of cubic feet per mile on the rule in thousands opposite the value of A in Table XXXI.

Read the quantity of cubic yards per mile on the rule in hundreds opposite the value of B in the table. Read the quantity of tons per mile on the rule in tens or hundreds, opposite the value of D in the table. Read the price per mile in sterling or dollars, in hundreds or thousands, from E or F.

Example 1. Find the above desiderata for standard-gauge sleepers 9 feet × 10″ × 6″, pitched 2′ 9″, at 2*s*. apiece.

Place the pitch 2·75 on the slide opposite to 5,280 on the rule.

Then the 1 of the slide is opposite to 1,920 No. on the rule.

„	3·75	„	(A) „	„	7,200 c. ft.	„
„	1·39	„	(B) „	„	267 c. yds.	„
„	5·86	„	(D) „	„	112·1 tons	„
„	100	„	(E) „	„	192*l*.	

[1] The quantity of cubic yards is required as a deduction from the ballast.

Example 2. Find the above particulars for narrow-gauge sleepers, 6′ 6″ × 7″ × 4″, pitched 2′ 3″, at 35 cents apiece.

Place the pitch 2·25 on the slide opposite to 5,280 on the rule.

Then the 1 of the slide is opposite to 2,350 No. on the rule.
,, 1·27 ,, (A) ,, ,, 2,985 c. ft. ,,
,, 4·69 ,, (B) ,, ,, 110·1 c. yds. ,,
,, 1·98 ,, (D) ,, ,, 46·5 tons ,,
,, 350 ,, (F) ,, ,, \$822

TABLE XXXI.—*Quantity, weight and cost table of railway sleepers for standard and narrow gauge.*

Dimension Ft. × in. × in.			A. Cub. feet in one sleeper	B. Cub. yds. in ten sleepers	C. Wt. in lbs. of one sleeper at 35 lbs. per c. ft.	D. Wt. in tons of 100 sleepers at 35 lbs. per c. ft.
4	6	4	0·667	0·247	23·3	1·04
5	7	4	0·972	0·360	34·0	1·52
6	7	4	1·167	0·432	40·6	1·82
6½	7	4	1·267	0·469	44·3	1·98
7	7	5	1·705	0·632	59·7	2·67
8	8	6	2·667	0·988	93·5	4·17
8	9	6	3·000	1·110	105·0	4·69
8	9	7	3·500	1·296	122·5	5·47
8	10	6	3·333	1·233	116·5	5·20
8	10	7	3·889	1·441	136·0	6·07
8	10	8	4·444	1·645	155·3	6·93
8	12	8	5·333	1·976	186·8	8·33
8½	8	6	2·833	1·048	99·3	4·42
8½	9	6	3·188	1·181	111·6	4·98
8½	9	7	3·719	1·377	130·1	5·81
8½	10	6	3·542	1·312	124·0	5·53
8½	10	7	4·132	1·530	144·8	6·45
8½	10	8	4·722	1·750	165·0	7·39
8½	12	8	5·667	2·095	198·0	8·85
9	8	6	3·000	1·110	105·0	4·69
9	9	6	3·375	1·250	118·0	5·27
9	9	7	3·938	1·456	138·0	6·15
9	10	6	3·750	1·388	131·1	5·86
9	10	7	4·375	1·619	152·8	6·84
9	10	8	5·000	1·851	175·0	7·81
9	12	8	6·000	2·221	210·1	9·37

E, cost of 1,000 sleepers in £ sterling at 1/6, 75; 1/9, 87·5; 2/0, 100; 2/3, 112·5; 2/6, 125; 2/9, 137·5; 3/0, 150; 4/0, 200; 5/0, 250.

F, cost of 1,000 sleepers in \$ at 35c., 350; 50c., 500; 70c., 700; 100c., 1000.

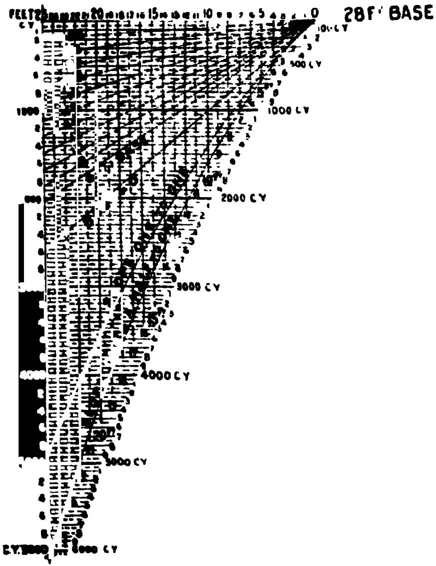
Earthwork

It is becoming very customary to use diagrams for measuring earthwork for preliminary estimates. They are usually drawn to large scale, and are correct to a fraction of a yard for level cuttings. To avoid folds, they are in a thin quarto or folio book, which becomes quite unwieldy. The little diagram given on Plate V. is quite close enough for a preliminary estimate. It has the advantage over tables—first, that it is all on one page; secondly, that it is applicable to any base and any slope. It will be observed that for any additional depth of cutting, the quantities for varying width of base increase as the ordinates of a triangle, but the quantities in the side-slopes increase as ordinates to a curve. The line marked base=0 is a datum line for slopes only; ordinates measured *upwards* from it to the curve give quantities *in the two side-slopes* for 100 feet length.

It is also a datum for the central portion alone; ordinates measured *downwards* from it to the line marked with the given width of base give the quantities in the central portion.

When the total quantity, central and sides, is required, the ordinate is measured clear through from the line marked with the given width of base upwards to the curve. This has to be done with the dividers, or with a slip of paper. If with the former, and much work has to be done, a piece of tracing-cloth should be gummed over the diagram to preserve it. The length of the ordinate is then applied to the vertical scale of cubic yards. When considerable work is to be done from the same width of base a piece of paper can be gummed down so as to cover all below the datum which is being used.

Although the side-slopes do not vary directly as the depth, they *do* vary for the same depth *directly* as the slope.* For instance, a 2 to 1 slope contains twice as much as a 1 to 1



Universal earthwork diagrams for level cuttings or embankments, constructed from prismoidal formula. Quantities in cubic yards per 100 ft. distance.

Note. If this diagram is in frequent use with dividers, a piece of dull-back tracing cloth, gummed over it by the four corners, will protect it.

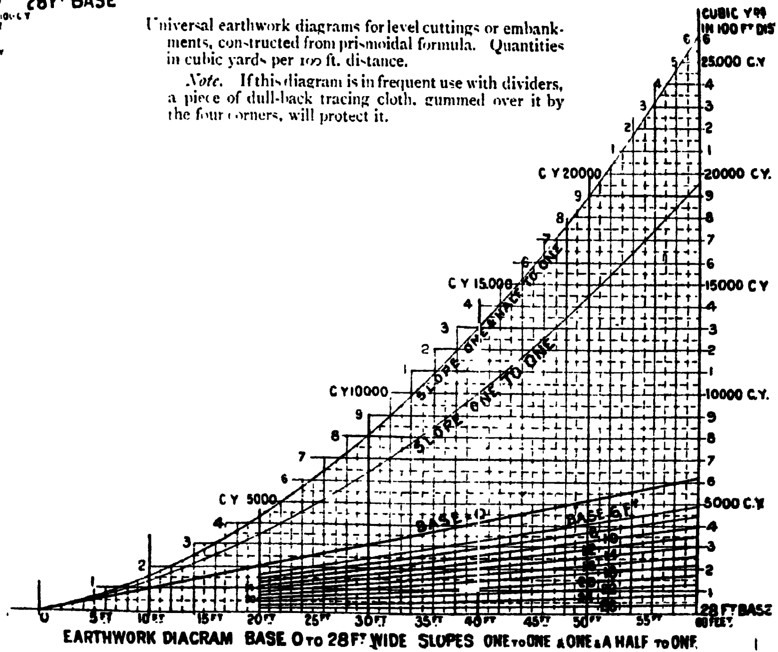


Fig. 76.

for the same depth. We can therefore obtain other values by a simple proportion.

The diagram is calculated from the prismoidal formula, which is a modification of Simpson's rules for land areas and marine displacements, and may be relied upon within 0·5 per cent. If used with a paper slip instead of dividers, it will stand a considerable amount of usage.

The smaller diagram, Fig. 76, is only an enlargement of that portion of the larger one between 0 and 25 feet of vertical depth, in order to obtain a clearer reading.

The diagram gives the contents in cubic yards in 100 feet length of level cuttings of any base from 6 to 28 feet, and having side-slopes of 1 to 1 or $1\frac{1}{2}$ to 1. For any less distance than 100 feet, such as Gunter's chains of 66 feet, or odd distances, the quantities given by scale are obtained by simple proportion. With the slide-rule, by placing the 1 of the slide opposite the 66 or other fraction of 100 feet on the rule, and reading opposite the full quantity on the slide, the odd quantity on the rule.

Example. To find the cubic contents of 100 feet length of embankment, base 18 feet, side-slopes 1 to 1, and 27 feet deep. On the larger diagram, tracing the base line marked 18 to where it intersects the vertical 27, we scale from said intersection up the vertical to the lower curve, and applying the quantity to the end vertical, we find it measures exactly 4,500 cubic yards, which is the required quantity in 100 feet length.

The cubic content of a level cutting of any length and any slope may be obtained from this diagram by a simple rule of three sum, or by direct scaling with proportional compasses.

Rule. Multiply the quantity in 1 to 1 slopes by the given ratio, and add to the product the quantity for the given base. If the slope is given in degrees from the vertical, multiply by the tabular tangent; if from the horizontal, by the cotangent of the slope.

Example. Required the cubic content of a piece of level cutting. Base 24 feet, height 27 feet, slopes $\frac{3}{4}$ to 1, length 13 feet. By diagram, the quantities are for the base 2,400, and for the side slopes 2,700.

$$\left(2400 + \frac{2{,}700 \times 3}{4}\right) \times \frac{13}{100} = 575 \text{ cubic yards.}$$

When the cuttings are in side-hill they sometimes have to be equalised, even on preliminary survey, for which a graphic method is given on Fig. 77, which represents a section of 'hog-backed' cutting ABCDHG, in which the crooked portion ABC is replaced by the straight line CFE; BE is drawn parallel to AC; to find E.

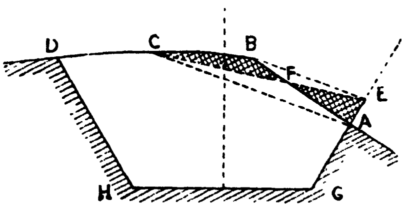


FIG. 77.

The triangles EBC, BEA are equal, being on the same base and between the same parallels; deducting the common portion EFB, the remainder EFA, which is added by the equalising line, is equal to the remainder FBC, which is subtracted. The crooked portion DCE is replaced by another equalising line in a similar manner. Finally the one sloping surface line is replaced by a horizontal equivalent as follows:

Let DE, Fig. 78, be the final equalising slope line. Assume a point *a* in the centre line of the section above DE, and mark *b*, *b'* by a parallel run up from HG. Find *b''* by a parallel run up from *b*E to D; halve the error *b'b''* in *b'''*, and *b''''* *a'* *b'''* will be the horizontal surface line of an equivalent level cutting. This should be checked by a parallel

run up from b'''' E to D, and if not quite correct, the error bisected a second time.

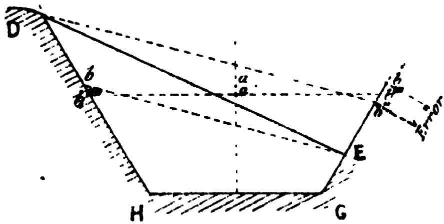


Fig. 78.

The diagram, Fig. 79, is a rough approximation curve constructed from a few actually measured areas, and may be used only within the limits given.

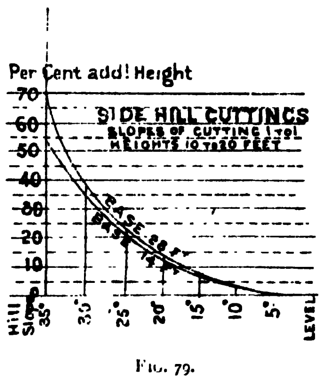


Fig. 79.

Example. What would be the equivalent level cutting when the slope of the surface of the ground (or its equivalent DE, Fig. 78) is 25°, and the central depth 18 feet, base 21 feet? The percentage here is 22, therefore the equivalent level cutting $= 18 \times \cdot 22 + 18 = 21 \cdot 96$ deep.

Iron Bridges

The diagram on Plate VI. has been prepared from the empirical formula of the late Mr. Trautwine, after checking several spans both of plate-girders and trusses. Designs vary greatly, especially in British construction, sometimes from unavoidable circumstances, more often from a desire for novelty on the part of the designer. It is therefore impossible to give any precise formula which will cover the engineer's 'personal equation;' but it is perfectly possible to give a formula and a diagram which will *represent a weight which a bridge of moderate span need not exceed, in order to be safe within a certain limit under given permanent and rolling loads*, and that condition is fulfilled by the diagram. The standard gauge covers a range from 4 feet $8\frac{1}{2}$ inches to 5 feet 3 inches, and the narrow gauge 3 feet or metre.

The formulæ are as follows:

For plate-girders up to 75 feet.

Weight in pounds per foot run of girders only $= 5 \times$ span in feet $+ 50 \sqrt{\text{span in feet}}$.

For open trusses up to 250 feet.

Weight in pounds per foot run of trusses only $= 4{\cdot}5 \times$ span in feet $+ 22 \sqrt{\text{span in feet}}$.

The weights of the complete bridge are all scaled for either gauge from the bottom to the curve bearing the proper designation; the weights of the trusses alone are scaled from the line marked platform only; the weights of the platform only are scaled from the bottom.

The following comparisons with the weights of actual structures will show to some extent the divergences to be expected from the formula.

Oak Orchard Viaduct, New York State, 23 spans of 30 feet each, by Mr. Chas. Macdonald, engineer. They are plate-girders 20″ deep, trussed by a centrepost and eyebars carrying a standard-gauge single-line railway. Actual weight 3·12 tons. Weight by diagram 5·00 tons.

PLATE VI.

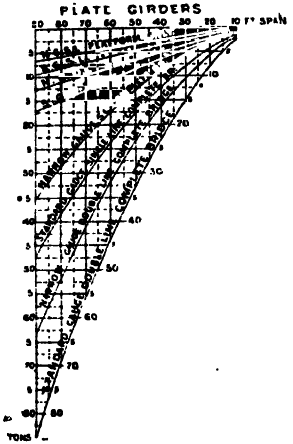


FIG. 81.

Universal Iron Bridge Diagram.

Note 1. The standard-gauge curve is 4' 8½", but will do also for 5' 3". The narrow-gauge curve is 3' 0", but will do also for metre. Calculated for a rolling-load of 1½ tons per foot run. Iron to bear 5 tons per square inch in tension.

Note 2. If this diagram is in frequent use with dividers, a piece of dull-back tracing cloth, gummed over it by the four corners, will protect it.

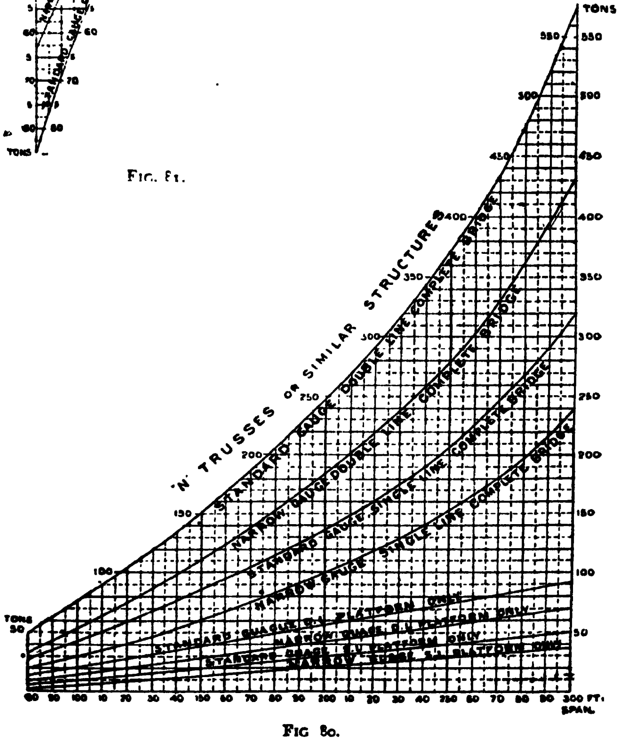


FIG 80.

Railway bridge over the Ohio at Cincinnati, Mr. I. H. Linville engineer. Deck span of 110 feet carrying standard-gauge single line. Braced on the Pratt system; actual weight 46·7 tons, weight by diagram 48 tons.

Double line, standard-gauge railway truss bridge at Harrisburg, Pa. 21 spans of 156′ 6″ each. Weight of a span by actual measurement 129 tons. Weight by diagram 119 tons.

Jhelum bridge, British India. Mr. Lee Smith engineer-in-chief. Metre gauge. Single line and two footways. Deck system with lattice bracing of single intersection, spans fifty in number, 97′ 6″ long. Actual weight of one span 42·4 tons; weight by diagram for single line 29 tons, for double line 50 tons. Mean 39·5 tons.

IRON TRESTLE PIERS

Such wide differences of design exist in iron trestles that it is almost impossible to form either a table or a diagram of their weights.

One of the largest of such structures, the *New Portage Viaduct*, of America, has towers of maximum height 203 feet. It is built for double-line standard gauge, and carries spans on either side of 118 feet. The weight of the trestle, consisting of four columns, and bracing weighs about 1,400 lbs. per foot of vertical height.

Another celebrated trestle, the *Kinzua Viaduct* in Pennsylvania, is built for single-line standard gauge. The maximum height of any tower is 278 feet. The tower is capped by girders of 38 feet span, and supports adjacent spans of 61 feet. The weights vary from 500 to 700 lbs. per foot of vertical height.

On the *Oak Orchard Viaduct* of twenty-three 30-feet spans on single trestles, called ‘bents,’ consisting of a pair of raking posts and bracing, carrying a single-line standard-gauge railway, the weight of the bents per foot of vertical height up to 75 feet was 160 lbs.

Stone Bridges of Coursed Rubble

The diagrams on Plate VII. for stone bridges are constructed from Mr. Trautwine's formulæ. The arches are semicircular. The depth of keystone is taken from the formula

$$\text{Depth of keystone in feet} = \frac{\sqrt{\text{rad.} + \text{half span}}}{4} + \cdot 2 \text{ feet.}$$

The thickness of the abutment is taken from the formula

$$\text{Thickness in feet at springing} = \frac{\text{rad. in ft.}}{5} + \frac{\text{rise in ft.}}{10} + 2 \text{ ft.}$$

The abutment is plumb on the face, and battered on the back from the thickness at the springings found by the formula to a thickness at ground level $= \frac{4}{5}$ the vertical height from ground to springing, which batter is continued down through the ground to the bottom of the foundations, 3 feet below ground level, and footings added.

The spandrel walls are $\frac{4}{10}$ of their vertical height where they join the wing walls and 2 feet 6 inches at cope.

The wing walls are also $\frac{4}{10}$ of their vertical height at base, and diminishing to 2 feet 6 inches at cope.

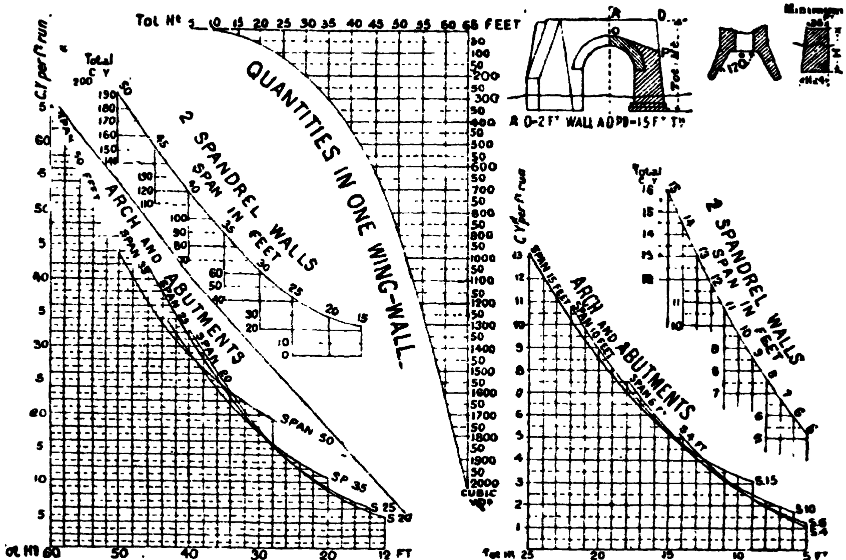
Brick Bridges

The formula for stone bridges will serve for brick bridges by taking half the quantity for the wing-walls. The quantities in the diagram agree with American practice for rubble stone, but wing-walls of brick are not nearly so wasteful of material.

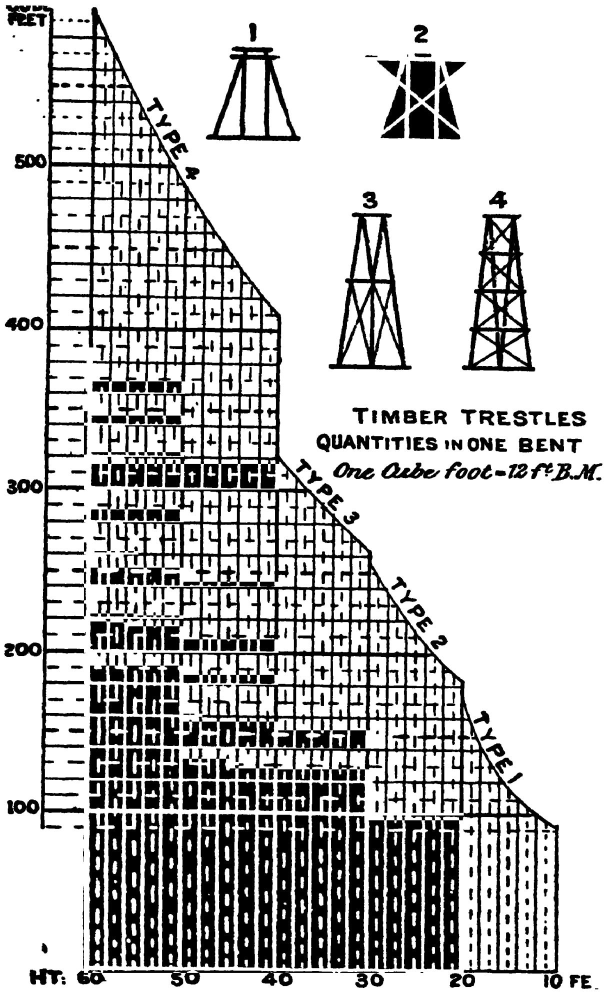
The two following examples are type bridges on the Eastern and Midlands Railway, England, and agree in the main as to quantity with those of most other railways in England for similar height, span, and width.

Example 1. 25-foot span over-bridge for double line; elliptical arch; total height 21·5 feet, width 18 feet; one counterfort to each abutment.

PLATE VI



Note.—If this diagram is in frequent use with dividers, a piece of dull-back tracing cloth, gummed over it by the four corners, will protect it.



Note.—If this diagram is in frequent use with dividers, a piece of dull-back cloth, gummed over it by the four corners, will protect it.

	Actual measurement, cubic yards	Diagram (with half quantity for wings), cubic yards
Arch, abuts., cforts., and parapets .	196·5	180
4 wings	147·9	140
Spandrels	9·3	41
	353·7	361

Example 2. 15-foot span under-bridge ; segmental arch ; total height 20 feet ; two counterforts to each abut ; width 23·5 feet.

	Actual measurement, cubic yards	Diagram (with half quantity for wings), cubic yards
Arch, abuts., cforts., and ppts. .	200·0	194
4 wings	126·0	123·5
Spandrel walls	4·0	21
	330·0	338·5

Timber Trestles

Plate VIII. gives quantities in one bent. Types 1, 2, and 3 are for a trestle carrying a single-line standard-gauge railway on a pair of stringers 14″ × 14″. The bents 12 feet apart centre to centre. The posts, sills, and caps 12″ × 12″. The sway-braces and wales 12″ × 6″. The quantities include the longitudinal bracing but not the sleepers (cross-ties).

Type 4 is for one of a pair of bents forming a pier for a Howe truss. Thus for a height of 45 feet the diagram gives 445 cubic feet in one bent, therefore the pier would contain 890 cubic feet. For narrow-gauge railways three-quarters of the quantities may be taken.

A Howe truss is a composite bridge commonly used in America, resembling in appearance the lattice-girder in England. The top and bottom chords and the diagonal bracing are of timber, and the stresses are distributed through the members to the piers by vertical tension-rods.

The following table is an extract from Trautwine's 'Pocket Book,' which, in an excellent article upon trusses, fully

TABLE XXXII.—*Howe Trusses of timber and iron. Weights approximately equal to iron trusses of same span.*

Clear span	Rise	No. of panels	An upper chord: No. of pieces	An upper chord: Size of each piece	A lower chord: No. of pieces	A lower chord: Size of each piece	An end brace: No. of pieces	An end brace: Size of each piece	A centre brace: No. of pieces	A centre brace: Size of each piece	A counter: No. of pieces	A counter: Size of each piece	End rod: No. of pieces	End rod: Size of each piece	Centre rod: No. of pieces	Centre rod: Size of each piece
ft.	ft.			ins.		ins.		ins.		ins.		ins.		ins.		ins.
25	6	8	3	5 × 6	3	5 × 12	2	5 × 8	2	5 × 6	1	5 × 6	2	1½	2	1
50	9	9	3	6 × 9	3	6 × 14	2	6 × 9	2	5 × 8	1	5 × 8	2	1⅞	2	1¼
75	12	10	3	6 × 12	3	6 × 14	2	6 × 11	2	6 × 8	1	6 × 8	2	2¼	2	1⅜
100	15	11	3	6 × 14	3	6 × 16	2	8 × 12	2	6 × 10	1	6 × 10	2	2½	2	1½

describes how the truss should be built, and gives the theory of static equilibrium in so simple a manner that for this, or for any ordinary structure of the kind, those who have never studied mathematics can easily work out the proper dimensions for any intermediate size. Composite bridges are also largely made of the N type, having the uprights of timber and oblique tension-rods. The total quantity of material is somewhat less with this form of bridge, but there is more ironwork in it. High timber viaducts are not much built now for several reasons, but mainly on account of their inflammability.

Howe trusses up to 50 feet span can be placed upon timber trestle piers, but if the latter are higher than 60 feet they require three bents to a pier, like the old viaduct at Portage, New York State, which was 234 feet high from bed of river to rail and contained 133,000 cubic feet of timber including trusses. The quantity curve increases very rapidly for trestles over the heights in the diagram. The maximum length of bents on the Portage Viaduct was 190 feet. Taking a proportion of $\frac{190}{60} \times 595$ (the quantity in the diagram for a bent 60 feet high), multiplying by 3, and allowing for timber in the trusses, the actual quantity would

be about $2\frac{1}{2}$ times more than the diagram. It is so very rare that timber trestles are built above 60 feet nowadays that it would be needless to extend the diagram.

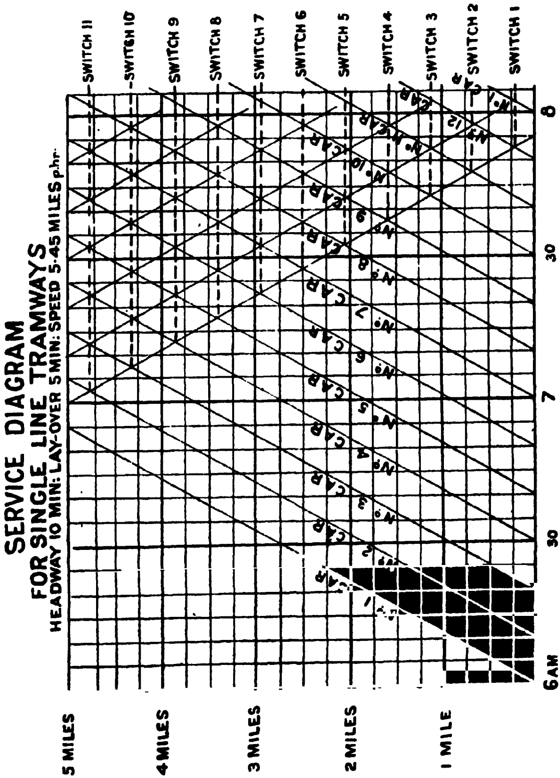


FIG. 82.

SERVICE DIAGRAMS

The service diagrams Figs. 82 and 83, for tramways and railways working single line with crossing stations, are

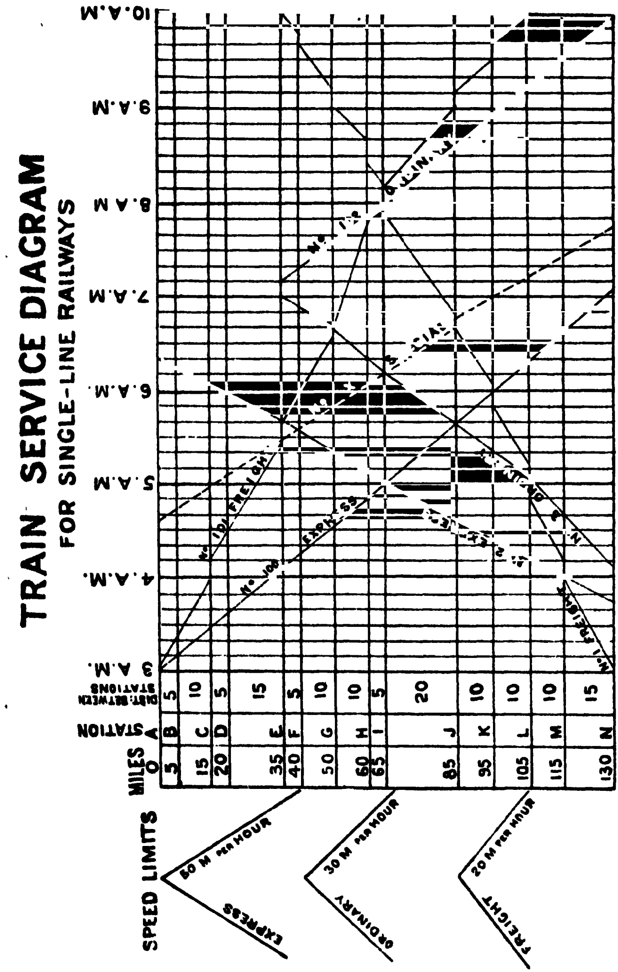

TRAIN SERVICE DIAGRAM
FOR SINGLE-LINE RAILWAYS
3 A.M.
4 A.M.
5 A.M.
6 A.M.
7 A.M.
8 A.M.
9 A.M.
10 A.M.
DIST. BETWEEN STATIONS
STATION
MILES
A B C D E F G H I J K L M N
0 5 15 20 35 40 50 60 65 85 95 105 115 130
5 10 5 15 5 10 10 5 20 10 10 10 15
SPEED LIMITS
EXPRESS
50 M PER HOUR
ORDINARY
30 M PER HOUR
FREIGHT
20 M PER HOUR
No. 100 EXPRESS
No. 101 FREIGHT
No. 1 FREIGHT

Fig. 83.

necessarily to a small scale. They should be drawn for working purposes on a sheet of double elephant, which will include the 24 hours for railway and the working hours for tramways. The value of the diagrams is that they show at a glance where the trains meet and pass, so that a special train can be interpolated at any time with no danger of mistaking its connections.

English railways worked upon the block system with the double line, or with the staff or tablet on single line, need nothing of this kind, but pioneer railways are generally worked merely by telegraph or even telephone, and the diagram forms an indispensable adjunct. It also serves the purpose with tramways of showing the number of cars required to work a certain service.

In diagram Fig. 82 we will explain the first round trip of No. 1 tramway car.

Leave depôt 6 A.M.; run through to terminus with line clear; arrive terminus 6.55; leave terminus 7.00; cross No. 2 car on No. 11 switch; cross No. 3 car on No. 10 switch, No. 4 car on No. 9 switch, and so on; arrive depôt 7.55; leave again at 8.

In diagram Fig. 83 we will follow No. 1 freight, whose waybill will be as follows. Leave N at 3 A.M.; switch off at M to allow No. 2 express to pass; leave M at 4.10; switch off at L for No. 3 ordinary; leave L at 5.15; switch at K for No. 100 express; leave K at 6.05, switch at J for No. x special; leave J at 6.45; switch at I for No. 101 freight and No. 102 ordinary; leave I at 8.10; switch at H for shunting from 8.25 to 8.40; ditto at G from 9 to 9.12 and at F from 9.40 to 9.45.

Centrifugal Force

The following rules are prepared on the assumption of gravity being 32·2 feet per second, and R the radius of rotation; they are sufficiently approximate when the thick-

ness of the body such as the rim of a flywheel is not more than $\frac{1}{3}$ of the radius from out to out; the radius being measured to the centre of gravity of the rim.

Let F=centrifugal force in lbs. per lb. weight of rotating body.

Let F′=centrifugal force in lbs. per ton of rotating body.

Let R=radius of rotation in feet.

Let N=number of revolutions per minute.

Let M=number of revolutions per second.

$F = \cdot 00034\ RN^2$ . . . . (1)

$F = 1 \cdot 224\ RM^2$ . . . . (2)

$F' = 0 \cdot 7616\ RN^2$ . . . . (3)

Example. What is the centrifugal force in lbs. of a body making 120 revolutions per minute, at a radius of 5 feet, and weighing 3 lbs.?

$F = 1 \cdot 224 \times 5 \times 2^2 = 24 \cdot 49$ lbs. per lb.
or for 3 lbs. $= 73 \cdot 47$ lbs.

Formulæ suitable for side-stresses on viaducts due to centrifugal force:

Let V=velocity in feet per second.

Let VV=velocity in miles per hour.

Let F=centrifugal force in lbs. per ton of 2,240 lbs.

Let FF=centrifugal force in lbs. per ton of 2,000 lbs.

Let R=radius in feet.

Let RR=radius in Gunter's chains of 66 feet.

$F = 69 \cdot 5 \dfrac{V^2}{R}$ . . . . (4)

$FF = 62 \cdot 1 \dfrac{V^2}{R}$ . . . . (5)

$F = 2 \cdot 268 \dfrac{VV^2}{RR}$ . . . . (6)

$F = 149 \cdot 7 \dfrac{VV^2}{R}$ . . . . (7)

$$FF = 2{\cdot}025 \frac{VV^2}{RR} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$FF = 133{\cdot}6 \frac{VV^2}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Example 1. What is the centrifugal force in lbs. per ton of 2,240 lbs. on a curve of 23·5 chains at a velocity of 31·3 miles per hour?

By slide-rule. Place the brass marker opposite 31·3 on the lower scale of the rule, and bring a 23·5 of the upper scale of the slide to coincide with the brass marker so placed. Read off the result 94·5 feet per ton on the upper scale of the rule opposite to the 2·268 of the slide.

Example 2. What is the centrifugal force in lbs. per ton of 2,000 lbs. at 60 miles per hour on a curve of 4 chains, 33 links?

Place the brass marker opposite 60 on the lower scale of the rule. Bring a 4·33 of the upper scale of the slide to coincide with the brass marker so placed. Read off the result, 1,681, on the upper scale of the rule opposite to the 2·025 of the slide.

Example 3. What is the centrifugal force in lbs. per ton of 2,240 lbs. at 40 miles per hour on a curve of 5° (1,146 feet radius)?

$$\text{By No. 7, } F = 149{\cdot}7 \times \frac{40^2}{1146} = 209 \text{ lbs.}$$

For table of certain values of F at different curvatures and speeds, see Tables VII. and VIII., p. 15.

To Reduce Thermometer Scales

Fahrenheit and Centigrade.

Boiling-point is at 100° Cent., and 212° Fahr. Zero Cent. corresponds with 32° Fahr., whence 180° Fahr.= 100° Cent.

Place the left hand 1 of the upper scale of the slide

under the 1·8 of the rule; then the slide will give Centigrade and the rule Fahrenheit, when the constant 32 is added to the latter.

$$
\begin{aligned}
\text{Thus } \cdot1^\circ \text{ Cent.} &= \cdot18+32=32\cdot18 \text{ Fahr.}\\
1^\circ \text{ Cent.} &= 1\cdot8+32=33\cdot8 \quad ,,\\
5^\circ \text{ Cent.} &= 9\cdot0+32=41\cdot0 \quad ,,\\
25^\circ \text{ Cent.} &= 45\cdot0+32=77 \quad ,,\\
100^\circ \text{ Cent.} &= 180+32=212 \quad ,,\\
250^\circ \text{ Cent.} &= 450+32=482 \quad ,,\\
35^\circ \text{ Fahr. } (32+3) &= 1\cdot66 \text{ Cent.}\\
300^\circ \text{ Fahr. } (32+268) &= 149 \text{ Cent.}
\end{aligned}
$$

or by rule $C=\frac{5\,(F-32)}{9}$.

Below freezing-point, the values Fahr. read on the rule are deducted from 32.

$$
\begin{aligned}
\text{Thus } -1^\circ \text{ Cent.} &= 32-1\cdot8=30\cdot2 \text{ Fahr.}\\
,, \quad -5^\circ \quad ,, &= 32-9 \;=23^\circ \quad ,,
\end{aligned}
$$

Fahrenheit and Réaumur.

Boiling-point is at 80° Ré. and 212° Fahr.

Zero Ré. corresponds with 32° Fahr.

Place the left hand 8 of the upper scale of the slide under the 1·8 of the rule, and read Ré. on the slide and Fahr. on the rule, adding the constant 32.

$$
\begin{aligned}
1^\circ \text{ Ré.} &= 32+\;2\cdot25=34\cdot25 \text{ Fahr.}\\
5^\circ \text{ Ré.} &= 32+11\cdot25=43\cdot25 \quad ,,\\
25^\circ \text{ Ré.} &= 32+56\cdot25=88\cdot25 \quad ,,
\end{aligned}
$$

or by rule $F=\frac{9R}{4}+32$.

Centigrade and Réaumur.

Zero coincides in these scales, so there is no constant.

Place the 8 of the slide under a 1 of the rule, and read Ré. on the slide, and Cent. on the rule.

Thus 1° Ré. = 1·25 Cent.
2·4° Ré. = 3·00 „
100° Ré. = 125° „

or by rule $C = \frac{5}{4} R$.

The Slide-rule as a Universal Measurer

Circular Measure of Angles.

Two movements of the slide are all that is needed for the length of any circular arc to any radius.

An arc of 1° to a radius of 100 = 1·745, that is to say is 1·745 per cent. of the radius whatever that radius may be. An arc of 10° is 17·45 per cent., and so on.

(a) *To obtain the value of any arc in percentage of the radius.* Place the left hand 1 of the slide opposite to the 1,745 of the rule; then opposite to the angle in degrees and decimals on the slide will be given the percentage on the rule.

Example. What is the percentage of an arc of 22° 30′, *i.e.* 22°·5? Placing the slide as directed we find opposite the 22·5 upon it 39·2, which is the required percentage.

(b) *To obtain the linear value of the same arc to any given radius.* Leave the brass marker at the percentage found on the rule so as not to lose it, and move the slide until a 1 upon it coincides with the given radius. Then move the marker to the percentage on the slide and read off the linear value upon the rule.

Example. With the 39·2 per cent. found as above, required the linear value of the said arc to a radius of 234 feet.

Placing the slide as directed with the left hand 1 opposite to 234 on the rule, move the marker to 39·2 on the slide and read off the result, 91·7, on the rule.

Circumferences of circles. Multiply 3·1416 by the diameter, or 6·2832 by the radius. When the argument is given in twelfths or sixteenths or sixtieths reduce them to decimals by

Tables XXXIV., XXXIX., and XL., respectively, or by the slide-rule as explained further on.

(d) *Areas of circles.* Multiply the square of the diameter by ·7854, or the square of the radius by 3·142.

(e) *Surface of cylinders.* Multiply the circumference found as above by the length.

(f) *Volume of cylinders.* Multiply the area found as above by the length.

(g) *Surface of spheres.* Multiply the square of the diameter by 3·1416.

Volume of spheres=diameter^3 × ·5236.

(h) *To reduce inches to decimals of a foot.* Place 12 on the slide opposite to 10 on the rule, and read the decimals on the rule opposite the inches on the slide.

(i) *To reduce fractions of inches to decimals of a foot* Place the denominator on the slide opposite the 833 on the rule, which will be regarded as ·0833, and then read off the decimals on the rule opposite to the numerator on the slide. Thus, to express $\frac{21}{32}$ of an inch in decimals of a foot, place the 32 of the slide opposite the 833 on the rule, which represents the decimal for one inch ·0833; then 21 on the slide will be found opposite to ·0547 on the rule.

(j) *When the fraction is a mixed one of inches and fractions of inches.* Find the decimal of the integer by the *first* process, and keeping the result on record with the brass marker find the decimal of the fraction by the second process, and add the two results together.

(k) *To reduce fractions of inches to decimals of inches.* Place the denominator on the slide opposite a 1 of the rule and read the decimals on the rule opposite the numerators on the slide.

(l) *To reduce feet to metres or* vice versâ. Place a 1 of the slide, representing 100 feet, under the 30·48 of the rule and read off feet on the slide and metres on the rule.

(m) *To reduce yards into metres or* vice versâ. Place a 1 of the slide, representing 100 yards, opposite to 91·44 on

the rule; read off yards on the slide and metres on the rule.

(n) *To reduce inches and decimals into millimetres or* vice versâ. Place a 1 of the slide to represent one inch opposite to 25·4 on the rule; read inches on the slide and millimetres on the rule.

(o) *To reduce kilometres into statute miles and* vice versâ. Place a 1 of the slide, to represent one statute mile, opposite 1·609 on the rule, and read miles on the slide and kilometres on the rule.

(p) *To reduce kilometres into geographical miles and* vice versâ. Place a 1 of the slide, to represent one geographical mile, opposite 1·853 on the rule, and read geographical miles on the slide and kilometres on the rule.

(q) *To reduce statute miles to geographical miles and* vice versâ. Place a 1 of the slide, to represent one geographical mile, opposite to 1·151 on the rule, and read geographical miles on the slide and statute miles on the rule.

Metric Square Measure.

1 square centimetre	=	0·155 square inch
1 ,, metre	=	10·7641 ,, feet
1 ,, ,,	=	1·19601 ,, yard
1 square kilometre	=	247·11 acres
,, ,,	=	0·38611 square miles

Metric Weights.[1]

1 centigramme	=	0·15432 grain
1 gramme	=	15·432 ,,
1 kilogramme	=	2·2046 pounds
1 tonne	=	2204·6 ,,
,,	=	0·9842 ton

Metric Cubic Measure.

1 decalitre = 0·35316 c. ft. . . . . . = 2·2009 British gals.
,, = 0·28378 U. S. struck bushel = 2·64179 U. S. liquid gals.

See also Specific Gravity, pp. 389, 390.

Table XXXIII.—*For converting Inches into Decimals of a Foot.*

In.	Feet	In.	Feet	In.	Feet	In.	Feet
0	·0000	3	·2500	6	·5000	9	·7500
1/16	·0052		·2552		·5052		·7552
1/8	·0104	1/8	·2604	1/8	·5104	1/8	·7604
	·0156		·2656		·5156		·7656
1/4	·0208	1/4	·2708	1/4	·5208	1/4	·7708
	·0260		·2760		·5260		·7760
3/8	·0313	3/8	·2813	3/8	·5313	3/8	·7813
	·0365		·2865		·5365		·7865
1/2	·0417	1/2	·2917	1/2	·5417	1/2	·7917
	·0469		·2969		·5469		·7969
5/8	·0521	5/8	·3021	5/8	·5521	5/8	·8021
	·0573		·3073		·5573		·8073
3/4	·0625	3/4	·3125	3/4	·5625	3/4	·8125
	·0677		·3177		·5677		·8177
7/8	·0729	7/8	·3229	7/8	·5729	7/8	·8229
	·0781		·3281		·5781		·8281
1	·0833	4	·3333	7	·5833	10	·8333
	·0885		·3385		·5885		·8385
1/8	·0938	1/8	·3438	1/8	·5937	1/8	·8437
	·0990		·3490		·5990		·8490
1/4	·1042	1/4	·3542	1/4	·6042	1/4	·8542
	·1094		·3594		·6094		·8594
3/8	·1146	3/8	·3646	3/8	·6146	3/8	·8646
	·1198		·3698		·6198		·8698
1/2	·1250	1/2	·3750	1/2	·6250	1/2	·8750
	·1302		·3802		·6302		·8802
5/8	·1354	5/8	·3854	5/8	·6354	5/8	·8854
	·1406		·3906		·6406		·8906
3/4	·1458	3/4	·3958	3/4	·6458	3/4	·8958
	·1510		·4010		·6510		·9010
7/8	·1563	7/8	·4063	7/8	·6563	7/8	·9063
	·1615		·4115		·6615		·9115
2	·1667	5	·4167	8	·6667	11	·9167
	·1719		·4219		·6719		·9219
1/8	·1771	1/8	·4271	1/8	·6771	1/8	·9271
	·1823		·4323		·6823		·9323
1/4	·1875	1/4	·4375	1/4	·6875	1/4	·9375
	·1927		·4427		·6927		·9427
3/8	·1979	3/8	·4479	3/8	·6979	3/8	·9479
	·2031		·4531		·7031		·9531
1/2	·2083	1/2	·4583	1/2	·7083	1/2	·9583
	·2135		·4635		·7135		·9635
5/8	·2188	5/8	·4688	5/8	·7187	5/8	·9688
	·2240		·4740		·7240		·9740
3/4	·2292	3/4	·4792	3/4	·7292	3/4	·9792
	·2344		·4844		·7344		·9844
7/8	·2396	7/8	·4896	7/8	·7396	7/8	·9896
	·2448		·4948		·7448		·9948

TABLE XXXIV.—*Time, Coinage, and Measurement in Decimals.*

Pence Months Inches	Shillings Years Feet	Yards	Pounds sterling
1	·0833	·0277	·00466
2	·1667	·0554	·00833
3	·2500	·0833	·01250
4	·3300	·1111	·01666
5	·4167	·1389	·02083
6	·5000	·1666	·02500
7	·5833	·1944	·02916
8	·6667	·2222	·03333
9	·7500	·2500	·03750
10	·8333	·2777	·04166
11	·9167	·3056	·04583

TABLE XXXV.—*Shillings in Decimals of a Pound.*

Shillings	Pounds sterling	Shillings	Pounds sterling	Shillings	Pounds sterling	Shillings	Pounds sterling
1	·05	6	·30	11	·55	16	·80
2	·10	7	·35	12	·60	17	·85
3	·15	8	·40	13	·65	18	·90
4	·20	9	·45	14	·70	19	·95
5	·25	10	·50	15	·75	20	1·00

TABLE XXXVI.—*Days, Hours, Minutes, and Seconds.*

Minutes Seconds	Hours Minutes	Days	Minutes Seconds	Hours Minutes	Days
1	·01667	·000694	6	·10000	·004166
2	·03333	·001389	7	·11667	·004861
3	·05000	·002083	8	·13333	·005555
4	·06667	·002777	9	·15000	·006250
5	·08333	·003472	10	·16667	·006940

TABLE XXXVII.—*Weeks, Months, and Years.*

Weeks	Months	Years	Weeks	Months	Years
1	·23077	·01923	6	1·3846	·11538
2	·46154	·03846	7	1·6154	·13461
3	·69231	·05769	8	1·8462	·15384
4	·92308	·07692	9	2·0769	·17307
5	1·15380	·09615	10	2·3077	·19230

TABLE XXXVIII.—*Days, Weeks, Months, and Years.*

Days	Weeks	Months	Years	Days	Weeks	Months	Years
1	·1429	·0329	·00274	6	·8572	·1973	·01644
2	·2857	·0657	·00548	7	1·0000	·2302	·01918
3	·4286	·0986	·00822	8	1·1429	·2630	·02192
4	·5714	·1315	·01096	9	1·2857	·2959	·02466
5	·7143	·1644	·01370	10	1·4290	·3290	·02740

TABLE XXXIX.—*For converting Fractions into Decimals.*

$\frac{1}{64}$	·0156	$\frac{17}{64}$	·2656	$\frac{33}{64}$	·5156	$\frac{49}{64}$	·7656
$\frac{1}{32}$	·0312	$\frac{9}{32}$	·2812	$\frac{17}{32}$	·5312	$\frac{25}{32}$	·7812
$\frac{3}{64}$	·0469	$\frac{19}{64}$	·2969	$\frac{35}{64}$	·5469	$\frac{51}{64}$	·7969
$\frac{1}{16}$	·0625	$\frac{5}{16}$	·3125	$\frac{9}{16}$	·5625	$\frac{13}{16}$	·8125
$\frac{5}{64}$	·0781	$\frac{21}{64}$	·3281	$\frac{37}{64}$	·5781	$\frac{53}{64}$	·8281
$\frac{3}{32}$	·0937	$\frac{11}{32}$	·3437	$\frac{19}{32}$	·5937	$\frac{27}{32}$	·8437
$\frac{7}{64}$	·1094	$\frac{23}{64}$	·3594	$\frac{39}{64}$	·6094	$\frac{55}{64}$	·8594
$\frac{1}{8}$	·1250	$\frac{3}{8}$	·3750	$\frac{5}{8}$	·6250	$\frac{7}{8}$	·8750
$\frac{9}{64}$	·1406	$\frac{25}{64}$	·3906	$\frac{41}{64}$	·9406	$\frac{57}{64}$	·8906
$\frac{5}{32}$	·1562	$\frac{13}{32}$	·4062	$\frac{21}{32}$	·6562	$\frac{29}{32}$	·9062
$\frac{11}{64}$	·1719	$\frac{27}{64}$	·4219	$\frac{43}{64}$	·6719	$\frac{59}{64}$	·9219
$\frac{3}{16}$	·1875	$\frac{7}{16}$	·4375	$\frac{11}{16}$	·6875	$\frac{15}{16}$	·9375
$\frac{13}{64}$	·2031	$\frac{29}{64}$	·4531	$\frac{45}{64}$	·7031	$\frac{61}{64}$	·9531
$\frac{7}{32}$	·2187	$\frac{15}{32}$	·4687	$\frac{23}{32}$	·7187	$\frac{31}{32}$	·9687
$\frac{15}{64}$	·2344	$\frac{31}{64}$	·4844	$\frac{47}{64}$	·7344	$\frac{63}{64}$	·9844
$\frac{1}{4}$	·2500	$\frac{1}{2}$	·5000	$\frac{3}{4}$	·7500	1	1·0000

TABLE XL.—*For converting Minutes into Decimals of a Degree, or Seconds into Decimals of a Minute* ($\frac{1}{60}$).

Min. Sec.	Degree Minute	Min. Sec.	Degree Minute	Min. Sec.	Degree Minute	Min. Sec.	Degree Minute
1	·0167	16	·2667	31	·5167	46	·7667
2	·0333	17	·2833	32	·5333	47	·7833
3	·0500	18	·3000	33	·5500	48	·8000
4	·0667	19	·3167	34	·5667	49	·8167
5	·0833	20	·3333	35	·5833	50	·8333
6	·1000	21	·3500	36	·6000	51	·8500
7	·1167	22	·3667	37	·6167	52	·8667
8	·1333	23	·3833	38	·6333	53	·8833
9	·1500	24	·4000	39	·6500	54	·9000
10	·1667	25	·4167	40	·6667	55	·9167
11	·1833	26	·4333	41	·6833	56	·9333
12	·2000	27	·4500	42	·7000	57	·9500
13	·2167	28	·4667	43	·7167	58	·9667
14	·2333	29	·4833	44	·7333	59	·9833
15	·2500	30	·5000	45	·7500	60	1·0000

Table XLI.—*For converting Seconds into Decimals of a Degree, or Thirds into Decimals of a Minute* ($\frac{1}{3600}$).

Sec. Thds.	Deg. Min.	Sec. Thds.	Deg. Min.	Sec. Thds.	Deg. Min.	Sec. Thds.	Deg. Min.	Sec. Thds.	Deg. Min.	Sec. Thds.	Deg. Min.
1	·0003	11	·0030	21	·0058	31	·0086	41	·0114	51	·0142
2	·0005	12	·0033	22	·0061	32	·0089	42	·0117	52	·0144
3	·0008	13	·0036	23	·0064	33	·0092	43	·0119	53	·0147
4	·0011	14	·0039	24	·0067	34	·0094	44	·0122	54	·0150
5	·0014	15	·0042	25	·0069	35	·0097	45	·0125	55	·0153
6	·0017	16	·0044	26	·0072	36	·0100	46	·0128	56	·0156
7	·0019	17	·0047	27	·0075	37	·0103	47	·0130	57	·0158
8	·0022	18	·0050	28	·0078	38	·0106	48	·0133	58	·0161
9	·0025	19	·0053	29	·0080	39	·0108	49	·0136	59	·0164
10	·0028	20	·0055	30	·0083	40	·0111	50	·0139	60	·0167

Example. What is 17° 11′ 29″ 47‴ in decimals of a degree?

47‴ = ·0130′ =		·0002°
29″ =		·0080
11′ =		·1833
	17	
	17°·1915	

Table XLII.—*Decimals of a Degree in Minutes and Seconds.*

·01	0′ 36″	·21	12′ 36″	·41	24′ 36″	·61	36′ 36″	·81	48′ 36″
·02	1′ 12″	·22	13′ 12″	·42	25′ 12″	·62	37′ 12″	·82	49′ 12″
·03	1′ 48″	·23	13′ 48″	·43	25′ 48″	·63	37′ 48″	·83	49′ 48″
·04	2′ 24″	·24	14′ 24″	·44	26′ 24″	·64	38′ 24″	·84	50′ 24″
·05	3′ 0″	·25	15′ 0″	·45	27′ 0″	·65	39′ 0″	·85	51′ 0″
·06	3′ 36″	·26	15′ 36″	·46	27′ 36″	·66	39′ 36″	·86	51′ 36″
·07	4′ 12″	·27	16′ 12″	·47	28′ 12″	·67	40′ 12″	·87	52′ 12″
·08	4′ 48″	·28	16′ 48″	·48	28′ 48″	·68	40′ 48″	·88	52′ 48″
·09	5′ 24″	·29	17′ 24″	·49	29′ 24″	·69	41′ 24″	·89	53′ 24″
·10	6′ 0″	·30	18′ 0″	·50	30′ 0″	·70	42′ 0″	·90	54′ 0″
·11	6′ 36″	·31	18′ 36″	·51	30′ 36″	·71	42′ 36″	·91	54′ 36″
·12	7′ 12″	·32	19′ 12″	·52	31′ 12″	·72	43′ 12″	·92	55′ 12″
·13	7′ 48″	·33	19′ 48″	·53	31′ 48″	·73	43′ 48″	·93	55′ 48″
·14	8′ 24″	·34	20′ 24″	·54	32′ 24″	·74	44′ 24″	·94	56′ 24″
·15	9′ 0″	·35	21′ 0″	·55	33′ 0″	·75	45′ 0″	·95	57′ 0″
·16	9′ 36″	·36	21′ 36″	·56	33′ 36″	·76	45′ 36″	·96	57′ 36″
·17	10′ 12″	·37	22′ 12″	·57	34′ 12″	·77	46′ 12″	·97	58′ 12″
·18	10′ 48″	·38	22′ 48″	·58	34′ 48″	·78	46′ 48″	·98	58′ 48″
·19	11′ 24″	·39	23′ 24″	·59	35′ 24″	·79	47′ 24″	·99	59′ 24″
·20	12′ 0″	·40	24′ 0″	·60	36′ 0″	·80	48′ 0″	1·00	1° 0′ 0″

Thousandths of a Degree in Seconds and Decimals of a Second.

·001	3″·6	·003	10″·8	·005	18″·0	·007	25″·2	·009	32″·4
·002	7″·2	·004	14″·4	·006	21″·6	·008	28″·8	·010	36″·0

TABLE XLIII.—*Of Cotangents of a few leading Angles with their corresponding Tangents for checking the Slide-rule.*

Tan of	—	Cotan of	Tan of	—	Cotan of
°		°	°		°
5	0·0874	85	60	1·732	30
10	0·1763	80	65	2·144	25
15	0·2679	75	70	2·747	20
20	0·364	70	75	3·732	15
25	0·466	65	80	5·671	10
30	0·577	60	85	11·430	5
35	0·700	55	86	14·300	4
40	0·839	50	87	19·081	3
45	1·000	45	88	28·636	2
50	1·192	40	89	57·29	1
55	1·428	35			

Rule. To find a tangent of a higher angle than is give by the slide.

$$\tan \Theta = \frac{1}{\cot \Theta} = \frac{1}{\tan (90° - \Theta)}$$

Example. Find tan 73° 20′.

The table shows that the decimal punctuation will b between 2·7 and 3·7.

$$\tan (90° - 73° \ 20') = \tan 16° \ 40'$$

Place the scale of tangents in its initial position, and fin tan 16°40′=·299.

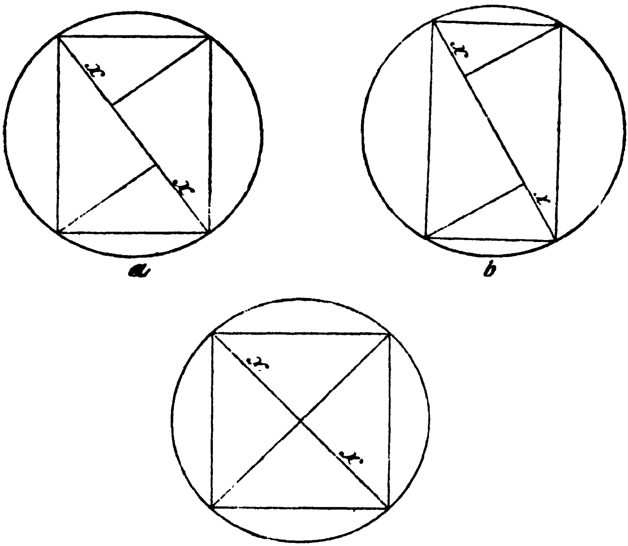
Reverse the slide and adjust the scale of numbers wit 299 over the 1 of the rule, and opposite to the other 1 c the slide will be found 3·34 the required tangent.

TABLE XLIV.—*Of some Higher Sines than are clearly given on the Slide-rule.*

Sine of	—	Cosine of	Sine of	—	Cosine of
°		°	°		°
89	·9998	1	85	·9962	5
88	·9994	2	80	·9848	10
87	·9986	3	75	·9659	15
86	·9976	4	70	·9397	20

Measurement of Tree Timber

In measuring felled logs, allowance is made first for squaring, secondly for bark. As to the first, instead of multiplying the area of cross section by the length, $\frac{1}{4}$ of the girth squared is taken as the area, and this is the quantity given in square feet in Hurst's 'Pocket Book.' It is about 28 per



a *b*

c

Fig. 84.

cent. less than the actual cross section. When the tree tapers considerably the two ends and middle are girthed and the average taken. A tree is not called timber unless the stem measures 24 inches in circumference.

Rule. Marketable area$=(\frac{1}{4}$ girth$)^2$; marketable cubic contents$=$length $\times$ $(\frac{1}{4}$ girth$)^2$.

By slide-rule. Find the $\frac{1}{4}$ girth in decimals of a foot by

Table XXXIII. (unless a decimal tape is used), square by Rule 4, p. 244, and multiply by Rule 2.

If the bark is on the tree, deduct as follows:

For oak, old and thick barked	$\frac{1}{10}$ of girth.
For oak, young and thin barked	$\frac{1}{12}$,,
For elm, pine, and fir	$\frac{1}{15}$,,
For ash and beech	$\frac{1}{30}$[1] ,,

Comparison of various beams cut from a log 2 feet in diameter.

		sq. ft.
a, the most serviceable beam . .	$x = \frac{1}{3}$ diam.,	area = 1·92
b, the stiffest beam	$x = \frac{1}{4}$ diam.,	area = 1·75
c, square timber	$x = \frac{1}{2}$ diam.,	area = 2·00
Marketable measure of log $\frac{1}{4}$ girth square . .		area = 2·47
Gross area of cross section of log		= 3·14

To obtain a 12″ piece of square timber, the tree must be, allowing for bark, 15″ diameter, or 4 feet in circumference, at its smallest end.

Railway Track

Weight per mile of single track, consisting of two rails and fastenings. For rails only: Weight in tons per mile of single track=area of rail in square inches × 15·7143; the weight in pounds per yard being ten times the area of cross-section in inches.

Molesworth's tables of Indian State railways produce results as follows, including allowance for waste in fastenings:

5 ft. 6 in. gauge.

Rails only	weight in tons	= area	× 15·65
Rails, fishplates, fishbolts, and spikes	,,	,,	× 16·65
Rails &c. as above, and bearing plates	,,	,,	× 17·86

Metre gauge.

Rails only	weight in tons	= area	× 15·65
Rails, fishplates, fishbolts, and spikes	,,	,,	× 16·65
Rails &c. as above, and bearing plates	,,	,,	× 17·48

[1] Hurst.

Example 1. What will be the weight of iron in a mile of single track of narrow-gauge railway with 40 pound rails and fastenings, but no bearing plates?

Place the 1 of the slide opposite the 4 of the rule, and opposite to 16·65 (the weight per mile corresponding to an inch of section) we find 66·6 tons.

Example 2. What section must a rail have so that the rails only, without fastenings, will amount to 100 tons per mile? Adopting the factor 15·7143, place the 15·71 of the slide opposite the 1 of the rule; then opposite the 1 of the slide will be found 6·36, the required sectional area; the weight per yard would be 63·6 pounds.

Note. The weights of the fastenings have not been given in detail; they cover the weight of ordinary fishplates, but not angled or sleeve fishplates.

Weight of Angle and Tee Iron.

$$W=(B+D-t)\,w.$$

Where W=weight in pounds per foot run.

B=breadth of one flange of angle, or clear breadth of head of Tee in inches.

D=breadth of other flange of angle, or extreme depth of Tee in inches.

t=thickness of iron in inches.

w=coefficient in table.

TABLE XLV.—*Multipliers for Weights of Structural Iron.*

t	w	t	w	t	w	t	w
$\frac{1}{16}$	·208	$\frac{5}{16}$	1·041	$\frac{9}{16}$	1·874	$\frac{13}{16}$	2·708
$\frac{1}{8}$	·416	$\frac{3}{8}$	1·25	$\frac{5}{8}$	2·082	$\frac{7}{8}$	2·916
$\frac{3}{16}$	·625	$\frac{7}{16}$	1·458	$\frac{11}{16}$	2·29	$\frac{15}{16}$	3·124
$\frac{1}{4}$	·833	$\frac{1}{2}$	1·666	$\frac{3}{4}$	2·50	1	3·333

Intermediate values of w, *e.g.* for 32nds of an inch, or decimals, can be interpolated by slide-rule.

Example 1. What is the weight of a lineal foot of Tee iron $3\frac{3}{4}\times4\frac{1}{2}\times\frac{11}{16}$?

$$4\tfrac{1}{4}+3\tfrac{3}{4}-1\tfrac{1}{8}=7{\cdot}32 \text{ inches, and } w=2{\cdot}29$$

Answer. $7{\cdot}32 \times 2{\cdot}29 = 16{\cdot}75$ pounds per foot.

Weight of channel and H iron may be found similarly, when the web and flanges are of the same mean thickness, by the formula $W=[D+2\ (B-t)]w$.

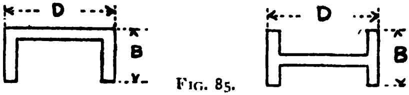


Fig. 85.

Example 2. What is the weight of channel iron in pounds per foot run, size $8'' \times 6'' \times \frac{1}{2}''$?

$$8+2\ (6-\tfrac{1}{2})=19\ ;\ w=1{\cdot}666.$$

Answer. $19 \times 1{\cdot}666 = 31{\cdot}66$ pounds.

Weight of Round and Square Iron.

Rule 1.—*Round iron.* Place the 2·61 of the upper scale of the slide under the middle 1 of the rule. Read the weight in pounds per lineal foot on the upper slide-scale, opposite the square of the diameter in inches on the rule.

Example. What is the weight of $2\frac{3}{8}''$ round iron per lineal foot? Adjusting the rule as described, we first find in Table XXXIX. the diameter in decimals 2·375, and by brass marker we find its square 5·64 on the upper scale of the rule. Under this latter figure we find the result 14·8 lbs. on the slide.

Rule 2.—*Square iron.* Instead of the number of 2·61 use 3·33, and proceed as before. For flat iron, use the product of the breadth by thickness instead of the square of the side.

For round and square cast iron use 2·43 and 3·097 respectively.

For steel, 2·66 and 3·397.
„ copper, 3·00 and 3·83.
„ brass, 2·84 and 3·63.
„ lead, 3·84 and 4·89.
, zinc, 2·40 and 3·06.

THE SLIDE-RULE AS A 'READY RECKONER'

TABLE XLVI.—*Wages and Salaries.*

	Multiplier
Pence per hour into shillings per day of 11 hours	0·917
,, ,, ,, ,, 10 ,,	0·833
,, ,, ,, ,, 9 ,,	0·750
,, ,, ,, ,, 8 ,,	0·667
,, ,, ,, week of 66 ,,	5·5
,, ,, ,, ,, 60 ,,	5·0
,, ,, ,, ,, 54 ,,	4·5
,, ,, ,, ,, 48 ,,	4·0
,, ,, £ per mo. of 26 d. of 11 hours	1·192
,, ,, ,, ,, 10 ,,	1·083
,, ,, ,, ,, 9 ,,	0·975
,, ,, ,, ,, 8 ,,	0·867
,, ,, ,, an. of 313 d. at 11 ,,	14·34
,, ,, ,, ,, 10 ,,	13·04
,, ,, ,, ,, 9 ,,	11·73
,, ,, ,, ,, 8 ,,	10·43
Shillings per day into £ per week of 6 days	0·3
,, ,, ,, month of 4 weeks	1·20
,, ,, ,, ,, $4\frac{1}{3}$,,	1·299
,, ,, ,, ann. of 313 days	15·65
,, week into £ per month of 4 weeks	0·2
,, ,, ,, ,, $4\frac{1}{3}$,,	0·216
,, ,, ,, ann. of 52 weeks	2·6
£ per week into £ per month of 4 weeks	4·0
,, ,, ,, $4\frac{1}{3}$,,	4·33
,, ,, ann. of 52 weeks	52·0
,, month (lunar) into £ per annum	13·0
,, (calendar) into £ per annum	12·0

See also tables, pp. 271, 272.

Example 1. How much per annum is an hourly wage of $3\frac{3}{4}$*d.* worth, at the rate of 9 hours per day, working every day except Sundays? Place the left hand 1 of the slide opposite the multiplier 11·73 on the rule, and opposite to the wage 3·75*d.* on the slide will be found 44*l.*

Example 2. A man has made 178*l.* 15*s.* in the year. What is the equivalent wage in shillings per day, if he had

worked steadily every day but Sundays? Place the left hand 1 of the slide opposite the multiplier 15·65 on the rule, and opposite to 178·75 on the rule will be found 11·42 =11*s.* 5*d.*

Example 3. A servant gets 17*l.* 3*s.* 5*d.* per annum ; what are his wages for 5 months 2 weeks and 5 days?

Find from the tables the decimal equivalents of the time and money as follows :

	£		Years
Rate in decimals .	17·000	5 months (Table XXXIV.)	·417
3 shillings (Table XLVII.)	·150	2 weeks (Table XXXVII.)	·038
5 pence (Table XLVII.)	·021	5 days (Table XXXVIII.)	·014
	17·171		·469

By slide-rule. Place the left hand 1 of the slide over the 469 on the rule, and read the answer 8·07*l.* opposite to 1,717 on the slide.

By table, 8·07*l.*=8*l.* 1*s.* 4$\frac{3}{4}$*d.*

From Table XLVII. read off ·05 as 1*s.* and divide the balance ·02 by the multiplier for one penny ·00417 ; the equivalent fraction would be $\frac{2000}{417}$. For division by slide-rule see p. 245. The result will be 4·8*d.*

English Money

Table XLVII.—*Decimal Multipliers.*

	£	*s.*	*d.*	*f.*
One farthing . .	·00104	·0208	·25	1
One penny . . .	·00417	·0833	1	4
One shilling . .	·05	1·0	12	48
One pound . .	1·0	20	240	960

Example. To reduce 131*l.* 13*s.* 9$\frac{1}{4}$*d.* to pounds and decimals by the slide-rule.

With the multiplier	·00104	we find	$\frac{3}{4}d$. .	. = ·0031
,, ,,	·00417	,,	9*d*.	. = ·0374
,, ,,	·05	,,	13*d*. .	. = ·65

Answer . . . £131·6905

Colonial and Foreign

Where the value is not at par under the gold standard, or where silver is not at 44d. *per ounce, the slide-rule will give the current value by rule* 1, *p.* 242.

Argentine Republic—see Chili.

Austria. (Par value.)

1*l*.	. . = 10·215 fl.	. . = 1,021·5 kreuzer
1*s*.	. = ·5107 ,,	. . = 51·07 ,,
1*d*.	. = ·0425 ,,	. . = 4·2 ,,

Brazil.

1*l*. .	. . = 8·925 milreis	. . = 8,925 reis
1*s*. .	. . = ·446 ,,	. . = 446 ,,
1*d*.	. . = ·037 ,,	. . 37 ,,

Canada and United States of America.

Par Value in Sterling.

1 £	=	4·87 dollars
1 shilling	=	24·35 cents
1 penny	=	2·03 ,,
1 farthing	=	0·51 ,,

Chili, Colombia, and Uruguay. (Par value.)

1*l*. .	. . = 5·340 peso	. . = 534 centavos
1*s*.	. . = ·267 ,,	. . = 26·7 ,,
1*d*.	. . = ·022 ,,	. . = 2·2 ,,

China.

Intrinsic value with silver at 44d. per oz. troy.

1*l*.	. . = 4·28 taels	. . = 428 conderin
1*s*.	. . = ·214 ,,	. . = 21·4 ,,
1*d*.	. . = ·018 ,,	. . = 1·8 ,,

FRANCE. (Par value.)

1*l*.	. .	25·220 francs	
1*s*.	. .	1·261 ,, . .	126·1 centimes
1*d*.	. .	·105 ,, . .	10·5 ,,

EMPIRE OF GERMANY. (Par value.)

1*l*.	. . =	20·420 marks	
1*s*.	. . =	1·021 ,, . . =	102·1 pfennig
1*d*.	. . =	·085 ,, . .	8·5 ,,

TABLE XLVIII.—*Indian Money at par* (1 *Rupee* = 2*s*.).

Rupees	Annas	Pie	Sterling
1	16	192	2 shillings
·0625	1	12	$1\frac{1}{2}$ pence
·005208	·0833	1	0·125 penny

1 lakh = 100,000 rupees. 1 crore = 10,000,000 rupees. 1 pice = 3 pie or $\frac{1}{4}$ anna. In Ceylon the rupee is divided into 100 cents.

Example a. What is the value of 473*l*. sterling in rupees at the exchange of 1*s*. $7\frac{3}{4}$*d*. per rupee? Using the multiplier in Table XLIX.,

$$10 : 4{,}730 :: 12{\cdot}15 : x$$

whence $x = 5{,}747$ rupees. For rule of three sum by slide-rule see p. 243.

Example b. What is the value of 72 pie at the exchange of 1*s*. $8\frac{3}{4}$*d*. sterling per rupee?

First operation: To obtain the par value. By Table XLVIII.,

$$1 \text{ pie} : {\cdot}125d. :: 72 \text{ pie} : x$$

whence $x = 9d.$

Second operation: To obtain the current value. By Table XLIX.,

$$11{\cdot}56 : 10 :: 9d. : y$$

whence $y = 7{\cdot}8d.$, nearly.

TABLE XLIX.—*Indian Money.*

MULTIPLIERS FOR THE CONVERSION OF STERLING CURRENCY INTO RUPEES AT VARIOUS RATES OF EXCHANGE.

V = *Rate of Exchange for the Rupee in Shillings and Pence.*

V	1*s.* 11$\frac{3}{4}$*d.*	1*s.* 11$\frac{1}{2}$*d.*	1*s.* 11$\frac{1}{4}$*d.*	1*s.* 11*d.*	1*s.* 10$\frac{3}{4}$*d.*	1*s.* 10$\frac{1}{2}$*d.*	1*s.* 10$\frac{1}{4}$*d.*	1*s.* 10*d.*	
£ ×	10·1053	10·2127	10·3226	10·4348	10·5495	10·6667	10·7865	10·9091	= Rs.
Rs. ×	·098958	·097917	·096875	·095833	·094792	·093750	·092708	·091667	= £
V	1*s.* 9$\frac{3}{4}$*d.*	1*s.* 9$\frac{1}{2}$*d.*	1*s.* 9$\frac{1}{4}$*d.*	1*s.* 9*d.*	1*s.* 8$\frac{3}{4}$*d.*	1*s.* 8$\frac{1}{2}$*d.*	1*s.* 8$\frac{1}{4}$*d.*	1*s.* 8*d.*	V
£ ×	11·0345	11·1628	11·2941	11·4286	11·5663	11·7073	11·8519	12·000	= Rs.
Rs. ×	·090625	·089583	·088542	·087500	·086458	·085417	·084375	·083333	= £
V	1*s.* 7$\frac{3}{4}$*d.*	1*s.* 7$\frac{1}{2}$*d.*	1*s.* 7$\frac{1}{4}$*d.*	1*s.* 7*d.*	1*s.* 6$\frac{3}{4}$*d.*	1*s.* 6$\frac{1}{2}$*d.*	1*s.* 6$\frac{1}{4}$*d.*	1*s.* 6*d.*	V
£ ×	12·1519	12·3077	12·4675	12·6316	12·8000	12·9730	13·1507	13·3333	= Rs.
Rs. ×	·082292	·081250	·080208	·079167	·078125	·077083	·076042	·075000	= £

Example c. To reduce 15 annas 7 pie to cents (100ths of a rupee).

16 annas : 100 cents :: 15 annas : x cents

whence $x=93\cdot7$;

192 pie : 100 cents :: 7 pie : y cents

whence $y=3\cdot7$ cents.

and answer, $93\cdot7+3\cdot7=97\cdot4$ cents.

JAPAN.

Intrinsic value with silver at 44*d. per oz. troy.*

1*l.* . . .	6·500 yen . . .	650 sen
1*s.* . . .	·325 ,, . . .	32·5 ,,
1*d.* . . .	·027 ,, . . .	2·7 ,,

MEXICO. (Par value.)

1*l.* . .	= 6·160 dollars . .	= 616·0 cents
1*s.* . .	= ·308 ,, . . .	= 30·8 ,,
1*d.* . .	= ·025 ,, . . .	= 2·5 ,,

NETHERLANDS.

1*l.* . .	= 12·00 florins . .	= 1,200 cents
1*s.* . .	= ·60 ,, . .	= 60 ,,
1*d.* . .	= ·05 ,, . .	= 5 ,,

The par value according to United States Treasury circular is about 1 per cent. more.

PERSIA.

1*l.* .	= 2·800 thomans .	= 28·0 banabats .	= 280 shahis
1*s.* .	= ·140 ,, .	= 1·4 ,, .	= 14 ,,
1*d.* .	= ·012 ,, .	= ·128 ,, .	= 1·28 ,,

PORTUGAL. (Par value.)

1*l.* . . .	= 4·500 milreis . .	= 4,500 reis
1*s.* . . .	= ·225 ,, . .	= 225 ,,
1*d.* . . .	= ·019 ,, . .	= 19 ,,

RUSSIA. (Paper currency.)

Current value in Whitaker's Almanack for 1890.

1*l.* .	= 6·3000 roubles . .	= 630 kopek
1*s.* . .	= ·3150 ,, . .	= 31·5 ,,
1*d.* . .	= ·0265 ,, . .	= 2·65 ,,

SPAIN. (Par value.)

1*l*.	= 9·7000 scudo	= 25·200 peseta	= 2,520 centimos
1*s*.	= ·4850 ,,	= 1·260 ,,	= 1,260 ,,
1*d*.	= ·0404 ,,	= ·105 ,,	= 105 ,,

SWEDEN. (Par value.)

1*l*. . .	= 18·200 crowns . .	= 1,820 ore
1*s*. . .	= ·910 ,, . .	= 91 ,,
1*d*. . .	= ·076 ,, . .	= 7·6,,

TURKEY. (Par value.)

1*l*. . .	= 110·0 piastres	
1*s*. . .	= 5·5 ,, . .	= 220 paras
1*d*. . .	= ·46 ,, . .	= 18·3 ,,

VENEZUELA.

1*l*. . .	= 25·220 bolivars . .	= 50·44 decimos
1*s*. . .	= 1·261 ,, . .	= 2·522 ,,
1*d*. . .	= ·105 ,, . .	= ·210 ,,

Example d. How many Chinese taels are contained in 10 Mexican dollars, the former at intrinsic value of 42*d*. per oz., the latter at par value?

First operation: To obtain the current value in Chinese taels of 1*l*. sterling.

$$42 : 44 :: 4{\cdot}28 : x = 4{\cdot}48$$

Second operation: To obtain the value of 10 Mexican dollars in taels.

$$6{\cdot}16 : 4{\cdot}48 :: 10 : 7{\cdot}11 \text{ taels}$$

Example e. How many Japanese yen at 44*d*. per oz. should one receive for 105 U.S. dollars at par value?

$$4{\cdot}87 : 6{\cdot}50 :: 105 : x$$
$$x = 140 \text{ yen } 44 \text{ sen}$$

CHAPTER IX

INSTRUMENTS

Levels and Levelling

The necessity for condensation has led to the insertion here of all that can be said within our limits upon the theory of levelling instead of giving it a separate chapter. It will be attempted to place the fundamental principles of levelling and the adjustments of different kinds of levels in a practical and simple light. The abstruse disquisitions upon possible sources of error in levelling to be found in proceedings of learned societies have no place here. The surveyor ought to be able to put his instrument in adjustment every morning without referring to a book, and considerable space has been required to make the reason of each adjustment plain. Secondly, he should be alive to the limits of error arising from taking long or unequal sights, curvature of the earth, &c., so that on the one hand he should be prepared to adopt extra precautions for special cases, and on the other hand not to waste time upon refinements which are not essential to his object.

The adjustment of the level is also the foundation of the adjustment of the theodolite and tacheometer. This is shown in Fig. 88, where the error in the line of sight is treated as a vertical angle which has to be eliminated similarly to the index error of a theodolite.

For these reasons, the lion's share of this chapter has fallen to the subject of levels.

Theory

Two points are said to be upon the same level when they are equidistant from the earth's centre: conseqently, a *level line* cannot be strictly speaking a *straight line*. It is a parallel to the curvature of the sea. A *horizontal line* is a parallel to a tangent to the earth's circumference, and therefore a straight line. In common parlance, the words level and horizontal are synonymous, and it would be pedantic to endeavour to keep their application wholly distinct, but the difference is mentioned because it comes into the question of adjustment for 'collimation.'

A level adjusted horizontally will not represent an object as high as it really is. This is proved by looking at the top of a vessel's masts when just appearing above the horizon. A level would report the mast-head to be at the same elevation as the observer's eye, whereas it would be perhaps fifty or 100 feet further from the earth's centre.

The object of the leveller is to determine the relative elevations, or heights above sea-level, of points upon the earth's surface, which is the same thing as their difference of distance from the earth's centre.

Light always travels in a *straight line*, unless diverted from its path by the medium traversed. The bending of light by the atmosphere is called refraction: see Glossary.

Levels are either adjusted with the line of sight, horizontal, and consequently subject to correction for both curvature and refraction, or else they are adjusted with the line of sight parallel to a chord of the earth's circumference to allow for curvature and refraction, as by Mr. Gravatt's method, described in 'Heather on Instruments.'

Refraction makes objects appear too high, consequently it counteracts to a small extent the effect of curvature. It varies with the state of the weather, but as regards levelling it is quite near enough to assume it at its average amount

at sea-level. That is ·095 of a foot at a distance of a mile, and varying directly as the square of the distance.

The Curvature of the earth at a mean radius is = ·667 foot at a distance of a mile, and also varies (within a limit of many miles) directly as the square of the distance. The allowance to be made for refraction and curvature at a distance of a mile is therefore ·572 foot, or for any distance in miles up to 100, more exactly, the correction *to be added*, is ·5717 D^2.

TABLE L.—*Correction for Curvature of the Earth.*

Distance in feet	Correction +	Distance in miles	Correction +
250	·00149	·25	·0356
500	·00511	·50	·143
750	·0116	·75	·321
1,000	·0205	1·0	·572
1,250	·0320	1·25	·893
1,500	·0460	1·50	1·286
1,750	·0620	1·75	1·750
2,000	·0820	2·00	2·287

Whether the line of sight is adjusted tangentially or to a terrestrial chord, if the back-sights and fore-sights are of equal length, there will be no error in the result on account of curvature. The correction to be added in each case is entirely dependent upon the excess in distance of the one sight over the other. Thus with a back-sight of 300 feet, and a fore-sight of 900 feet, the correction of ·002 foot for the back-sight would have to be taken from the correction ·017 for the fore-sight, leaving an additive correction of ·015 foot. But if the back-sight is longer than the fore-sight the correction will be subtractive. Now with the ordinary 14″ level, at a distance of 500 feet *an almost imperceptible* movement of the bubble from the centre of its run will produce a difference in the reading of ·03 foot, amounting as it does to about one second of vertical arc, or the twentieth part of one of the small subdivisions usually marked upon the level bubble.

There are not many men who would have the hardi-

hood to swear to their levels to *one-hundredth* at the distance of 500 feet with a 14-inch level; but it will be seen from the table that *both instrument and man must be true to half a hundredth* every time at a distance of five hundred feet, or else they may as well leave curvature and refraction out of their calculations. It was partly (1) to obtain this minute accuracy, and partly (2) to set the line of sight in the exact optical axis of the telescope, that Mr. Gravatt invented his elaborate 'three peg' adjustment. To go through a tedious process to obtain the first condition is a waste of time when it can be added for special exactitude from the table, but besides that, the condition is only really fulfilled for the

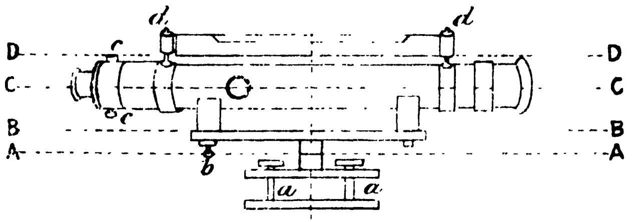


FIG. 86. DUMPY LEVEL.

A. A. Plane of rotation; *a. a.* adjusting screws to same.
B. B. Horizontal bar; *b.* adjusting screw to same.
C. C. Line of sight; *c. c.* adjusting screws to same.
D. D. Bubble tube; *d. d.* adjusting screws to same.

length of base used in the adjustment: it produces an equal and opposite error at a midway point to what is produced by a horizontal adjustment at the further point. This is evidently the case from the fact that no line of sight can be made to follow the curvature of the earth. The second condition is not essential to correct levelling, but it can be fulfilled by a perfectly simple and rapid process, as will be presently shown, but a few words are first needed on the organic principle of the level whether Y or dumpy.

The spirit-level, like the plummet, is a device for utilising the law of gravity to establish a horizontal or perpendicular

line. Either of them can, by means of a square, do the work of the other.

If we fill a bottle nearly full of water and cork it, the air-space is always at the top because the water is heavier. If we turn it on its side on a level table the air-space will if very small form a bubble which will stop in any position along the side, because, the sides of the bottle being parallel, no one part is higher than the other.

The level-tubes of good spirit-levels are very carefully ground to a true curve, so that a movement of the tube in a vertical plane is equal to the right or left for equal vertical angles. The worst kind of level is that which has hardly any curvature and is everlastingly getting off the centre with inappreciable vertical movements. A bubble-glass which is not perfectly uniform in curvature requires more care than a perfect one, but correct levelling can be done with it also.

Suppose now a bubble-tube with legs like the striding bubble of a theodolite standing upon a table. If the bubble be at the middle it does not prove the table level. It only proves that either the table is level or else it has a slope which is equal to a corresponding inequality in the length of the legs. For if we reverse the bubble-tube end for end, the bubble may be displaced from the centre. If the bubble remains in the centre when reversed it proves *firstly* that the table is horizontal; *secondly*, and in consequence, that the legs of the tube are equal. *We cannot reverse a bubble upon a sloping plane.* The first thing therefore in the adjustment of the level is to make the *plane of rotation* AA (Fig. 86) horizontal, which we can do whether the line of sight is horizontal or not, and it is therefore here mentioned first.

The correctness of the bubble is the basis of all the adjustments. The common expression, 'correcting' or 'adjusting' the bubble, is a misleading one. The only correction suitable to a defective bubble-tube is to break it

up, because nothing can correct an imperfect grinding. When the bubble-tube is truly ground, the bubble is always at the top of the curved surface, and just as a plummet gives the vertical, so the bubble gives a true horizontal line. It is the different parts of the instrument, such as the plane of rotation and line of sight, which have to be adjusted to the bubble, and not the bubble to them. The test of accurate grinding is by marking and measuring the travel of the bubble within equal angular movements in opposite directions. *When the bubble-tube may be turned completely round without disturbing the bubble from the centre of its run it proves that the plane of rotation is horizontal. It is not necessary for practical levelling that the line of sight should coincide with the optical or focal axis of the telescope tube;* it is sufficient for it to be sensibly horizontal within the range of the focussing screw, and therefore parallel to the plane of rotation.

When both the plane of rotation and line of sight are horizontal, if the telescope tube is not parallel to them, the line of sight cannot be in the optical axis and will be theoretically thrown out of its horizontality by actuating the focussing screw. This fact is met by another, which is that the adjustment is made for the longest distance that the telescope will read correctly, and in that position the movement of the focussing screw is not sufficient to produce sensible error. It is only when the distances are very short that the effect would be appreciable, and then the divergence of the line of sight has not sufficient distance in which to accumulate sensible error.

The travel of the diaphragm in a vertical plane is so small compared with the field that if the hairs are displaced as far as they will go and the instrument adjusted to horizontality by the method described on p. 296, it will give the same difference of level when set up midway between the stakes or close to either of them.

The method given here will, however, include an almost

perfect coincidence of the line of sight with the optical axis. It makes all four lines, AA, BB, CC, DD, in Fig. 86 perfectly horizontal. It takes a quarter of an hour the first time, and five minutes when the pegs are driven and their difference of level known.

The adjustment of the plane of rotation is analogous to

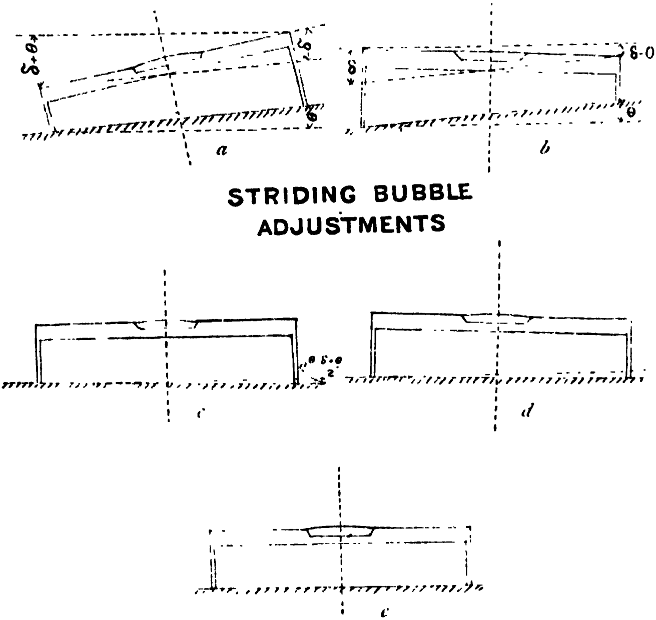


Fig. 87.

that of the striding bubble on a table alluded to on p. 290, and will be therefore illustrated in that manner. Referring to Figs. *a*, *b*, *c*, *d*, *e*, the line with hatching represents the plane upon which the level stands. In the case of a theodolite it would be the plane passing through the trunnions. In the case of the Y or dumpy it is AA, Fig. 86, the plane of

rotation. It is supposed to be inclined to the horizontal by an angle $=\theta$. The bubble-legs are drawn of unequal length to represent in an exaggerated manner the error of adjustment which may be in these legs or in the little legs *d*, *d*, Fig. 86, and for adjustment of which capstan-headed screws are provided. The angle of error by which the feet are supposed to be out of parallel with the axis of the bubble (that is with a tangent to the upper curved surface of the bubble-tube) is δ. It does not matter what proportion each error bears to the other ; they can at once be removed.

In Fig. *a* the striding bubble is placed so that the two angles augment the divergency of the bubble from the centre of its run, which becomes $=\theta+\delta$. In Fig. *b* the level is turned end for end and the error becomes $\delta-\theta$.

Now if we bring the bubble from its position in Fig. *a* to the centre of its run, as shown in Fig. *c*, correcting the total error $\delta+\theta$, half by the screws on the bubble, and half by altering the inclination of the plane (in the transit by the capstan-headed screws under the trunnion), we shall obtain the following equation.

Placing the striding bubble in the position of augmented error, Fig. *a*, and deducting $\frac{\delta+\theta}{2}$ from δ gives $\frac{1}{2}(\delta-\theta)$.

Deducting $\frac{\delta+\theta}{2}$ from θ gives $\frac{1}{2}(\theta-\delta)$ or $-\frac{1}{2}(\delta-\theta)$,

that is to say, the error of the bubble and the plane in Fig. *c* will each then be equal and opposite.

If, now, we reverse the striding bubble into the position Fig. *d*, the reduced error of the bubble will be $\delta-\theta$, and if we bring it to the centre of its run by equally dividing the error as before, the error will be eliminated, as in Fig. *e*.

Example. The error of the plane was $+5°$, and the bubble $7°$. Placing the latter so as to augment the error,

$$\delta+\theta=12° ; \frac{\delta+\theta}{2}=6° ; \ 5°-6°=-1°=\theta'$$

$$7°-6°=+1°=\delta'$$

Reversing, as in Fig. *d*, combined error $= -2°$, and again levelling with halved correction,

Probably the correction will not be accurately halved the first time, and if very much out of adjustment it will take three or four trials.

The ordinary tradesman's level is planed or ground on its base to a true parallelism with the axis of the bubble, and can only be adjusted, if out of order, by re-planing.

1. *To adjust the plane of rotation for either* Y *or dumpy levels.* Bring the bubble to the centre of its run over a pair of parallel-plate screws. Turn the telescope 90°, and repeat over the opposite pair of screws. Bring it back to its first position, and retouch the parallel-plate screws. Turn the telescope 180°, so as to be over the first pair of screws, but end for end. If the bubble is off the centre, correct half the error by the screws *d*, *d*, and half by *a*, *a*, Fig. 86. Repeat until the bubble remains central in each position. Turn the telescope 90°, and retouch the parallel-plate screws so as to bring the bubble to the centre, after which it ought to revolve completely in a central position. If it will not, it proves that the plane of rotation is not accurately ground, or that it has become worn by sand or what not, so that it is impossible to produce horizontality in all directions over its surface. This, of course, could only be remedied by a maker, or the instrument could only be used by adjusting the parallel-plate screws at every sight.

2. *To remove parallax in* Y *or dumpy levels.* This is an adjustment of the eyepiece to bring the cross hairs into the common focus of the eyepiece and object-glass. Sometimes, with very short-sighted people, the little brass tube into which the eyepiece fits has to be ground down.

The adjustment is performed by directing the telescope upon a distant object, and first focussing the object with the focussing screw, then the cross hairs with the eyepiece, until the object is seen perfectly distinctly, and the hairs are clear and do not appear to shift when the eye is moved.

3. *To place the line of sight approximately in the optical axis of the telescope. Dumpy levels.* Hold the staff about thirty feet off, with a sheet of white paper against its back.

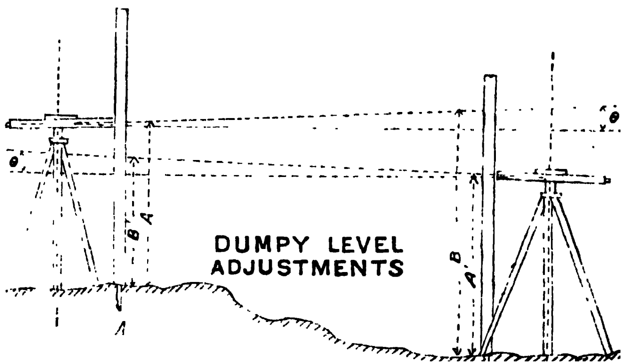


FIG. 88.

Direct the staff-holder to mark with a pencil the top and bottom of the circular field made upon the paper. If possible, let the staff be held against a wall to steady it. Bisect the space between the pencil marks, and mark the centre. Bring the axial hair to coincidence with this mark by the screws *c, c,* Fig. 86. The hair will be then in the centre of the shutter, and if the shutter is concentric with the tube, which it is in all properly constructed instruments, and if the glass is perfectly ground, it will also be in the optical axis. In any case it will be quite near enough to avoid all

error, since it is not from this adjustment that the line of sight is made horizontal. When this is done, the screws *c*, *c* need never be touched unless the hairs are broken. An ivory scale fastened against a wall will enable the foregoing to be done without assistance.

4. *To make the line of sight horizontal.* See Fig. 88. Drive two pegs on nearly level ground about a hundred paces apart. Set up the level so close to one of them that, when the levelling staff is held upon it, the eyepiece will be about half an inch away from it.

Level the instrument very carefully, and look through the *object-glass* at the staff, swaying the latter gently until it comes into the focus of the eyepiece about half an inch from the face of the staff. Mark the staff at the centre of the field, which will be about one-eighth of an inch in diameter. Book the height, which we will call instrument-height A. Then remove the staff to the distant peg, and with the bubble in the middle, read the height there in the usual way, and call it staff-reading B.

Next carry the instrument to the distant peg, and set it up, carefully levelled, close to the staff, taking a similar reading through the object-glass, which call instrument-height A′. Remove the staff to the first peg and read it, calling it staff-reading B′. Now referring to Fig. 88, if the line of sight had been parallel to the bubble, it would be represented by the two horizontal lines in the figure, and the difference of the readings in each position would be the same, that is to say $A-B'=B-A'$.

When, however, the line of sight is inclined to the axis of the bubble, as shown on the figure, it makes a vertical angle of elevation or depression$=\theta$. In the case illustrated, being an angle of elevation, the first difference of readings is augmented, and the second difference of readings is diminished by that angle.

When, therefore, the difference of readings at A, that is $A-B'$, is less than the difference of readings at B, that is

B − A′, the line of sight ‘throws’ upwards, and when more it ‘throws’ downwards.

We can express this more simply by the following :

Rule. When the sum of the instrument-heights A + A′ is $\begin{matrix}\text{less than}\\ \text{more than}\end{matrix}$ the sum of the staff-readings B + B′, the line of sight throws $\begin{matrix}\text{upwards}\\ \text{downwards}\end{matrix}$, and the half difference is to be $\begin{matrix}\text{deducted from}\\ \text{added to}\end{matrix}$ the second staff-reading.

This last clause needs a little explanation. The difference of the two differences (B − A′) − (A − B′) clearly measures double the angle θ. The above may be written B + B′ − (A′ + A), and this is twice the error. In the illustration, as the line of sight threw upwards, it has to be deducted.

The crucial test of this method is that wherever the level is placed when the adjustment is complete, whether touching either of the staves or midway between them, the difference of level recorded is precisely the same.

The angle θ is caused by the line of sight not being parallel to the plane of rotation, and having determined its amount, we eliminate the *whole of it* by the screw *b*, Fig. 86, by screwing it up or down until the reading on the staff is corrected by the amount which measures the angle θ on the staff. This will of course disturb the bubble, and we bring the bubble back to the centre of its run entirely by the screws *d*, *d*. It will be noticed that we have *kept the plane of rotation horizontal all the time*, and we have now all four lines horizontal.

The correctness of this adjustment can be seen by treating the Y level in the same manner, for it will be found when completed that the Y's can be thrown open, and the telescope reversed end for end without disturbing the centrality of the bubble.

We could have produced horizontality in the line of

sight by correcting the final error by the screws *c*, *c*, and neglecting the third adjustment altogether, but the method as given is the best.

When, however, there is no screw *b*, Fig. 86, under the horizontal bar, as is frequently the case with the smaller dumpy levels, the parallelism of BB with AA can only be corrected by an instrument-maker. We cannot then use the third adjustment on p. 295. We make the first and second adjustments and then proceed directly to the fourth, making the final correction with the collimation screws *c*, *c*, Fig. 86. If the level is properly made, the parallelism of BB with AA will be quite near enough to bring the line of sight sufficiently close to the focal axis for all practical purposes.

Example 1. (Line of sight throwing upwards.)

Readings at Station A.

Ht. of inst. at A. . . .	(A) 5·23	
Staff-read. at B.		5·15 (B)

Readings at Station B.

Ht. of inst. at B. . . .	(A′) 4·63	
Staff-read. at A.		5·03 (B′)
Sum of staff readings . . .		10·18 (B + B′)
Sum of inst. heights		9·86 (A + A′)
Difference		·32
Half difference		·16

Reading of true horizontal line from B=5·03—·16=4·87. To which reading the cross hairs are to be brought by the screw *b*.

Example 2. (Line of sight throwing downwards.)

Readings at Station A.

Ht. of inst. at A. . . .	5·08 (A)	
Staff-reading at B.		5·16 (B)

Readings at Station B.

Ht. of inst. at B. . . .	4·95 (A′)	
Staff-reading at A.		4·55 (B′)
Sum of inst. heights . . .	10·03 (A + A′)	
Sum of staff-readings . . .	9·71	(B + B′)
Difference . . .	·32	
Half difference . .	·16	

Reading of the true horizontal line of sight from B= 4·55 + ·16=4·71.

The difference being the same in these two examples is a mere coincidence.

The difference of level between the two pegs might have been determined by setting up the instrument midway between them, and taking readings alternately on one and the other; but the method as given takes no longer, and needs no measurement.

When, however, the difference of level is known, the instrument need only be set up beside one of them.

When the adjustments are completed, particular care should be given that all the screws are tight; if not the adjustment will last but a very short time, but if carefully made it will probably not need touching after a month's steady work. Every day before starting, five or ten minutes at the peg will suffice to show that all the adjustments are in order.

Adjustments of the Y Level

1. To adjust the plane of rotation horizontal, see p. 294.

2. To remove parallax, see p. 294.

3. To place the line of sight in the optical axis.

Direct the telescope on some clearly defined point, and

intersect it with the cross-hairs, revolve the telescope half round in its Y's (being careful not to rotate it on its plane). If the object is not still intersected, correct one half by the screws *c*, *c*, Fig. 86, and the other half by the parallel-plate screws (the horizontality of the plane of rotation has nothing to do with this adjustment). Repeat until the intersection is the same in all positions.

4. To make the line of sight horizontal repeat No. 1 adjustment and open the Y's. Reverse the telescope end for end. If the bubble is not still central, correct half the error by the screws *d*, *d*, and half by *b*. Repeat until the telescope can be reversed without disturbing the bubble. DD and BB will then be both horizontal.

5. To place the bubble axis in the same *vertical plane* with the axis of the telescope.

An error in this respect is detected by the fact of the bubble not retaining its central position when the telescope is turned a little way in the Y's. The two ends of the bubble are not quite in line, consequently as the telescope is turned, one end rises a little before the other, and this error is corrected by the capstan-headed screw at the *side* of the screws which correspond to *d*, but are only found in Y levels and not shown on the figure.

The Y level has the advantage of requiring no peg-adjustment for collimation. The dumpy level is handier for small sizes. English surveyors prefer it on the ground of its supposed superiority to the Y level in retaining its adjustment. This of course is a consideration when they are in the habit of sending their level to the makers for adjustment. The writer does not hold that view, but believes the Y level will retain its adjustment just as well, last longer without re-grinding of the axis, and is adjusted in the field in a few minutes. The peg adjustment of the dumpy in the usual form of the text-books is a great bug-bear to young engineers, so it brings much grist to the instrument-makers' mill.

The Y level is almost exclusively the type adopted in America.

A modification of Y level has been recently introduced by Messrs. Cooke and Sons, of York. The principal difference is that the telescope is contained in a shorter external tube terminating in sockets into which the telescope fits very exactly; it is reversed by withdrawing it most carefully, and inserting it end for end to obtain the adjustment No. 4, on p. 296. Never having used this instrument, the writer does not wish to speak decidedly about it. Coming from that firm, the workmanship would no doubt be excellent, and that always means an instrument which will retain its adjustment well. On the one hand it must be less liable to wear in the bearings, and on the other hand, more awkward to reverse it when it needs adjusting.

The wear of ordinary Y's can be corrected by the screw *b*, Fig. 86.

LEVEL-STAVES

Two types of these are used: those in which the sight is taken on a sliding vane or target, painted black and white so as to obtain very precise intersection with the cross hairs. On the target is a vernier which reads with graduations on the staff. This type is still somewhat used in America, but hardly at all in England. It can be read at a greater distance than the graduated rod, but requires an assistant who can be depended upon to book the readings correctly.

Graduated rods are usually marked by lines across at every hundredth of a foot, the spaces being alternately black and white.

When the staff is used for telemetry, it should have a device for ensuring that it is held at right angles to the line of sight, or else plumb according to the manner of working. For colliery work, an illuminated staff has been successfully employed having the figures painted on glass, and a lamp carried in a thin casing at the back of them.

Different Methods of Keeping the Fieldbook

The 'rise and fall' Method

This is the most rigorous plan. The reduction of intermediate sights forms a check upon the turning-points. The form of fieldbook is as under:

Back-sight	Intermediate	Foresight	Rise	Fall	Red. level of intermediate	Red. level of turning-points	Distance	Total distance	Remarks
4·65						100·00			B. M. on mile-stone
	2·23		2·42		102·42				
	11·74			9·51	92·91				
	9·37		2·37		95·28				
8·43		11·13		1·76		93·52			Turning-point on peg
	5·22		3·21		96·73				
	1·13		4·09		100·82				
		7·84		6·71		94·11			
13·08		18·97							
		13·08				5·89			
						100·00			Check

By keeping the reduced levels of the intermediate separated from the turning-points, the independent check of back-sights and fore-sights is more conveniently applied at any time, and it should be done at the end of every page.

The Collimation Method

This is almost exclusively used in North America, and to a large extent in the Colonies. It is much quicker, and will receive a little more explanation because it is the foundation of the tacheometer fieldbook. It has the disadvantage of greater liability to error in the reduction of the intermediate sights, each calculation being independent; but as it enables the railway surveyor to put in his gradient stakes without first sitting down to reduce his levels, its advantages are deemed to outweigh the one disadvantage.

The back-sight by this method becomes a plus sight, and the only one. All the intermediate sights and the fore-sight are minus sights.

In Fig. 89 the operation commences with a back-sight

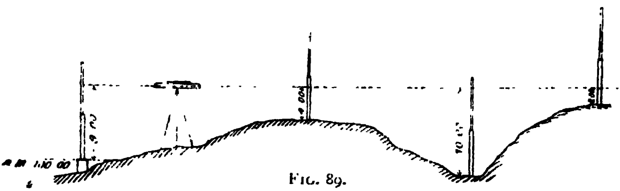


FIG. 89.

to determine the elevation of the line of sight, commonly called 'collimation;' the staff-reading of 9·00 feet being *added* to the known or assumed elevation of the fiducial point on which the staff is held at starting. The collimation being known, the elevation of any other point which can be seen is equal to the collimation height *less* the reading on the staff, and the reduction of the levels is relieved of the column of rise and fall.

Form of Fieldbook.

+	−	−T. P.	Collimation	Red. level of intermediate	Red. level of turning-points	Distance	Total distance	Remarks
9·00			109·00		100·00			B.M. on mile-stone
	4·10			105·00				
	10·00			99·00				
11·00		2·00	118·00		107·00			
		7·00			111·00			
20·00		9·00			11·00			
9·00					100·00			Check
11·00								

The best rules for precise levelling are -

First. To adjust the line of sight truly horizontal, *i. e.* tangential to the earth's circumference.

Second. Not to allow any sight to exceed 250 feet distance with a 14-inch level or less with a shorter one.

Third. To keep the back-sights and fore-sights as nearly as possible at equal distances.

Fourth. When back-sight and fore-sight are at unequal distances to correct by table—by *adding to every sight* the tabular amount where it is not less than ·001. It will be quite near enough to measure the distances with a passometer.

THEODOLITES

The plain theodolite for ranging base lines and curves is the foundation of most tacheometers. The two types in use are the Y and the transit. For ordinary work the transit is very much to be preferred on account of the liability of the line of sight to be shaken off its position when

reversing the telescope in its Y's; the time taken in reversing; and the danger of leaving the clips loose and dropping the telescope out when shouldering the instrument.

When telemetric work is the chief use of the instrument the Y type has two advantages which bring it more into competition with the transit.

1st. Within a considerable range of vertical arc, a telescope of twice the focal length usually supplied can be safely and *steadily* carried in specially constructed Y's.

2nd. The adjustment for collimation is made more rapidly.

On the Hawaiian survey, the author used a seven-inch Y theodolite by Elliott Brothers, carrying an eighteen inch telescope with eyepiece magnifying forty diameters. It was furnished with stadia-hairs reading 1 per 100, and a movable micrometer hair for long distances. It was a heavy instrument, but, being of such long range, did not want so much shifting. It was carried by one man, over very bad country, and was quite satisfactory.

The adjustment of the Y theodolite is the same as the Y level for parallax collimation and bubble. The zero is then brought to coincide with the zero of the vernier of the vertical arc when the bubble is at the centre of its run. This is done by means of a small screw fastening the vernier of the vertical limb to the vernier plate over the compass-box.

The adjustment of the horizontal limb is the same as that for the transit theodolite.

Stanley's New Patent Telemetrical Theodolite

Stanley's New Telemetrical Theodolite, Fig. 90, has the following improvements: A tribrach stage instead of four parallel screws, which permits the instrument being used on a wall without the legs, and prevents all possibility of straining the centre.

The instrument has a mechanical stage for shifting the centre with exactness over the desired spot.

The telescope supports and the centre are in one casting, also the size of the bearing on the centre has been increased.

Instead of webs, which are a constant anxiety to the engineer or surveyor, platinum-iridium points are substi-

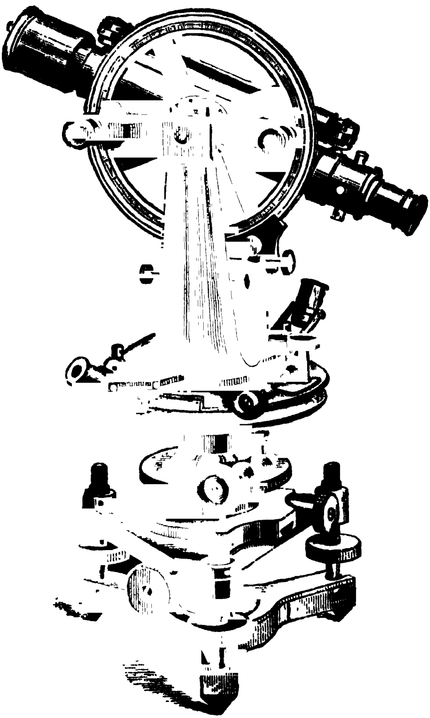


FIG. 90

tuted. These neither rust nor break, and allow of any dust being brushed off with a camel-hair pencil without interfering with the adjustment.

The eyepiece is also fitted with two vertical adjustable points for measuring distances.

A long trough needle is supplied with each instrument in place of the usual small circular compass.

The head of the tripod is of an improved form, giving greater rigidity.

The transit-theodolite is the basis of the author's tacheometer, and the following description will embrace all that is required for the use and adjustment of the ordinary transit, together with the special points of difference in his particular design.

GRIBBLE'S 'IDEAL' TACHEOMETER

The author has been fortunate enough to be able to experiment a good deal in instruments without so much outlay as most inventors in that line. He has been able to dispose of his theodolites in foreign countries and then get new ones. The 'ideal' is the fifth instrument specially designed by him for preliminary survey. There is really nothing original about its principle, and yet in its actual form and combination of parts it is unlike any other instrument he has met with.

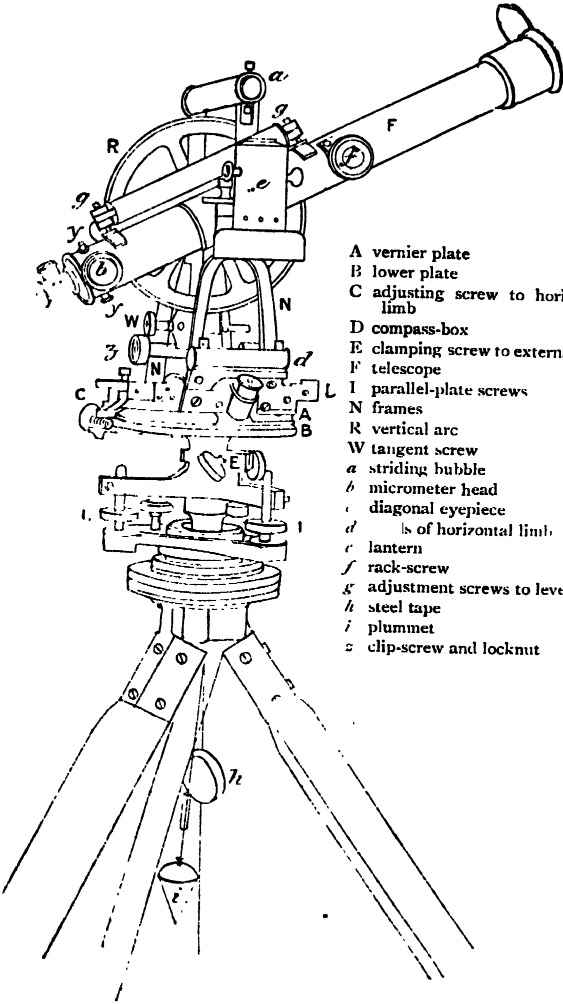
The principal features of novelty are :

First. A decimal subdivision of the ordinary degree of 90 to the quadrant.

By this means all the advantages of the centesimal graduation are obtained whilst retaining correspondence with the published astronomical ephemeris.

Tables to five places of decimals for the trigonometrical functions, together with logarithms of numbers to five places, can be procured from Messrs. Ascher & Co., Bedford Street, London. They are compiled by Dr. C. Bremiker, and cost 1*s.* 6*d.* bound in cloth. They are the only tables extant for this graduation. Transits can be adapted by merely changing the vernier.[1]

[1] It is the original graduation of Briggs in 1633, followed by Roe and Oughtred, in which they endeavoured to get rid of the senseless sexagesimal subdivision, which survives in spite of them.



A vernier plate
B lower plate
C adjusting screw to horizontal limb
D compass-box
E clamping screw to external axis
F telescope
I parallel-plate screws
N frames
R vertical arc
W tangent screw
a striding bubble
b micrometer head
c diagonal eyepiece
d ls of horizontal limb
e lantern
f rack-screw
g adjustment screws to level
h steel tape
i plummet
z clip-screw and locknut

Slide-rules graduated decimally can also be obtained, at the same price as for the sexagesimal subdivision, from either Messrs. Tavernier & Gravet at Paris or Messrs. Davis & Sons, Derby.

Curve-ranging and tacheometry are especially facilitated by this graduation : see Chapter VII.

Secondly. The advantages of a glass diaphragm are obtained without the usual drawbacks. The loss of light is reduced to a minimum, and the diaphragm can be removed without disturbing the micrometer diaphragm, if it should be found necessary to clean it. Its adjustment does not require the delicate handling which is needed for the micrometer hair, so that any one who can adjust a transit can adjust this instrument with greater ease and in less time.

Thirdly. The instrument is a combination of the stadia principle within the limits of legibility of the figures upon the staff, together with the micrometer principle for sights at longer range ; the level staff being adapted for use either on the one principle or the other.

Fourthly. The telescope is of unusual power ; probably no instrument has yet been constructed of the same lightness and portability with a magnifying power of fifty diameters. It is consequently equally well adapted to astronomical observations and long-range sights of the staff.

Fifthly. It combines minuteness of levelling power with lightness of construction by making the vertical arc 6 inches, and the horizontal arc 5 inches diameter. Both arcs read with the vernier to ·01 degree, which is equal to 36 seconds, but the vertical arc can be estimated by the eye to 18 seconds. In tacheometry it is generally sufficiently accurate to read the horizontal line to the nearest minute, which is more than obtained by the five-inch circle, and the saving in weight is considerable. The trunnion standards also are stayed.

Sixthly. It has a wide range of efficiency.

As an astronomical telescope, its power enables a good observation to be taken of the phenomena of Jupiter's

satellites, or the culmination of one of the small stars in the British Association Catalogue, used in conjunction with a lunar transit for determination of the longitude.

As a micrometer telescope of long range it will determine distances with considerable accuracy at several miles. As a tacheometer it is of superior power to any the writer has yet met with.

As a handy theodolite for ranging base-lines and railway curves, it has none of the complications of the Wagner-Fennel Tacheometer, the Eckhold Omnimeter, or the Porro type of telemeter. It has the appearance of a medium sized simple transit theodolite and can be more easily adjusted.

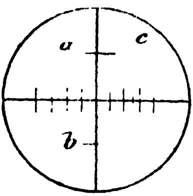


Fig. 97.

The appearance of the diaphragm is not so complicated as the usual tacheometer. Only the axial lines are drawn across the glass, the rest are replaced by ticks which cannot possibly lead to confusion.

The ticks *a* and *b* are the stadia lines. The vertical ticks, or comb as they are called, are spaces of 100 micrometer units, or 1,000 units on the micrometer vernier.

A three-screw or 'tribrach' stage has been adopted to avoid strain on the pivot and expedite the adjustment. A centering arrangement was at first included, but has been left out of the last instrument made. Where the chief use is for short bases with the chain, or curve-ranging, the centering clamp is a convenience, but it weighs from 4 to 5 pounds, and in tacheometry the bases are long, and the instrument is not so frequently shifted. Any small errors from the width of the staff as a picket, or from inaccurate centering, are eliminated by the observation for azimuth.

The addition of another lens as first introduced by Porro to make the centre of anallatism (see Glossary) coincide with the vertical axis has been abandoned. The constant

of the instrument is from 1·75 to 2·0 feet, according to size. Practice has proved that no time need be lost nor any mistake made in adding this constant. It is not worth any extra expense or loss of light in order to eliminate it.

Instead of the ordinary compass-box, a trough-needle is substituted, which saves the headway under the telescope, and answers the purpose of obtaining the magnetic variation at the commencement of the work equally well.

A word of caution is needed about the use of the tribrach stage. A very grave defect exists in the form frequently supplied to theodolites, namely, that the clamping plate is not left attached to the parallel-plate screws when the instrument is put away in the box. It forms a separate piece. In addition to the extra trouble in packing away, it is a great danger, inasmuch as there is no warning to the surveyor, who may chance one day to leave his clamp open. With the old-fashioned screw head, his instrument may be loose and turn once or twice round without falling, but one slip of memory with the clamping plate and it would be sudden death to his instrument the moment he shifted it. To remove this danger, a screw is put into the clamping-plate, which has to be removed if the transit has to be set on a wall. Comparatively few surveyors use it thus, and it is generally preferable when taking observations extending over several days to drive very solid pegs 4″ × 4″ into the ground and cut notches for the feet of the tripod. At any rate, it is so very seldom that the instrument requires to be taken out of the clamping-plate, that it is a most mistaken plan to risk the safety of the instrument in order to make it easily removed. In the 'Ideal' tacheometer, the instrument packs into its case in two pieces.

For astronomical observation, the usual diagonal eye-piece and lamp for illuminating the axis are provided.

The plummet is suspended from a short chain, which is a fixture with the instrument. It is preferable to a rigid hook, which is liable to get bent. The hook of the chain

is strong enough to carry a heavy weight for steadying the instrument in a high wind. The distance of the hook from the centre of the trunnion forms with that from the initial point of a 5 feet steel spring tape 1·50 feet, so as readily to take the height of instrument as an independent check.

The box of the steel tape forms for ordinary use the plummet, but for special accuracy a pointed plummet is also provided.

Adjustments of the 'Ideal' Tacheometer

Supposing that the micrometer hair has broken, we will commence with

1. *Putting in a spider hair.* Prepare the rectangular frame shown in Fig. 93 of copper wire, $\frac{1}{16}$ diameter or thereabout, soldered together. Catch a field spider and let

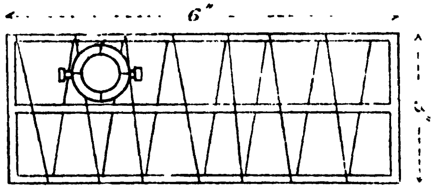


Fig. 93.

him drop by his web from one end, then wind round the frame till full. Preserve it at the bottom of the level or transit case in a little casing with blocking strips to keep the web from rubbing off. It will then be ready for use at any time. The points where the web touches the frame should be tipped with shellac to fix them.

Place the diaphragm on the table with a strong light bearing upon it. Tip with shellac the faint lines cut on it to mark where the web should go. Superimpose the web by delicately turning the frame until the web is in its position. Hold it there till the shellac cakes, very gently

pressing the web, so that it shall be quite taut. Remove the frame, and the web will be left in position.

2. *To replace the diaphragms.* To put in the glass diaphragm requires no description. To replace the movable diaphragm, hold it by a pair of tweezers while inserting the capstan-headed screws. Turn the screws until the hair is as nearly vertical and as near the middle of the tube as can be estimated by the eye. Leave the micrometer screw till the last thing. Slide the movable hair to one side so as to be out of the way.

Insert the eyepiece.

3. *To remove parallax.* See p. 294.

4. *To make the line of sight correspond with the zero of the horizontal limb.* Set the instrument as nearly level as can be done with the eye, then clamp the lower plate B, and, having unclamped the vernier-plate A, direct the telescope on some well-defined object, and bring it into coincidence with the point of intersection of the axial lines on the diaphragm. Take the reading on the horizontal limb AB; suppose it to be 20°·00; then move the vernier-plate A half-round, turn the telescope over, and again intersect the object, taking the reading on the horizontal limb AB—suppose it 200°·04; take the difference between this and the first reading + 180° (which in the present case would be 200°, and the difference ·04°); halve this difference, and subtract it from the second reading when it is greater than the first reading + 180°, and add it when it is less; this is the mean reading (= 200°·02). Set and clamp the instrument to this mean reading, and intersect the object by means of the screw which moves the glass diaphragm, sideways. Repeat this operation until the readings taken with the instrument in these two different positions, face right and face left, differ from one another by 180°.

5. *To make the line of sight correspond with the zero of the vertical limb.* It is not necessary that the line of sight should also be identical with the optical axis. See remarks

on levels, p. 291. Parallelism is sufficient. Level the instrument carefully on the external axis, by means of the levels *d*, *d* on the horizontal limb AB; next take a pair of verticals—*i.e.* on faces right and left to any well-defined terrestrial object; set the vertical circle R to the mean of these readings, and clamp it; now intersect the object, using the two screws *z*, *z* which clip the vertical circle R to the stud in the telescope frames N, N, ***and not*** the tangent screw W. When the readings on the right face agree with the left face, the index error will be o. The clip-screws *z*, *z* are provided with capstan-headed locknuts, which should then be screwed home, as these screws are not used except for adjustment. The leather caps on the nuts of the clip-screws are to prevent their being moved by mistake instead of the tangent screw. One of them need never be touched. It can be marked, and only the other one used for releasing the telescope, so that the locknut on the untouched clip-screw will bring the vertical limb to its proper position when it is replaced without fresh adjustment.

Example. With the telescope in its normal position observed the top of church spire. Vertical angle $+11°\cdot13$. Rotating the horizontal limb and reversing the telescope, vertical angle $= +11°\cdot19$. Set the vertical limb at $11°\cdot16$ by the tangent screw W, and make the line of sight intersect the object by means of the clip-screw, *z*. Then reverse the telescope, and rotate the horizontal limb to their original positions, and the vertical arc will still read $11°\cdot16$. If not quite exact repeat the operation.

6. ***To make the plane of rotation of the vernier plate horizontal.*** Clamp the vertical limb R at o°. Tighten the clamp-screw E, unclamp the vernier-plate A, and turn it round until the telescope is immediately over two of the parallel-plate screws I, I; bring the bubble in the telescope-level P to the middle of its run by the screws I, I; turn the vernier-plate 180° so as to bring the telescope

again over the same screws, but with its ends in a reverse position. If the bubble of the telescope level does not remain in the middle of its run, bring it back to that position, half by the parallel-plate screws I, I, and half by the screws *g*, *g*. This operation must be repeated until the bubble remains accurately in the centre of its run in both positions of the telescope; now turn the vernier-plate A until the telescope is directly over the third parallel-plate screw, and bring the bubble to the middle of its run by turning that screw. The bubble should now retain its position while the vernier-plate is turned completely round, showing that the plane of rotation is truly horizontal or parallel to the axis of the bubble.

7. *To test the accuracy of the external axis.* Clamp the vernier-plate to the lower plate by turning the clamp-screw C, and loosen the clamp-screw E; move the instrument round its external axis, and if the bubble retains its central position during a complete revolution there is no error. If one exists it is only to be remedied by the maker. If, however, adjustment 6 is correct, the work will be correct.

8. *To adjust the levels on the vernier plate.* These are only guides for approximate adjustment of the instrument when it is being set up. They are set true when the other adjustments are completed.

9. *Horizontality of the axis of the telescope.* This is done with the striding bubble. See full description under levels, p. 292.

10. *To test whether the vertical axial hair has been placed in a vertical line.* This adjustment, is not required in the 'Ideal' tacheometer, but is added for the use of those who have ordinary transits. It is not essential to correct work if the intersection of the hairs is always observed, but it is convenient to be able to intersect an object with any part of the vertical line, and it is also useful for telling if a staff &c. is held vertical.

Fix the intersection of the axial lines on some sharply

total measured length, and at every 100 feet measuring up ordinates equal to the micrometer values. Between the hundreds, the values are interpolated by ordinates to the curve down to the tens, and the units are then interpolated by simple proportion with the slide-rule.

The curves are put in with ordinary railway sweeps.

This will be found quite as accurate as more frequent observations in the field; as a matter of fact, the curves will soften several asperities due to eccentricities of observation.

Printed profile paper with faint scale lines is convenient for this purpose.

The 'Ideal' reads the level staff with the stadia up to 1,000 feet; beyond that the micrometer is used.

A close approximation to the actual magnifying power of the telescope may be very simply obtained in the following manner. Set it up at a distance of 20 feet from the anterior focus and cause a levelling-staff to be held there perfectly steady. The stadia will then subtend ·2 upon the staff. Direct the lower stadia hair to some convenient figure such as 5·00 feet, then the upper hair will be at 5·20. Open the other eye and look with the one eye through the telescope, and the other unassisted at the staff, until the actual staff appears to be superimposed upon the magnified portion and you seem to *see the one through the other.* Book the two positions of the stadia hairs on the actual staff. For instance, if the eyepiece magnified 20 diameters, the stadia hairs would appear to cover $20 \times {\cdot}2 = 4$ feet of the natural staff. The lower would then appear to be at 3·10 and the upper at 7·10 feet.

If there are no stadia hairs, take the top and bottom of the field and proceed similarly.

The Sketch-board Plane-table

Fig. 95 represents the present improved type of cavalry sketching-board as modified from Col. Richard's original design by Capt. Willoughby Verner, whose field-sketching

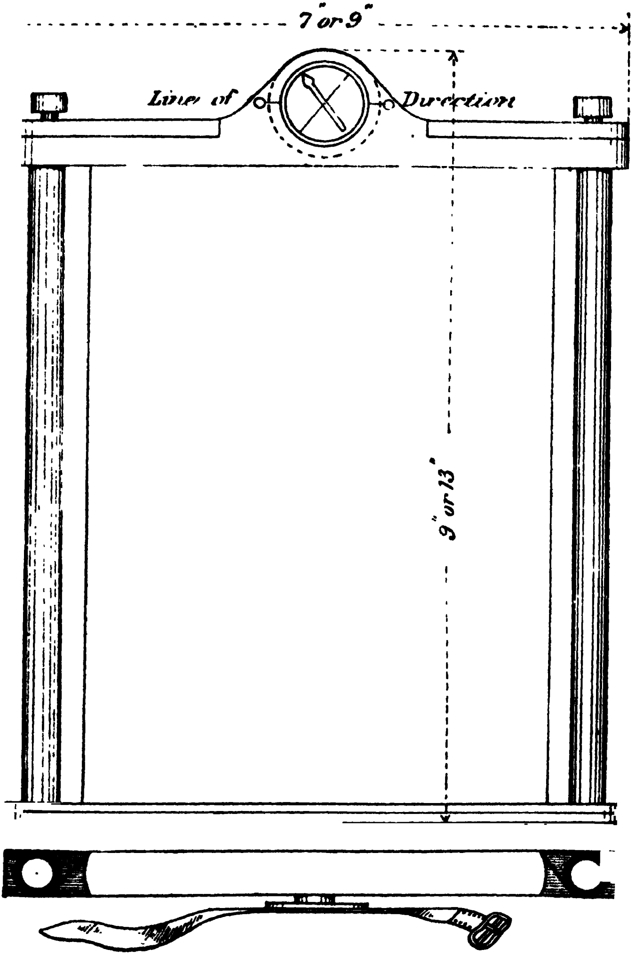


FIG. 95.

and reconnaissance in Egypt form an interesting feature of the Soudan Campaign.

The paper is in a continuous roll, wound round the brass rollers, and turned down under them so as to be always quite tight. It is to a large extent waterproof, being specially prepared, and graduated in quarter-inch squares.

The line marked 'line of direction' is that in which the paper is unrolled so that when the line of march does not deviate very much from it, the work will not run off the board.

The thin line drawn across the compass-box is a cut in the glass, the object of which is to indicate, by turning the compass-box round, the position of the needle, when the board is in the line of direction. The needle, being a loose one, is first observed before starting, the board being set to the line of direction. The compass-box is then turned until the cut in the glass coincides with the needle, and serves ever after to bring back the board into the line of direction. This is further explained on p. 41.

The heads to the brass rollers fit tight in the wood and keep the paper stretched by their friction. In the cross section the buckle is shown attached to a dovetailed slide and pivot. When used on horseback, the strap fastens the board to the arm, and when fastened to a tripod, the dovetailed plate runs into a corresponding slide on the tripod. The seven-inch by nine-inch size is more of a sketchboard and the nine-inch by thirteen-inch size is more of a plane-table, but both sizes are used in either way effectually.

Pocket Altazimuths and Compass-Clinometers.

Strictly speaking an altazimuth is an instrument which gives the altitude and azimuth from one adjustment of its line of sight.

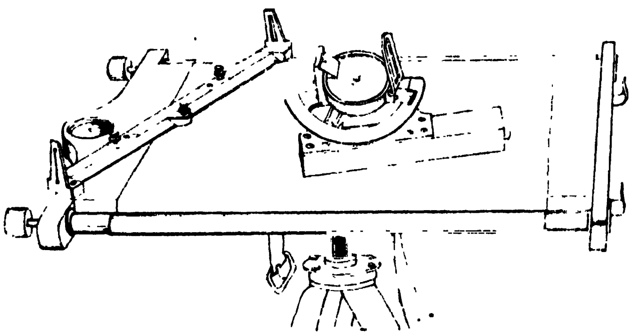
The first altazimuth was a stationary transit instrument ; what is commonly called now a wall transit.

Three-screw transit theodolites are now adapted to being

placed on walls ; but the term altazimuth has also become applicable to pocket instruments, in which the azimuth is found by the magnetic bearing and the angle of elevation or depression is read by means of a plummet or a level.

Another variety might be designed combining the pocket sextant with the measurement of altitudes. Possibly there may be such already.

Messrs. Casella make an altazimuth in which the magnetic bearing and altitude by plummet are both read by a microscope and the observation is assisted by a telescope.



Messrs. Elliott make Colonel O'Grady Haly's compass-clinometer in which the same operations are performed without any lens power.

All these instruments are practically two instruments in one, to save the time and trouble of taking out first one and then another from separate sling-cases.

Hand instruments are better without telescopes. Work requiring telescopic accuracy is better done with a tripod.

A combination of the box sextant with the altitude measurer would have the advantage of greater precision in the horizontal angles and fully as much despatch. On the other hand it would be subject to cumulative error, which

the compass is not. It would be free, however, from magnetic deviation, which sometimes renders the compass wholly unreliable.

The plummet makes a slow clinometer and not a sure one either. If the instrument is not held in a vertical plane, the plummet is apt to stick against the side.

The writer has used nothing of the kind of late years, but confines himself to an arrangement of his own, consisting of a combination of Captain Abney's reflecting level with a prismatic compass.

This is illustrated in Fig. 96. The Abney level is now so well known that it only needs a passing description (see Fig.

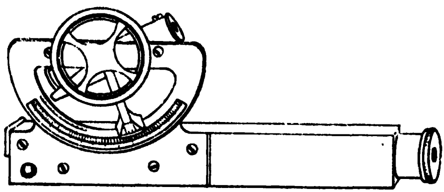


Fig. 97.

97). Its principle is somewhat that of the vertical arc of a theodolite, only it is in miniature and without lens power. A small telescope has been tried but not with success. A reflector occupies half the field of the sighting-tube, adjusted at an angle so that when the bubble is at the centre of its run, its reflection is seen in the centre of the tube.

Whatever angle is given to the line of sight, the bubble can be brought to the centre of its run without removing the eye from the tube, and the little handle which moves the bubble tube is furnished with an arm and vernier, which indicates the angle of altitude upon a graduated semicircle. The range of view of the bubble through the tube is only up to 60°, but the author uses his instrument also up to 90° for getting the batter of a wall or the side slope of a steep g by placing it on a straight edge, and then bringing

the bubble to the centre of its run without looking through the tube. He therefore has the circle graduated up to 90°, and often uses it as a 'plumb-level.'

The prismatic compass forms the handle, which turns the bubble, in place of the usual little brass wheel. Its disc is only $1\frac{1}{4}$ inch, but it has a graduation from 0 to 360° and also a quadrantal graduation for working by latitude and departure. The prism magnifies so well that the graduation of single degrees can be easily subdivided by estimation to quarter degrees. It is made by Messrs. Elliot Bros., and does them credit.

The further use of this instrument is explained in 'Route Surveying,' pp. 61, 64, &c.

PASSOMETERS

The passometer is a register of the number of paces taken at any time by a walker; its use has been explained on p. 54, &c. The graduation of passometers is often very clumsy. The arrangement in Fig. 98 is recommended as being easily readable. The right-hand small dial reads up to 20,000, which is over ten miles. The large hand gives the fifties up to 2,000, and the left-hand small dial the units between the fifties.

The instrument in Fig. 98 is reading 0,558 paces. The hands of passometers are generally set loose, so that they can be adjusted to zero. For surveying this is a mistake, because they soon get out of teaching with one another. A permanent fit is best—that is to say, with square bearings instead of conical, like the hands of a watch.

The passometer should be attached to the centre of the person; if placed on one leg it will only count half paces. The best place is hanging from a waistcoat button, with the hook well buttoned in to be secure, and just kept from shaking up and down by the edge of the waistcoat. If too free it will occasionally made a double count. When the counting

has to be suspended for a while, the passometer can be turned upside down.

The pedometer and passometer can be made to work together, by adjusting the former until it records a quarter

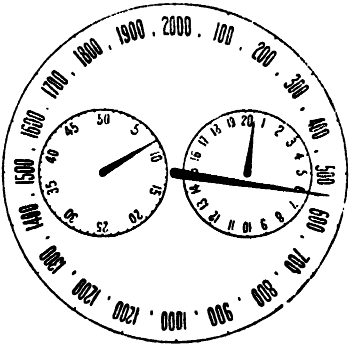


FIG. 98.

mile against the average number of paces to that distance by the latter. The ordinary adjustment of the pedometer to length of pace in inches is not always exact.

SEXTANTS

Both the Hadley sextant and the box sextant are so well known that only the adjustments will be here given, which are taken from Mr. Heather's excellent little work on 'Instruments.'

To examine the error arising from the imperfection of the dark glasses. View the sun through the dark glass at the end of the telescope, removing the shades; make a contact with the reflected image of the sun and its direct image seen through the unsilvered part of the horizon glass. Then, removing the dark glass, set up one shade glass after the other, and book any alterations of angle due to each succes-

sive combination. No adjustment can be made for this error; when registered it has to be applied at every observation.

The adjustments consist in setting the horizon glass perpendicular to the plane of the instrument, and in setting the line of collimation of the telescope parallel to the plane of the instrument.

To adjust the horizon glass. While looking steadily at any convenient object, sweep the index slowly along the limb, and, if the reflected image do not pass exactly over the direct image, but one projects laterally beyond the other, then the reflectors are not both perpendicular to the face of the limb. Now the index glass is fixed in its place by the maker, and generally remains perpendicular to the plane of the instrument; and, if it be correctly so, the horizon glass is adjusted by turning a small screw at the bottom of the frame in which it is set, till the reflected image passes exactly over the direct image.

To examine if the index glass be perpendicular to the plane of the instrument. Bring the vernier to indicate about 45°, and look obliquely into this mirror, so as to view the sharp edge of the limb of the instrument by direct vision to the right hand and by reflection to the left. If, then, the edge and its image appear as one continued arc of a circle, the index glass is correctly perpendicular to the plane of the instrument; but if the arc appears broken the instrument must be sent to the maker to have the index glass adjusted.

To adjust the line of collimation. 1. Fix the telescope in its place, and turn the eye-tube round, that the wires in the focus of the eye-glass may be parallel to the plane of the instrument. 2. Move the index till two objects, as the sun and the moon or the moon and a star more than 90° distant from each other, are brought into contact at the wire of the diaphragm which is nearest the plane of the instrument. 3. Now fix the index, and altering slightly the position of the instrument, cause the objects to appear on the other wire, and if the contact still remain perfect the line of col-

limation is in correct adjustment. If, however, the two objects appear to separate at the wire that is further from the plane of the instrument, the object end of the telescope inclines toward the plane of the instrument; but if they overlap, then the object end of the telescope declines from the plane of the instrument. In either case the correct adjustment is to be obtained by means of the two screws, which fasten to the up and down piece the collar holding the telescope, tightening one screw and slackening the other, till, after a few trials, the contact remains perfect at both wires.

The instrument having been found by the preceding methods to be in perfect adjustment, set the index to zero, and if the direct and reflected images of any object do not perfectly coincide, the arc through which the index has to be moved to bring them into perfect coincidence constitutes what is called the index error, which must be applied to all observed angles as a constant correction.

To determine the index error.—The most approved method is to measure the sun's diameter, both on the arc of the instrument properly so called, to the left of the zero of the limb, and on the arc of excess to the right of the zero of the limb. For this purpose, firstly, clamp the index at about 30′ to the left of zero, and, looking at the sun, bring the reflected image of his upper limb into contact with the direct image of his lower limb, by turning the tangent screw, and set down the minutes and seconds denoted by the vernier; secondly, clamp the index at about 30′ to the right of zero on the arc of excess, and, looking at the sun, bring the reflected image of his lower limb into contact with the direct image of his upper limb by turning the tangent screw, and set down the minutes and seconds denoted by the vernier underneath the reading before set down.

Then half the sum of these two readings will be the correct diameter of the sun, ***and half their difference will be the index error.*** When the reading on the arc of excess is

the greater of the two, the index error then found must be added to all the readings of the instrument, and when the reading on the arc of excess is the less, the index error must be subtracted in all cases.

To obtain the index error with the greatest accuracy, it is best to repeat the above operation several times, obtaining several readings on the arc of the instrument, and the same number on the arc of excess, and the difference of the sums of the readings in the two cases, divided by the whole number of readings, will be the index error ; while the sum of all the readings divided by their number will be the sun's diameter.

Example.

Readings on the arc of the instrument	Readings on the arc of excess
35′ 0″	29′ 25″
35′ 5″	29′ 35″
35′ 10″	29′ 20″
105′ 15″	88′ 20″
88′ 20″	105′ 15″
Number of reading } 6) 16′ 55″ Difference	6) 193′ 35″ Sum
	32′ 15·8″ Sun's diameter
2′ 49″ Index error	

The Adjustments of the Box Sextant

On the upper surface of the instrument close to the rack-screw is a little hole with the square head of a screw inside it. This is a screw for adjusting the horizon glass in the plane of the instrument by observing the reflected image of the sun.

There is also another screw at the side of the instrument for removing index error by taking readings on the arc of the instrument, and on the arc of excess as described in the adjustments of the Hadley sextant, and then by correcting with the screw so as to make both readings the same.

Both screws are turned by a little key, which is removed for the purpose from its position in the arc-plate opposite the rack screw.

The Solar Compass

The solar compass is an instrument for determining the true bearing of any object, instead of the magnetic. It performs automatically the operation of finding at any time of day and in any latitude the azimuth of the sun. It cannot of course be used in cloudy weather, and should not be used when the sun is less than one hour above the horizon or less than one hour from the meridian. It stands on a tripod of the same size, and is itself somewhat larger than the ordinary surveyor's compass.

It is furnished with vertical arcs to set it to the latitude of the place, and a pair of sight vanes.

Unless the adjustments are very carefully attended to errors are likely to arise, greater than those due to local magnetic deviation.

The price ranges somewhat between that of a surveying compass and a plain transit.

The Heliostat and Heliograph

These instruments are both sun-signals ; the principle is the same in both. A mirror reflects a sunbeam from one point to another. The heliostat is only intended to give a continuous ray, whereas the heliograph is provided with a spring to the mirror, by which it is made to give little jumps, causing it to flash short or long flashes corresponding with the dots and dashes of the Morse code. The heliostat is capable of being used for signalling also, by alternately covering and uncovering the mirror. Both instruments are furnished with sighting vanes on jointed arms, commonly called jiggers, to set the mirror in line with the station to be signalled. The heliograph is also furnished with a duplex

mirror, so that when the sun is behind the instrument it can still be used.

The best heliographs are Galton's sun signal, fitted with a telescope, and Mance's heliograph. Both are expensive instruments, and need not enter into the outfit of the preliminary surveyor.

The sketch on next page is taken from Capt. Wharton's 'Hydrography,'[1] and illustrates the principle of all such instruments. It is a heliostat made by a ship's blacksmith. The standard is about 2½ feet high. 'In soft ground the ends of the legs can be pressed into the earth, and on rocky ground stones placed against the legs will hold the instrument steady. The arm *m*, of light iron, is carried separately, and slips over the shaft of the standard, clamping when required with a screw.

'Into a circular socket in head of standard shaft the leg of the frame holding the mirror is shipped ; this is also to be tightened by a retaining screw. The mirror, which can be of any size from 2 to 6 inches or more in diameter, revolves on its retaining screws as an ordinary toilet-table glass, and can be held in any position by the screws.

'The ring, of flat wood, is made as light as possible, so as to exert less strain in wind. Across it are nailed crossed strips of copper with a white cardboard disc, about an inch in diameter, fastened to their centre.

'The rod that carries this ring slips up and down in a hole at the end of the arm, and is clamped by a retaining screw.

'In the centre of the back of the mirror a hole of about ¾ inch diameter is scraped in the tinfoil, being careful to leave a sharp edge. A similar hole is cut out of the wooden back of the glass frame. This we shall call the blind spot.

[1] A cheap instrument of this description is made by Potter of London.

'To direct the flash to an object, bring the mirror vertical, and looking through the hole in the centre, revolve the arm until in the direction of the object nearly, clamp it, and adjust the disc-rod as nearly as may be for elevation or depression. Then, slightly loosening the screw, clamping the arm, finally adjust the latter so that the object, as regarded through the hole in the mirror, is obscured by the

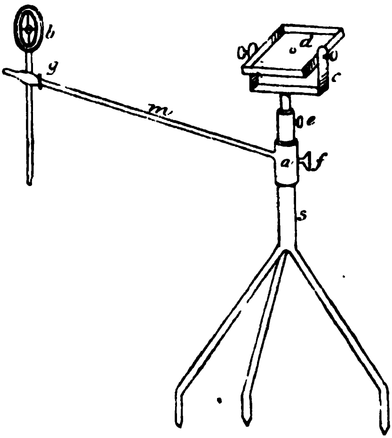


FIG. 90. Captain Wharton's 'Hydrography.'

a, sliding collar carrying arm *m*, revolving round *s*; *b*, wooden ring, painted black, with iron wires and white cardboard centre, sliding vertically by means of rod through arm *m*; *c*, iron frame to hold mirror, fitting into socket in top of standard *s*; *s*, iron standard with fixed tripod legs; *d*, blind spot in mirror; *e*, screw for clamping in iron frame; *f*, screw for clamping arm; *g*, screw for clamping ring rod.

white cardboard disc in centre of the ring. By turning the mirror so that the dark shade caused by the blind spot is thrown on to the disc, the flash will be truly directed, and must be kept so by slight alterations of the position of the mirror, which should therefore be clamped only sufficiently to hold it steady and yet admit of gentle movement. The shadow of the blind spot should be slightly smaller than

the disc, so as to ensure having it truly in the centre of the latter.

'The mirror must be of the best glass, with its faces parallel, or the shadow of the blind spot will be very indistinct when the mirror is at a large angle, and also the beam of light will be dispersed before it has traversed many miles.

'It is well to have the mirror a fair size, say 6 inches square, as in practice it will be found generally necessary, in order to save time, after once adjusting the flash, to leave a man to keep it on while the surveyor is taking his angles; and although a man will soon pick up the knack, a larger mirror will allow for eccentricities on his part, and also, on a dull day, a faint flash will be detected from a large mirror, where a small one would not carry any distance.

'On a bright day a flash from a 3 inch by 2 inch mirror has been seen 55 miles and more.

'In hazy weather, angles have been got when the place from which the flash was sent was entirely invisible; and

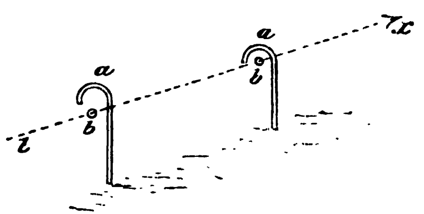


FIG. 100.
a, *a*, bent wire; *b*, *b*, brightened bullets; *l*, *x*, line of sight to the Tugela.

thus whole days have been saved by this simple contrivance.

'Only those who have spent hours, or even days, in straining their eyes to see a distant mark can appreciate the value of a heliostat.'

The following is an extract from an article in 'Science

for All,' by Major C. Cooper King, of the Royal Military College, Sandhurst, and explains a handy makeshift shown on Fig. 100.

'At Ekowe, in the campaign in Zululand, the whole apparatus had to be improvised. . . . Two wires with the upper parts bent into the form of a semicircle, and with cross wires uniting the bent end with the upright part, were tried. The sights were composed of brightened bullets in the centre of the cross wires, and when these rods were set in the ground and aligned with the flash from a common looking-glass, no difficulty was experienced in communicating.'

Sun signalling has the great advantage of being independent of background.

Telemeters and Range-finders

The term telemeter, which was introduced by surveyors, has been appropriated to some extent by electricians and others; and it is probable that the better term tacheometer, which has been quite adopted by the French, will soon become general.

The simplest form of telemeter is the plane-table, which is a graphic triangulation to attain the same end as the optical telemeter.

In every case the telemeter measures by a kind of triangulation, varying from the long base, bearing a considerable proportion to the distance to be measured, down to the base of a few inches or at most a few feet, from which distances of over a mile are measured.

Then, again, telemeters vary in size from the Weldon locket range-finder, about $\frac{3}{4}$ inch in diameter, to the 6 feet and 7 feet 6 inch telescope-telemeters of Clarke and Struve.

Organically they may be divided into three classes.

1. Those in which the measured base forms an integral part of the instrument itself. Such are Adie's 18 inch and 3 feet telemeters, described in Heather's 'Instruments;'

Piazzi Smyth's 5 feet telemeter, Colonel Clarke's 6 feet, and Otto Struve's $7\frac{1}{2}$ feet telemeter.

The instrument is held square with the line of sight. The object whose distance has to be measured must be sharply defined. One end of the instrument receives a reflected image at right angles, and the other end a reflection at an angle forming the complement of the angle subtended by the length of the instrument at the point observed.

With so great a multiplication as this, it is evident that the base must be very exactly measured in order to graduate the instrument.

Adie's telemeter is of brass, and covered with leather in order to diminish changes of length under varying temperatures; but all this class suffer from this cause. They are not much used now.

2. Those in which the measured base is at the point observed, generally consisting either of a graduated staff or a pair of discs connected by a rod; the naval surveyors call it a 10 feet pole, the Americans a target

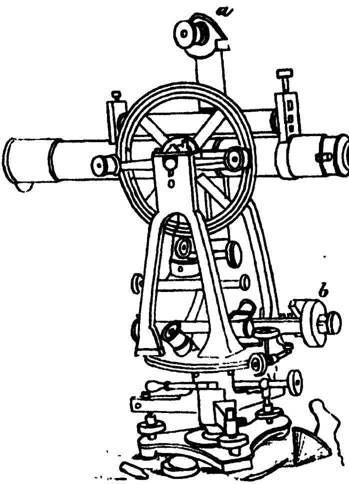
In this class there are two subdivisions.

a. Those which have a fixed base and varying angle, as the Rochon micrometer, furnished with a reflector by which the images of the discs are made to coincide, as in a sextant, and the angle subtended is read in terms of distance.

Messrs. Elliott's army distance-measuring telescope is another of this class. The fixed base is the assumed height of infantry, 5 feet 11 inches, or cavalry 8 feet 10 inches. Two micrometer wires are fixed in the diaphragm, and are actuated by the micrometer head so as to exactly intercept the object. The micrometer head is graduated differentially in terms of distance; one side for the cavalry base, the other for the infantry. Targets set at those respective distances on a pole would of course give more exact results.

Eckhold's omnimeter, Fig. 101, also made by Messrs. Elliott Bros., is a combination of this and the first class alluded to. The instrument is a transit theodolite in prin-

ciple, but it is furnished with a powerful telescope and a long and powerful microscope (*a*), reading a graduated baseplate (*b*). At one operation, by observing top and bottom of a 10 feet pole, the distance and elevation are given by a rule-of-three sum. The instrument has given much satisfaction, both in India and the Colonies; it is, however, more complicated than the stadia principle, takes longer to adjust, and is not more accurate. The author is



informed by Messrs. Elliott that an improvement upon this instrument is in course of development by Mr. W. N. Bakewell.

b. Telemeters which have a variable base and a fixed angle. This is the stadia principle, and the commonest instruments of this type are the stadia or telemetric theodolite and the tacheometer. The former instrument is illustrated on p. 307 in a new form by Mr. W. F. Stanley. Theodolites of all sizes are now frequently fitted with stadia-

lines. The tacheometer of Messrs. Troughton & Simms is graduated to 400 primary degrees and a decimal subdivision. Tables of the trigonometrical functions are published for this graduation. Any kind of decimal graduation affords great facility both to astronomical calculations and telemetry.

For the author's system of graduation, see p. 307.

Another feature is the obtaining of what is termed anallatism (see Glossary), or unchangeableness, at the vertical axis. There is very little practical advantage from this device, which involves another lens. A micrometer screw can of course be added if desired, but it does not usually form an integral part of the tacheometer.

An instrument of this kind with more novelty about it is the Wagner-Fennel tacheometer.

All the instruments previously described under this class require the calculations of vertical height to be made by tables or by slide-rule, but this instrument gives them direct, without even the rule-of-three sum required by the Eckhold omnimeter.

The measurement is by stadia hairs, but the movement of the telescope in a vertical plane instead of being recorded upon a graduated circle causes a vernier to slide backwards or forwards along a scale fixed parallel to the line of sight.

This vernier actuates a pair of other scales, one horizontal and the other vertical, thus giving directly the horizontal and vertical components of the inclined distance measured by the stadia.

The instrument is highly ingenious, but it is not so suitable for the ordinary work of the railway engineer, such as setting out curves, &c., and it is not adapted to astronomical observations.

3. The third class of telemeters is that in which the base is measured on the ground, at the observer's station. The Hadley sextant, though not, strictly speaking, a tele-

meter, is greatly used as such by naval hydrographers, sometimes in conjunction with a Rochon micrometer.

The Dredge-Steward omnitelemeter, made by J. H. Steward of London, illustrated and described in 'Engineering,' of August 20, 1886, is a good instrument of this class. It is in appearance and in principle very much like a box sextant, stands on a light tripod, and measures the angle formed at the distant point by the pair of rays from the ends of a base run out by steel tape at the observer's station. It has this special feature, that the base need not be run out square, which saves time when there are obstructions.

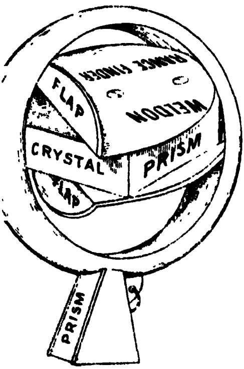


FIG. 102.

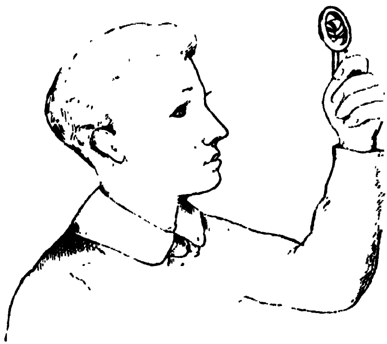


FIG. 103.

Perhaps the most suitable of any of this class of instruments to the ordinary surveyor is the Weldon range-finder, the patent of Colonel Weldon, R.A. There is indeed a new range-finder coming out from Woolwich, but the military authorities are not as yet communicative upon the subject.

Figs. 102, 103 represent the watch-size 'Weldon;' there is also a 'locket' size. The following description is mainly from the pamphlet written on the subject by Captain Willoughby Verner, R.A.:

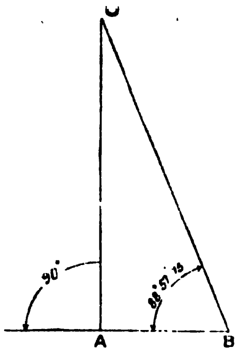


FIG. 104. Range-taking with a direction point (using first and second prisms).

A, position of observer.

O, object of which the range is required.

D, direction point (as distant as possible).

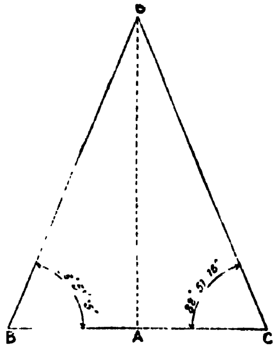


FIG. 105. Range-taking without a direction point (using second prism only).

A, position of observer marked by first picket.

O, object of which the range is required.

B, second picket.

C, third picket.

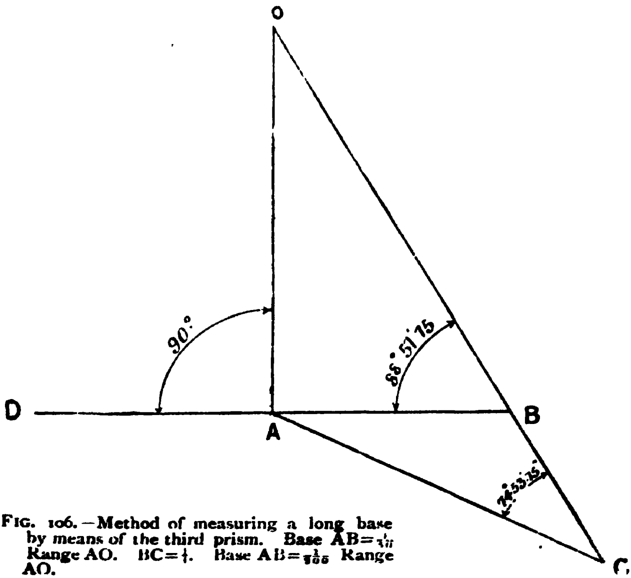


FIG. 106.—Method of measuring a long base by means of the third prism. Base AB=1/50 Range AO. BC=¼. Base AB=1/500 Range AO.

The Weldon range-finder consists of three prisms of crystal, accurately ground to the following angles :

1. 90°.
2. 88° 51′ 15″.
3. 74° 53′ 15″.

The range of an object, as at O, is taken by observing the angles OAD, OBD, at the base of a right-angled triangle ABO in Fig. 104, the measured base AB of which $=\frac{1}{30}$ of the distance or range AO. In this case the first prism, 90°, and second prism, 88° 51′ 15″, are used.

A second method, and one equally important, is by observing the angles at the base of an isosceles triangle as BCO in Fig. 105, when the measured base BC is $\frac{1}{25}$ of the distance or range AO. In this instance the second prism (88° 51′ 15″) only is used.

In order to measure the base AB or BC accurately and rapidly the third prism of 74° 53′ 51″ is used ; but this is merely a convenience and not a necessity, except under very exceptional circumstances.

It will be at once seen by those who have had any experience of range-finding that there are several objections to this apparently simple process. They may be summarised as follows :

1. Difficulty of obtaining any definite mark at right angles to the object to reflect the latter upon, when employing a base of $\frac{1}{30}$.

2. Difficulty of always finding ground suitable for measurement of base as regards view, general configuration, and space, whatever base may be employed.

These difficulties are more or less common to instruments of this class, but the Weldon possesses the two great advantages over most of them of exceeding portability and reliable, permanent reflectors.

The writer possesses one of these useful little instruments, but has not obtained results with it equal to those on record. Considerable practice is needed in

using them, both in judging a position for measuring a base and in taking the observation. The right-angled prism is practically an optical square. In Fig. 104 the range is taken thus. Choose a good direction point D or else put in a ranging rod at D making its reflection coincide with the object by means of the right-angled prism. Then using the 88° prism retreat along AB leaving a mark at A to keep yourself in line; when the 88° prism shows a coincidence of D with O, B is reached. AB is measured either by paces or tape, and multiplied by 50=the range.

In the official trials at Aldershot in 1883, the average error was only 34 yards for each range, whilst in India in 1885 it was 35 yards.

The chief use of this class of hand instrument is on reconnaissance, when neither plane-table nor theodolite can be carried. It is more accurate in the hands of a practised man than the sketching-board used on the portable stand, described in 'Route Surveying,' p. 47, and occupies no more space than a watch. The sketching-board, however, has counter advantages of permitting the surveyor to make some representation of the country as he goes. Captain Willoughby Verner advocates the use of the range-finder in conjunction with the sketching-board. The writer has not yet been able to get more correct results with it than with the sketching-board used alone, on its light tripod, as a triangulation, measuring bases with the latter not less than $\frac{1}{25}$ of the distance, but taking them in any convenient direction and any convenient length, which cannot be done with the range-finder. A great drawback to this class of instrument, and one which puts it out of competition with telemetric theodolites wherever the latter can be used, is the time occupied in running out the base. For taking ranges in battle military officers are content to step the base. Captain Verner mentions an amusing incident connected with 'field-firing' of a certain corps, in which a portly sergeant was told off to give the range with the range-

finder, but did not understand it, neither did he wish to show his ignorance. He was doubling along in rear of his section, and was seen after each 'rush' to raise his range-finder to his eye, after the manner of a spy-glass, and call out whatever his fancy moved him. He had orders to *give ranges constantly* and obeyed them.

A rapid method of using the range-finder with the accuracy needed on route-survey would be to measure the two angles simultaneously by two observers, each holding one end of a 100 feet steel tape. With the 90° prism in the hand of one, and the 88° in the hand of the other, distances up to 5,000 feet could be measured, or, by each holding the 88° prism, up to 2,500 feet. It would require a boy to hold up the sag of the tape. Each observer should hold his handkerchief under the range-finder, for the other observer to make the conjunction of image with the distant object through the prism.

The Weldon is now reduced in price, and hardly more expensive than an ordinary optical square, the duties of which are well performed by the 90° prism, and it is not at all liable to derangement.

Mr. Steward also makes a range-finder called 'The Simplex,' which is similar in principle of action with the Weldon, only the angle is taken with a glass mirror, and it requires adjustment each time. It is practically an optical square with an alternative position of the mirror at a lesser angle.

The Bate Range-finder

This is a binocular combined with two reflectors. The *modus operandi* is the same as with the Weldon and similar instruments.

It has the decided advantage of optical power combined with a wide field, so that for a hand-telemeter, used by one man, it is one of the best of this class.

Like the Dredge-Steward, it allows of considerable latitude in measuring the base, and instead of the very small

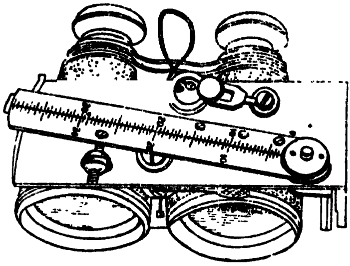


FIG. 107.—The Bate Range-finder, closed.

telescope of the latter, is furnished with a fine binocular, suitable for general use.

The angle subtended by the base measured is expressed in multiples by putting in a gauge between two graduated

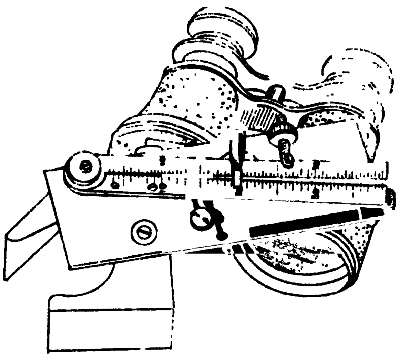


FIG. 108. The Bate Range-finder, open.

limbs, thus measuring their divergence, or, in other words, plotting the angle in the instrument itself. There is no

calculation beyond the multiplication of the measured base by the multiplier indicated by the gauge.

It costs about ten guineas, and, considering what there is in it, is not an expensive instrument.

HYPSOMETRIC INSTRUMENTS AND HYPSOMETRY

The Barometer. This instrument has become so familiar to the public as a weather-glass, and to engineers and travellers as a height-measurer, that it is hardly necessary to show an external view of it. Fig. 109 represents the internal

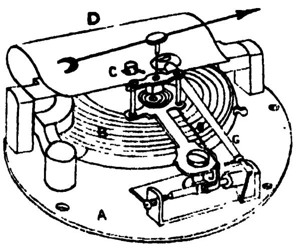


FIG. 100.

mechanism. The vacuum chamber B is of German silver, firmly attached to the base-plate A. The sides of the vacuum are compressed with a force equal to the weight of a three-year-old child when the air is pumped out, and are kept from collapsing by the powerful spring D. The varying pressure of the atmosphere produces tiny pulsations in the vacuum, which are very greatly multiplied by levers G and J, and chain Q. The lever G acts also as a compensator for changes of length in the mechanism due to temperature, but this does not dispense with the corrections to the hypsometric formula due to difference of temperature, which are given on p. 350, or to the actually recorded difference of level by any aneroid. The altitude scale corresponding

to equal variations of atmospheric pressure is a differential one, like water pressure on a lock gate, illustrated at p. 111. By means of a device termed a worm, similar to that used for equalising the tension of the spring in watches, the differential movement of the index is in the surveying aneroid changed into a uniform one, so as to admit of the use of a vernier scale. The 5 inch aneroid is the *only* satisfactory type for the surveyor. It reads with the vernier to feet, not that it is *reliable* to single feet at one reading by any means. It needs repetition of readings, compensation for temperature, and every other precaution, and then is only reliable in proportion to the number of readings taken, and the range between highest and lowest results.

It is clumsy and heavy, and it would be a great boon if a small aneroid could be obtained, say 2½ or 3 inch, which would give equally good results.

The reason that this has not been attainable is because there must be a *large vacuum* and a *small range* of altitude in order to get a sensitive instrument which will not 'hang' at all when rapidly changing from rise to fall and back again.

Travellers sometimes specify to instrument makers such absurdities as a 2 inch aneroid reading to single feet, and having a range up to 15,000 feet, and absolutely compensated.

A surveyor needs a maximum of exactitude over small differences of level. He should have a 5 inch aneroid with a range of 3,000 to 5,000 feet for all the ordinary work of reconnaissance.

For mountain-climbing he should have a 2 inch watch aneroid and a boiling-point thermometer.

The following results are added to show the range of discrepancy in two aneroids, one a 3 inch and the other a 5 inch, both of them eight guinea instruments by the same makers, of the first rank in London.

Distance, A to B, 0·55 mile. Difference of elevation by Ordnance benchmarks, 69·0 feet. Mean temperature (average of all the times), 68° Fahr.

Three-inch Aneroid.

		feet		feet
First time.	Up	72	Down	60
Second ,,	,,	77	,,	63
Third ,,	,,	77	,,	57
Fourth ,,	,,	50	,,	72
Fifth ,,	,,	66	,,	68
		5)342		5)320
Average		68·4		64
Corr. for temperature		2·3		
		70·7		
Range of error from true level				+ 10·3
				− 10·3
Range of error between two successive readings				22
Range of final mean of means of 'ups' from true elevation				1·7
Range of final mean of means of 'downs' from true elevation				2·7

Five-inch-Aneroid.

		feet		feet
First time.	Up	67	Down	65
Second ,,	,,	78	,,	68
Third		66		63
Fourth		63		71
Fifth	,,	64	,,	56
		5)338		5)323
		67·6		64·6
Corr. for temperature		2·2		
		69·8		
Range from true level				+ 11·3
				− 11·3
Range between two successive readings				10
Range of final mean of means of 'ups' from true elevation				0·8
Range of final mean of means of 'downs' from true elevation				2·2

Distance A. to D, 1·1 mile. Difference of elevation by Ordnance benchmarks 113·2. Mean temperature, 68° Fahr.

Three-inch Aneroid.

		feet		feet
First time.	Up . .	112	Down . .	107
Second ,,	,, . .	117	,, . .	97
Third ,,	,, . .	125	,, . .	92
Fourth ,,	,, . .	93	,, . .	123
Fifth ,,	,, . .	103	,, . .	106
		5)550		5)525
		110		105
Corr. for temperature		3·6		
		113·6		

Range of error from true level	+ 15·4 − 18·4
Range between two successive readings . . .	− 33
Range of final mean of means of 'ups' from true elevation	0·4
Range of final mean of means of 'downs' from true elevation	4·6

		feet		feet
First time.	Up . .	103	Down	110
Second ,,	,, . .	117	,, . .	113
Third ,,	,, . .	107	.	100
Fourth ,,	,, . .	107	.	108
Fifth ,,	,, . .	107	.	100
		5)541		
		108·2		106·2
		111·8		

Range of error from true level	+ 7·3 − 10·2
Range between two successive readings . . .	7·0
Range of final mean of means of 'ups' from true elevation	1·4
Range of final mean of means of 'downs' from true elevation	3·4

Distance, A to C, 0·776 mile. Correction for temperature added to each reading. Difference of elevation from Ordnance benchmarks, 129·8 feet.

Five-inch Aneroid.

		feet		feet
First time.	Down . .	130·2	Up . .	134·4
Second ,,	,, . .	131·1	,, . .	132·2
Third ,,	,, . .	132·3	,, . .	121·4
Fourth ,,	,, . .	128·3	,, . .	132·4
Fifth ,,	,, . .	107·0	,, . .	128·9
Sixth ,,	,, . .	124·8	,, . .	124·8
Seventh,,	,, . .	117·6	,, . .	136·5
Eighth ,,	,, . .	133·9	,, . .	123·6
		8)1,005·2		8)1,034·2
		125·7		129·3
Range of error from true difference . . .				+ 6·7
				- 22·8
Range between two successive readings . .				21·9
Range of final mean of means of 'ups' from true elevation				0·5
Range of final mean of means of 'downs' from true elevation				4·1

The wide readings will surprise those who are not familiar with the subject, and who have been led to believe otherwise by those who are proficient in the sale, less versed in the construction, and least of all conversant with the use of aneroids. *It will be noticed that the five-inch in the above tests has from a quarter to one-half the range of error of the three-inch between any two successive readings.* This is a much more important point to the surveyor than the accuracy of the final mean of means. In the three-inch the range of error for the half-mile was two-thirds of that for the mile, whereas in the five-inch the range of error for the mile was less than that for the half-mile. It is frequently the case that errors of five and ten feet will be registered in going up and down a hillock whereas the height of a mountain may be correctly given within two or three feet.

It will also be understood from this why stress is laid in

Chapter II. upon using the Abney level with the aneroid for the smaller rises and falls of route-survey, and making as many repetitions as possible of the aneroid readings which determine the maximum and minimum elevations. These repetitions should all be entered in the fieldbook as carefully as the original ones, and a symbol placed beside the mean of means on the plan to indicate the degree of accuracy which had been obtained. It will be observed, further, that in every case the mean of the 'ups,' whether the journey commenced by going up or by going down, was more correct than the mean of the 'downs.' Having used a number of instruments in different parts of the world, the writer has always found this the case. The pressure of the atmosphere puts a strain upon the spring D. When going uphill, the spring is relieved, and when going downhill it is again depressed. It is natural that the resilience of the spring should be freer in action than its compression, on account of the intermediate mechanism needed to procure the action.

Practical Suggestions in Procuring and Using Aneroids.

1. Procure a first-class instrument.
2. Learn its peculiarities and eccentricities.
3. Have it examined every year.
4. Register atmospheric changes by a second instrument at camp.
5. When reading, hold horizontally, and tap several times.
6. Take as many repetitions as possible.
7. Where discrepancy, not due to atmospheric disturbance in the district, exists between 'ups' and 'downs,' prefer the 'ups.'
8. Always apply the temperature correction.
9. Three 'ups' will generally give a mean result correct to 3 feet, or two 'ups' to 6 feet, in difference of level of 50 to 500 feet.

The Kew test is useful, and should be specified in ordering valuable instruments, but even that does not prove the instrument in every way.

A first-class aneroid is as hard to get as a horse without a blemish. A seasoned vacuum and perfect mechanism inside, together with clear open graduation outside, and a sling-case which will open and shut *quickly* and *safely*, are the chief points to aim at. Five-inch aneroids are now made in aluminium to save weight, but the price is very high.

HYPSOMETRY,

or height-measuring, is a term applied to determination of the level above the sea, by the barometer or boiling-point thermometer, as distinguished from levelling with the spirit level.

Both operations rest upon the same fundamental principles of the weight and pressure of the atmosphere, commonly known as Mariotte's and Charles's laws. These are, firstly, that at a uniform temperature the pressure of any gas varies inversely as its volume, and secondly, that at a uniform pressure the expansion produced by a given increase of temperature is the same for all gases.

The standard mercury barometer is now but little used for hypsometric purposes on account of the inconvenience in transporting it. The aneroid barometer just described has superseded it, first because of its exceeding portability, secondly because of its superior sensitiveness. It is subject, however, to derangement of its delicate mechanism, not alone from accident, but even from changes of climate; and the boiling-point thermometer, which measures the atmospheric pressure without any mechanism, is generally added to the outfit of anyone who wishes to make extended and reliable hypsometric observations. It must not be supposed that the use of any of these methods is sufficient to ascertain elevations with the accuracy of a spirit-level. They do no more than give the difference of atmospheric

pressure at two different situations. When the condition of the atmosphere is steady in the district, that is to say when a standard barometer at either of the stations remains stationary during the period of observation, the difference of pressure at the two situations forms a means of correctly ascertaining the difference of elevation. When, however, there is a disturbance of the air-pressure, no reliance can be placed upon the results unless by also recording the movements of a stationary barometer at the two stations by independent observers.

With all their draw-backs hypsometric instruments are indispensable, because they are the only means of approximately determining differences of elevation *en route*. The real makers of them are few, and hardly ever put their names on them. It is not sufficient to order them from a good instrument-maker or even to have them tested at Kew. The best proof of their excellence is in their daily use for a few weeks under conditions of varying temperature, elevation, and methods of transportation. All aneroids are marked 'compensated,' and instrument makers will often tell their customers that no further correction is needed on account of this compensation -a statement which always shows that they do not understand the principle of the instrument.

The compensation of the mechanism of the aneroid causes it to record the *correct difference of atmospheric pressure* at different times or in different places, and it is therefore not subject to the correction which is applied to mercurial barometers to correct them for the expansion or contraction of the column of mercury.

But the *difference* of atmospheric pressure between two different elevations is *less at high temperatures than it is at low ones*. If the barometer recorded 30″ pressure at sea level when the thermometer stood at 32° Fahr., at 1,000 feet elevation with the same temperature the pressure would be nearly 28″·88. But if the mean temperature were 72° Fahr., the pressure would be about 28″·96. In a table

graduated with altitudes corresponding to differences of atmospheric pressure at 32° Fahr. mean temperature, the pressure of 28″·96 would only give 923 feet, and the altitude in the table would have to be multiplied by 1·083 to give the true altitude. Hence it is much more convenient to know the multipliers for temperature applicable to the aneroids as they are actually constructed, for they are not graduated from 32° Fahr., but from 50° Fahr., by Airy's formula, or from 53° Fahr. by the author's mean formula of English and American standards.

The compensation of an aneroid is nothing more than a device analogous to that in a watch by which the expansion or contraction of the mechanism of the instrument itself due to changes of temperature is compensated. Nothing can however prevent the expansion of the air produced by any increase of temperature from acting differently upon the aneroid under its altered condition, and nothing can prevent the altered specific gravity of the air due to different altitudes and latitudes from likewise differently affecting the instrument. English aneroids are graduated with an altitude scale corresponding to a mean temperature of 50° calculated by Airy's formula or 53° by the author's mean formula; consequently, as shown in Plate IX. Fig. 111, the altitudes indicated by the instrument at lower temperatures are too great and should be treated with the multipliers given, and *vice versâ* at higher temperatures. Each of the subdivisions has a value of ·002 and reads thus: 1.000; 1·002; 1·004, and so on. The temperature corrections given in the textbooks are arranged from 32° Fahr., so that a double calculation is required. This diagram can be used with the slide-rule.

When the altitudes are calculated from readings of the barometer, or boiling-point thermometer, formulæ are used based upon the researches of Guyot and others, but varying considerably in the coefficients adopted by different experimenters in different countries.

The tables given by Mr. Francis Galton in the textbook

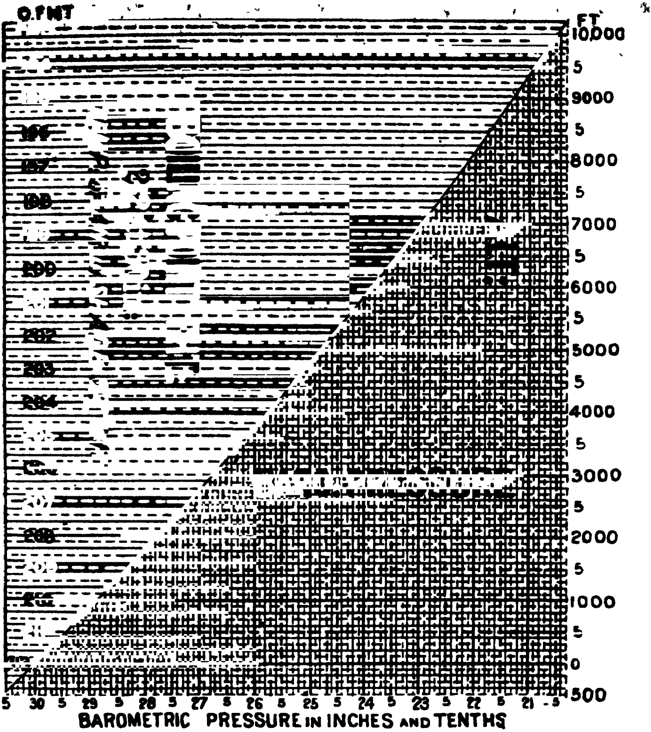
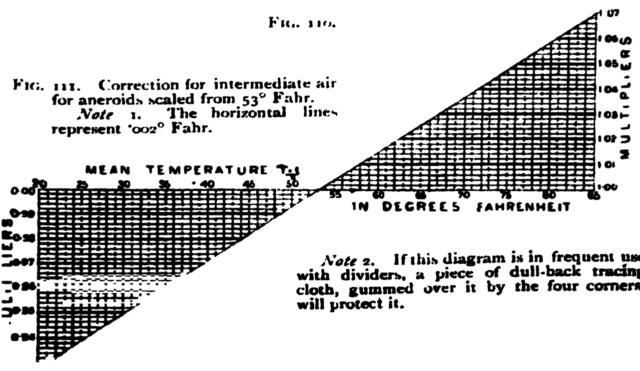


FIG. 110.

FIG. 111. Correction for intermediate air for aneroids scaled from 53° Fahr.

Note 1. The horizontal lines represent ·002° Fahr.



Note 2. If this diagram is in frequent use with dividers, a piece of dull-back tracing cloth, gummed over it by the four corners will protect it.

of the Royal Geographical Society are calculated by Loomis, and differ both from those of Airy, from which most English barometers are graduated, and still more from those of Col. Williamson, the American authority.

The formula of the latter is as follows:

$$H = \frac{30 D \times T}{B}$$

where H = difference of height in feet between two stations.
D = difference of barometric pressure between the two stations.
T = tabular number corresponding with mean temperature.
B = mean barometric pressure.

A short approximate rule is given by Mr. J. H. Belville of Greenwich Observatory, which is nearly correct between temperatures of 50° and 60° Fahr.

$$S : D :: 55{,}000 : H$$

where H = difference of height in feet between stations.
S = sum of barometric readings.
D = difference of barometric readings.

Examples will be given to show the divergence of the three authorities, and a series of constants, K, for each degree of temperature from zero to 102° Fahr., by which, with a modification of Belville's rule, a mean result will be obtained between those of the English and American formulæ and of so simple a kind that it can be worked by the slide-rule. It is as follows:

$$H \text{ (difference of height)} = \frac{K.D}{S}$$

where K is the coefficient for mean temperature given in Table LI., D = difference of barometric pressures, and S their sum.

TABLE LI.—*Value of* K *in Formula* $H = \frac{K \cdot D}{S}$.

Deg. M. T. Fahr.	K.	Log K.	Deg. M. T. Fahr.	K.	Log K.
0	48753	4·68800	52	54785	4·73866
1	48869	4·68903	53	54901	4·73958
2	48985	4·69006	54	55017	4·74049
3	49101	4·69109	55	55133	4·74141
4	49217	4·69211	56	55249	4·74232
5	49333	4·69314	57	55365	4·74323
6	49449	4·69416	58	55481	4·74414
7	49565	4·69517	59	55597	4·74505
8	49681	4·69619	60	55713	4·74596
9	49797	4·69720	61	55829	4·74686
10	49913	4·69821	62	55945	4·74776
11	50029	4·69922	63	56061	4·74866
12	50145	4·70023	64	56177	4·74956
13	50261	4·70123	65	56293	4·75045
14	50377	4·70223	66	56409	4·75135
15	50493	4·70323	67	56525	4·75224
16	50609	4·70423	68	56641	4·75313
17	50725	4·70522	69	56757	4·75402
18	50841	4·70621	70	56873	4·75491
19	50957	4·70720	71	56989	4·75579
20	51073	4·70819	72	57105	4·75667
21	51189	4·70918	73	57221	4·75755
22	51305	4·71016	74	57337	4·75843
23	51421	4·71114	75	57453	4·75931
24	51537	4·71212	76	57569	4·76019
25	51653	4·71309	77	57685	4·76106
26	51769	4·71407	78	57801	4·76193
27	51885	4·71504	79	57917	4·76281
28	52001	4·71601	80	58033	4·76367
29	52117	4·71698	81	58149	4·76454
30	52233	4·71794	82	58265	4·76541
31	52349	4·71891	83	58381	4·76627
32	52465	4·71987	84	58497	4·76713
33	52581	4·72083	85	58613	4·76799
34	52697	4·72179	86	58729	4·76885
35	52813	4·72274	87	58845	4·76971
36	52929	4·72369	88	58961	4·77056
37	53045	4·72464	89	59077	4·77142
38	53161	4·72559	90	59193	4·77227
39	53277	4·72654	91	59309	4·77312
40	53393	4·72748	92	59425	4·77397
41	53509	4·72843	93	59541	4·77482
42	53625	4·72937	94	59657	4·77566
43	53741	4·73030	95	59773	4·77650
44	53857	4·73124	96	59889	4·77735
45	53973	4·73218	97	60005	4·77819
46	54089	4·73311	98	60121	4·77903
47	54205	4·73404	99	60237	4·77986
48	54321	4·73497	100	60353	4·78070
49	54437	4·73589	101	60469	4·78153
50	54553	4·73682	102	60585	4·78236
51	54669	4·73774			

Comparison of Author's Formula with English and American Authorities

Example 1.

	inches.
Reading of barometer at lower station . .	26·64
,, ,, upper ,, . .	20·82
Thermometer at lower station	70°
,, ,, upper ,, ...	40°

Bar. sum $= 26{\cdot}64 + 20{\cdot}82 = 47{\cdot}46$	$T + t$ . . $=$	110°
Difference $= 26{\cdot}64 - 20{\cdot}82 = \ 5{\cdot}82$	M. T. . $=$	55°
	K, sec table	55,133

$$H = \frac{K.D}{S}$$

Log K	$= 4{\cdot}74141$
+ Log 5·82 . . .	$= 0{\cdot}76492$
	5·50633
− Log 47·46 . . .	$= 1{\cdot}67633$
Log alt. 6760·9 ft. .	3·83000

By slide-rule. Place the 47·46 on the slide opposite to the 55,133 on the rule; find the result 6,760 on the rule opposite to 5·82 on the slide.

The same by the American rule, from the standard work by Lieut.-Col. R. S. Williamson of the U. S. Army, as quoted by Trautwine.

Rule. Height in feet=

$$\frac{\text{difference D of barometer} \times \text{table No. for MT} \times \text{constant } 30}{\text{mean reading of barometer}}$$

whence height=6748·5 feet.

The same by the Geographical Society's rule by Francis Galton, F.R.S. See 'Hints to Travellers,' p. 185.

Part I. for 26·64 inches.	24,506·2
20·82 ,, ...	18,066
	6,440·2
$\frac{6,440{\cdot}2}{900} \times (70° + 40° - 64)$	329·1
	6,769·3

Example 2.

Reading of barometer at lower station . .	30·646
,, ,, upper ,, . .	23·66
Thermometer at lower station . . .	77·5
,, ,, upper ,, . . .	70·3

Sum. bar. 30·046 + 23·66 = 53·706	T + t = 147·8
Diff. ,, 30·046 − 23·66 = 6·386	M. T. = 73·9
	K, see table 57,337

$H = \frac{K . D}{S}$

Log K	. . .	4·7584350
Log D	6·386 . .	0·8052289
		5·5636639
Log S	53·706 . .	1·7300228
Log alt.	6,817·8 . .	3·8336411

By slide-rule as before 6,820.

The same by the American rule is 6,829 feet and by Galton's rule 6,805·6 feet.

Example 3.

Bar. lower station	28·00
,, upper ,, ...	22·00
S	50·00
D	6·00

Temp. lower station	60° Fahr.
,, upper ,, ...	40 ,,
	100
M. T.	50 K = 54,553

$$H = \frac{K . D}{S} = \frac{54,553 \times 6}{50} = 6,546·36$$

By the American rule	6,527·5
By Galton's rule	6,552·7
By Airy's tables	6,571·5

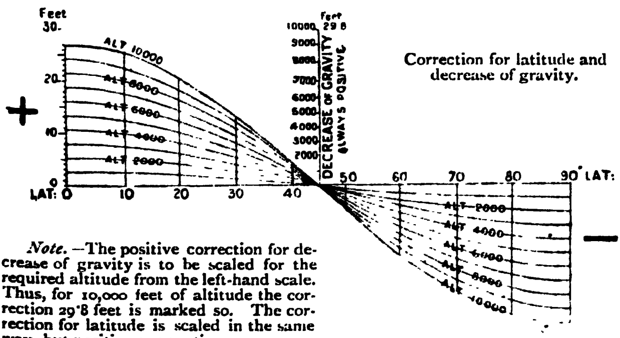
Examples 1 and 2 are taken from 'Trautwine' and 'Hints to Travellers.' Example 3 is formed of barometric pressures intermediate between the other examples, and the result from Airy's tables is taken from a pamphlet on

the aneroid by Houlston & Sons, lent to the writer by an instrument maker as being the standard used in graduating aneroids. The author's two surveying aneroids graduated 1 to 10,000 feet, the other to 5,000, have their zero at 31 inches and are graduated almost exactly in the same way. The former reads 9,325 feet at 22 inches. Airy's tables above alluded to show an altitude of 9,347·5 at that pressure and mean temperature of 50° Fahr.

The American rule would give 9,214·4 feet, Mr. Galton's rule 9,318·4, the author's rule 9,263·6, an almost exact mean as before between the English and American authorities.

The usual corrections for temperature of intermediate air are given in tables having 32° Fahr. M. T. as the starting point, and consequently are not capable of being applied to ordinary aneroid readings without a double calculation.

In the diagram Fig. 111 multipliers are given having 53° Fahr. mean temperature as their starting point, that



Note.—The positive correction for decrease of gravity is to be scaled for the required altitude from the left-hand scale. Thus, for 10,000 feet of altitude the correction 29·8 feet is marked so. The correction for latitude is scaled in the same way, but positive or negative.

Fig. 112.

being the basis of the graduation of the ordinary aneroid according to the author's mean formula. By using these multipliers, the correction can be made with one calculation,

and in fact for ordinary work by the slide-rule in the field as close as most instruments will read.

Example 4. At a mean temperature of 70° Fahr. the aneroid showed an altitude of 1,265 feet. The multiplier is 1·037.

Place the left hand 1 of the slide opposite to 1·037 on the rule and we have opposite to 1,265 on the slide 1,310 on the rule, or more correctly, by figures 1,311.

As a remembrancer for every 10 degrees of temperature $\frac{\text{above}}{\text{below}}$ 53° Fahrenheit $\frac{\text{add}}{\text{subtract}}$ 2·2 per cent. $\frac{\text{to}}{\text{from}}$ the registered difference of level by the aneroid.

The diagrams on Fig. 112 for change and decrease of gravity are added more to exhibit at a glance the amount of such corrections than for actual use, because it is rarely of any practical importance to know them. The local atmospheric disturbances may be far greater within a small area than the equivalent of these corrections in barometric pressure, so that to apply such refinements as these would be a sheer waste of time.

THE BOILING-POINT THERMOMETER [1]

'The boiling-point apparatus (Fig. 113) consists of a thermometer A graduated from 180° to 215°; a spirit-lamp B, which fits into the bottom of a brass tube C that supports the boiler D, and a telescopic tube E which fits tightly on to the top of the boiler. The thermometer is passed down the tube E from the top until within a short distance from the water, *which it should never touch*, and is supported in that position by an india-rubber washer F. The steam passes from the boiler up the tube E, and escapes by the hole G. To pack this instrument for travelling, withdraw the thermometer and put it into a brass tube, lined with

[1] From 'Hints to Travellers' by John Coles, Esq., Royal Geographical Society

india-rubber having a pad of cotton wool at each end; take off the tube E, shut it up and put the small end into the boiler D, which it fits, then withdraw the spirit lamp B, screw the cover over the wick, and replace it in C. The whole of this apparatus fits into a circular tin case 6 inches long and 2 inches in diameter.

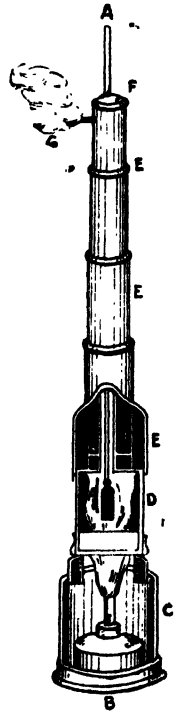


FIG. 113.

'*To use the boiling-point thermometer.* Take the apparatus to pieces, pour some water into the boiler D, the less the better, as it will boil the quicker (about one quarter full is quite sufficient); then put the instrument together as shown in the drawing, taking care that the thermometer is at least half an inch clear of the water, and light the spirit-lamp; as soon as the water boils, the steam, ascending through the tube E, will cause the mercury to rise; wait until the mercury becomes stationary, and then read the thermometer; at the same time take the temperature of the air in the shade with an ordinary thermometer.

'When purchasing this instrument be careful to see that the lamp is large enough to hold a good supply of spirit; it is a common fault to make it too small. A small screw which may be made of tin to fold up is most useful to place on the windward side, and at a very low temperature is almost indispensable, as the heat is otherwise carried off too rapidly for the water to boil properly.'

The following rule for finding the height due to temperature of the boiling point, has been prepared in a similar

way to that for barometric pressure by adopting a coefficient which produces mean results between the English and American authorities. It is for a mean temperature of intermediate air$=53°$ Fahr. instead of freezing point, as customary in the textbooks. This has been done to make the coefficient agree with the graduation of ordinary aneroids, and for other temperatures the coefficient 540 has to be multiplied by the multiplier on diagram Fig. 111 Plate IX.; thus, at 32° the coefficient becomes $\cdot954 \times 540 = 515$, or at $82° = 540 \times 1\cdot064 = 574\cdot6$.

Rule. Let B=temperature of boiling point in degrees Fahr. deducted from 212°.

H=height of station above level of sea.

K=540 for a mean temperature of intermediate air of 53°—and varying as explained above.

$H = K.B + B^2$.

Example.

Boiling points	211·37
,, ,, ...	210·14
Mean temperature	82° Fahr.

Required the difference of elevation :

$$H = 540 \times 1\cdot064 \times 0\cdot63 + 0\cdot63^2 \quad . \quad . = 362\cdot37$$
$$H' = 540 \times 1\cdot064 \times 1\cdot86 + 1\cdot86^2 \quad . \quad . = 1{,}072\cdot14$$
$$\text{Ans. diff. in feet} \quad . \quad . \quad . \quad . = 709\cdot77$$

The value of the boiling-point thermometer consists in the fact that it is a perfectly simple machine, there is no wheelwork to get out of order, no vacuum to play off its caprices. Otherwise it performs the same duty as the aneroid. Its results depend upon the assumption that water boils at 212° at sea-level. This is not always true; water boils at different temperatures in different latitudes and also varies under different conditions of the atmosphere. In fact the boiling-point is nothing more than

a register of the atmospheric pressure, like the aneroid. Water can be put under an air pump at sea level, and made to boil nearly at freezing-point. The boiling-point thermometer is only suitable for the first pioneer work in very hilly country, such as finding a pass for a railway through a chain of mountains. It is not frequently used even for that purpose.

The surveyor generally contents himself with two aneroids rather than spend time in boiling-thermometers. It is not, however, such good work, because the best aneroids are fickle under varying conditions.

Office Instruments

At a pinch the surveyor can get along with hardly anything more than a pair of compasses and a straight-edge.

For a short survey, where impedimenta are a great objection, with a pocket case of instruments and a slide-rule he can do all the protracting, contouring, scale-making, and gradient-drawing that he wants. What he misses most is a box of good railway sweeps. They are bulky, and it is seldom he can afford space for them.

The author has combined in one box, 19 inches long by 11 inches wide by 8½ inches deep, a complete repertory of all the drawing instruments he requires, and though quite bulky, he has contrived to make room for it wherever he has gone. A description may be useful.

At the bottom is a full set of railway sweeps, 1 to 240 inches radius. A boxwood rolling parallel ruler with the edges graduated as a protractor and loaded in the middle with lead to steady it.

On the lower tray is a colour-box and water-dish, receptacle for liquid ink, liquid carmine, and liquid prussian blue in their own bottles. Small hammer, wire-cutting nippers, 'Yankee notion' screw-driver-bradawl, store of pencils, rubber, and colour-brushes, needles, copper-wire, brass

screws, paperclips, small sponge, chamois-leather, palette, lancet, surgical needle.

On the upper tray are, six-inch German silver circular protractor with short arm and vernier, reading to ·01 degree; three ply magnifiers for ditto. Complete set of drawing instruments, including proportional compasses and trammels.

In the lid wallet are one 15 inch, 60° set square, one 12 inch, 45° set square; two French curves, card of crow-quills, one 9 inch paper protractor.

On the lid-flap are one set of ivory scales 10 to 60, one universal scale, all 12 inch. They are laid out in a row in elastic loops, so that they can be withdrawn without needing to search for the right one. Also one small 45° and one small 60° set square.

It is not only a great convenience, but the best plan of keeping one's instruments to have them all together in a box where each one has its own place, and becomes 'conspicuous by its absence.'

A tee-square is much the best means of protracting angles, where the sheets are not longer than can be reached by it. The steel tee-square with bronze head sold by Charles Churchill & Co. of Finsbury, though an expensive instrument, is well worth its price. It is a protractor with a 36 inch arm, and a tee-head. It has a clamping screw with a large head to it, so that it is adjusted in a moment to any angle. It is used with an ordinary set square.

Barnett's diagraph, which only costs 11*s*., is another and ingenious instrument for rapid plotting, by the same makers.

Where no very great accuracy is aimed at, a paper protractor pinned down in one corner of the board, an ordinary tee-square, and a single jointed two-foot rule with a pretty stiff joint form a very cheap and efficient method of protracting.

The tee-square is run up to the protractor, and the 2-foot rule is set to the angle. It is then run down to the station from which it is to be laid off by the tee-square. It is useful

for laying off bracing of trestles; equal and opposite battering walls &c., because the 2-foot rule has only to be reversed to give the opposite angle.

It is also very useful for making strain-diagrams—although that is outside our province—because it is not so liable to shift as a parallel ruler.

The eidograph and pantagraph are occasionally a great help to the surveyor, but they can hardly be called satisfactory instruments. Reduction of plans is much better done by dividing them into squares, and aided with proportional compasses does not take much longer than by the eidograph.

The planimeter is a most valuable instrument to those who have extensive computations of acreage to make. It

FIG. 114.—Amsler's Planimeter.

does the whole work by recording, on a pair of indicator wheels, the travel of a roller round the periphery of any figure. Stanley's computing scales, which are much cheaper than the planimeter, are an efficient substitute for it and do not take very much longer.

THE SLIDE-RULE

This ancient but not antiquated instrument is not nearly as much appreciated in this country as it ought to be. Thoroughly scientific as it is in principle it has until recent years suffered from the imperfection of mechanical science which prevented the attainment of the same accuracy in graduated scales which is possible with figures.

The increased skill in making dividing-lathes has enabled the mechanician to produce an instrument of precise accuracy

only limited by its length and the consequent visibility of the graduation.

The slide-rule has been formerly known as Coggeshall's rule or the carpenter's slide-rule, and has been regarded as a rough though labour-saving makeshift.

Ever since its invention by Oughtred it has been misunderstood, until recently the French and Italians have brought it into something of the estimation which it deserves. Oughtred is said to have kept the instrument by him many years out of a settled contempt for those who would apply it without knowledge, having 'onely the superficial scumme and froth of instrumental trickes and practices, and wishing to encourage the way of rationall scientialists, not of ground-creeping Methodicks.' [1]

A glance at the catalogue of rules of all sorts, sizes, and prices from 6*s.* up to 10*l.*, manufactured by the firm of Tavernier-Gravet, will show that the French have appreciated this instrument more than we.

A very decided improvement on the cheaper form of boxwood rules has, however, been made by Messrs. Davis & Son of Derby and London, by overlaying celluloid upon hard wood. It looks like ivory and costs no more than the 10-inch Mannheim slide-rule. The graduation is quite equal to that of the French rule. There are some points of advantage in the writer's opinion in favour of the latter, but which might be easily adopted by the English makers if they were so disposed.

In Preliminary Survey the slide-rule is simply invaluable, and it is astonishing how quickly its manipulation is acquired, especially by those accustomed to read graduation of any kind.

If it were only for the one operation of reducing the optically measured distance to horizontal and vertical co-ordinates, and the horizontal distance to latitude and departure by two shifts of the slide, no surveyor should be

[1] Rev. W. E. Elliott, 'The Slide-rule.'

without one; but that is only one of its numerous functions, some of which are described in the chapter on Graphic Calculation.

To do justice to all the applications of the slide-rule to the various branches of technology would require a small volume for that purpose alone.

The printed directions supplied with the slide-rule only give general principles, and therefore instructions are given in this manual for a considerable number of those operations which are most commonly required upon survey. Repetition is of course unavoidable, but inasmuch as the use of the instrument may not be continuous, if its special application to some particular rule is forgotten it will not be used at all. It was therefore considered advisable to give numerous examples with the various rules.

The instructions for using the slide-rule are principally contained in Chapter VIII., but they are also necessarily interspersed all over the book. They are partly for sexagesimal degrees, although in many places decimal degrees are given along with minutes and seconds; Tables XL., XLI., and XLII., moreover, give a ready reduction of minutes and seconds to decimals of a degree. Nothing has been worked out with the centesimal degree, that being so much of a novelty that everything would have had to be in duplicate.

MM. Tavernier & Gravet, and Messrs. Davis & Son, make slide-rules graduated for the ordinary degree divided decimally, and they can be used for minutes without any reduction, because the subdivisions are merely at 3′, 6′, 9′, 12′, &c., which are ·05°, ·1°, ·15°, ·20°, and may be used either way. Instead of the mark for single minutes, there is one for ·01 degree.

The slide-rule may be used for long rows of figures by placing them in batches of threes; but its chief value is for all those small calculations up to an accuracy of $\frac{1}{300}$, which only require one adjustment of the slide; for in these the operation is performed in less time than it would take to

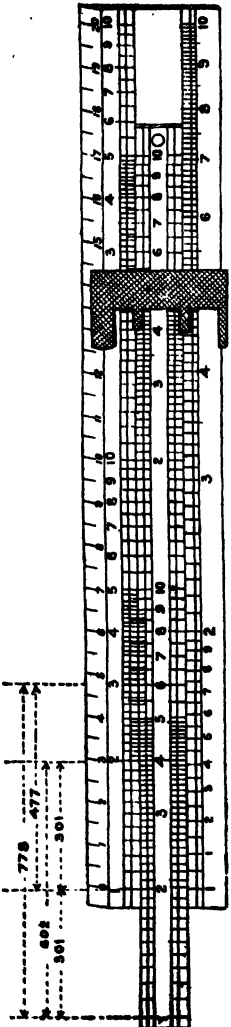


FIG. 115.—The Slide-rule.

look up the first factor of the sum in the table of logarithms, or to put them down on paper for ordinary reckoning.

The instrument, in its present form, will perform *accurately, and without any mental effort whatever*, a whole mass of tiresome, small calculations in less than half the time it would take a very quick man to work them out by any method he might choose.

In the field every surveyor knows how books and tables with soiled pages and fluttering leaves become lessons of patience and self-control to him, but the slide-rule enables him to reserve these moral exercises for other occasions. All graphic operations are essentially approximate, but it is possible to arrive at as close an approximation as may be needed for the purpose in view when the object and the principle are both clearly understood, and so to give perfectly correct results within the limits prescribed.

The principle of the slide-rule is that of *graphic logarithms*.

The organic formula of logarithms is Log $(A \times B) = \log A + \log B$. We can obviously perform this operation by scaling as well as by figures. For if we add the tabular logarithm of A to

that of B by scale or by figures, the sum of the two logarithms will represent the product of the two numbers. Referring to Fig. 115, the slide has been shown withdrawn one primary division to the left; and it will be noticed that the 1 of the upper scale of the rule is over the 2 of the slide, the 2 over the 4, the 3 over the 6, and so on. If the slide had been shown in its initial position, all the figures would have been in correspondence, *because they are equal logarithmic scales*. That is to say, the instrument maker has constructed the space 1 to 2, by any convenient scale of equal parts, =301 units, because the logarithm of 2 is ·301. Similarly the space from 1 to 3 is made equal to 477 units. The space from 1 to 6 is 778 units for the same reason.

By retreating the slide 301 units to the left in the manner shown we add to the whole scale of the rule an amount= log 2, and consequently represent graphically the multiplication of every figure on that scale by 2. The 3 of the rule coincides with the 6 of the slide because on the one 477 units are added to 301 units on the other, making 778 units, which is the log 6.

The intermediate graduation is made in the same manner, each line being ruled off in the instrument maker's dividing lathe at a distance equal to its tabular logarithm.

The upper scale of the rule is doubled, so that if we give to the left hand 1 the value of 1, the middle 1 will be 10 and the right hand 1 will be 100.

The lower scale is only single in the first place because with less range it has a more open graduation so that by it some figuring can be done twice as closely as by the upper. In the second place it will be observed that all its figures are under their doubles on the upper scale. As twice the log means the square of the number all the lower figures are the square roots of the upper and we can perform involution and evolution without moving the slide at all.

Where the lower figures are given their indicated value

and are under the right hand half of the upper scale, the figures of the latter must be given ten times their indicated value; thus 6 is under 3·6, which must be styled 36 and so on.

The scales of sines and tangents are constructed from the tabulated logarithmic sines and tangents; but with a radius of 100, of which the logarithm is 2. Tabular values are always figured to a radius whose logarithm is 10, so that the integral number 8 is deducted from all the tabular values. The process of calculation is the same as with numbers, only the sines and tangents cannot be read lower than 34′ 23″ nor the tangents above 45°. Smaller angles and higher tangents can be obtained in another way as explained at p. 247, Ex. 4, and p. 274.

The use of the brass marker or index is both to retain a number found by one process whilst the slide is being shifted to make a second calculation and also to perform involution and evolution without using the slide.

THE STATION-POINTER

This instrument is a treasure to hydrographers. By it they locate their position at sea from three fixed points on shore, very rapidly and exactly. AA, BB, and CC are three arms having a common centre O round which two of them can be moved in any direction; DE is a circle graduated from zero to 180° on each side of the central fixed arm which is permanently set to zero. The two other arms are provided with verniers, and when angles are taken with the sextant to the three fixed points forming two angles, one on each side of a central point, these two angles are laid off on each side of zero on the station pointer, which is then laid down on the chart so that each of the three arms points to one of the fixed points. The centre of the instrument will then be at the position of the point of observation on the chart, which is then pricked off through the pin-hole left for the purpose.

This is the same problem as that described in Route Surveying, p. 48, performed by the plane-table.

The only case when the station-pointer will *not* locate the station from three known points is when the station happens to be in the circumference of a circle described about the three points. If a tracing is made of the three arms it will be found that the centre of the graduated limb can be moved into any position in the circumference, and the three arms will still go through the three points.

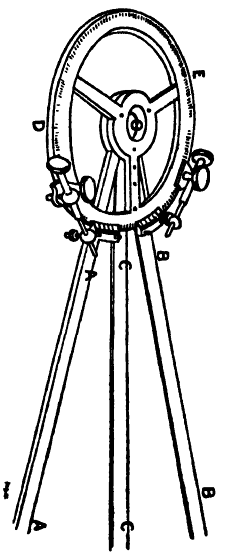


FIG. 116.

It is always practicable to choose points which will not fall into this position.

The same thing can be done by plotting the bearings on tracing paper and superimposing it upon the chart, working them round until all three intersect the points. This is, of course, not so rapid or correct.

BOOKS

The Nautical Almanac is the most indispensable portion of the surveyor's library. The chapter on astronomy together with the glossary will explain all that is needed for its use on survey.

Whitaker's Almanack is a substitute which is quite sufficient for the more ordinary calculations of the geographical position. A couple of the shilling edition should be taken, so that those leaves can be cut out which supply the data for observations, and carried about without taking up any room.

Raper's Navigation is one of the best works on nautical astronomy in the English language. It is bulky, and contains a great many more subjects than the surveyor requires. It has, however, a full supply of mathematical tables, and if the surveyor has not a copy of Chambers's or Weale's mathematical tables this work is the best he can procure.

Chambers's Practical Mathematics is a very suitable book for those who do not intend to go deeply into the subject. It abounds in examples and illustrations, and would doubtless clear up many points that it was impossible in a treatise like the present one to handle thoroughly on the subject of geodetic astronomy.

Chambers's Mathematical Tables are more comprehensive than those of Weale's series. Either will do quite well for the surveyor's purpose.[1]

Dr. Crelle's Calculating Tables are a triumph of German patience. They are a multiplication table in 500 pages of quarto. Three figures by three figures are multiplied by simple inspection. Larger sums are performed by dividing them up into threes. Every extensive survey should be provided with them for office use. Ordinary small surveys need nothing more than the slide-rule. The place of Crelle's tables in the economy of calculation comes between the logarithmic tables and the slide-rule, being quicker than the former for three or four figure calculations and closer than the slide-rule. But even when they are well understood they do not approach the rapidity of the slide-rule for small calculations.

Trautwine's Pocket Book. This wonderful compendium of general engineering knowledge had reached its 27th thousand three years ago. There is no one book which the writer would place beside this as a *vade-mecum* for the engineer who travels abroad either to America or the Colonies.

[1] Dr. Bremiker's Mathematical Tables are for ordinary degrees divided decimally. Ascher and Co., Bedford Street. Price 1*s*. 6*d*.

Molesworth's Pocket Book is likewise a remarkable collection of useful tables and formulæ. It is not so explanatory as Trautwine but less bulky and expensive. Engineers going to India will find in it special information upon engineering as practised in that country, which Sir Guildford Molesworth has been in the best possible position to collect.

Spon's Shilling Pocket Book of Engineering Formulæ contains, along with other most useful information, a table of sines and tangents which are closer than Molesworth's and are sufficient for ordinary curve-ranging.

Hints to Travellers. This handbook of the Royal Geographical Society is as remarkable for the modesty of its title as the value of its contents. It is mainly devoted to geodetic astronomy and route-surveying, but it has chapters on all the subjects of interest and importance to travellers, including geology, natural history, anthropology, &c., &c. It is the cheapest five-shillings' worth that the traveller can put into his outfit. The portion on surveying is by Mr. John Coles, the accomplished Instructor to the Royal Geographical Society and Curator of the Library.

Fieldbooks. The form of fieldbook recommended for tacheometry has been given on p. 178. The printed books can be had through Messrs Elliott Bros. They are marked No. — on the outside, and have an eyelet hole for holding the pencil.

Hold-alls. The best way to preserve pencil and rubber is to keep them and a 10-inch slide-rule in a canvas hold-all, buttoned on to the coat, against which it lies quite flat, and each article when used is replaced in its own proper place. A red and a blue pencil should be added for detail-sketching, or else red-ink and blue-ink fountain-pens.

MS. books. These should contain about 100 pages. Half a dozen should suffice; well bound to resist damp climates.

Stationery and drawing paper. A good supply of ruled profile paper will be found most useful, not only for cross-

sections and profiles, but also for estimates, diagrams, and different kinds of scale-drawings.

Tracing paper is not of much use in a trying climate. Tracing cloth with one dull side is best, as pencilling can be done upon it as easily as on tracing paper. A tough writing paper suitable for foreign postage and for using with the manifold writer will be found the most generally useful. Ink-pellets soluble in water make a fair substitute for ordinary writing fluid, and are no dearer. A gold pen saves its cost over and over again.

The Author's Outfit of Surveying Instruments

The following complete list of a surveying outfit, such as is recommended for an extensive survey, costs from 100*l.* to 120*l.*

'Ideal' tacheometer.—5-inch horizontal and 6-inch vertical limb. Telescope of 12-inch focal length and eyepiece magnifying 50 diameters, having micrometer screw head fitted on right-hand side of eyepiece. Fixed stadia hairs 1/100, and vertical moving hair worked by micrometer screw graduated to 1/10,000 inch. Perforated axis and lantern. Diagonal eyepiece.

Level staff.—One ordinary 3-draw Sopwith 16 feet staff, with 2 vanes fixed 10 feet apart for micrometer readings at long distances, and portable table for same.

Field-books.—One dozen specially printed fieldbooks, arranged for stadia measurements and space for sketching.

One Verner's large size military sketch-board-plane-table and metal tripod-stand, the scales being adapted to railway work.

Six dozen strips of impervious paper for ditto, ruled ¼-inch with co-ordinate lines.

One 100-feet steel chain and 2 sets arrows.

One 100-feet steel tape, divided into feet and hundredths.

One 100-feet linen ditto, divided into feet and tenths.

One bill hook, with long handle.

One small axe.

Three ash ranging rods.

All the above are in two iron-bound deal cases, forming one mule's burden.

One 12-inch Y level.
One Weldon range-finder.
One Abney level, with *prismatic* compass.
Two surveying aneroids, one reading to 5,000 feet, the other to 15,000 feet, fixed altitude scales and verniers.
One keyless semi-chronometer watch.
One pedometer and one passometer.
One large case of drawing instruments as described.
One pocket case of drawing instruments.
'Hints to Travellers.'—Stanford.
Bremiker's Mathematical Tables.
Chambers's Practical Mathematics.
Nautical Almanac.
Whitaker's Almanack (2).
Dr. Crelle's Calculating Tables.
MS. Books (2).
Tracing cloth with dull back, 24 yards by 30 inches.
Profile paper, ruled to scale. Drawing pins, pens, pencils, liquid ink, liquid blue, liquid carmine.

APPENDIX

PLANE TRIGONOMETRY

FUNCTIONS OF RIGHT-ANGLED TRIANGLES

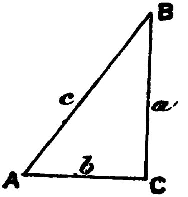


FIG. 117.

$$\sin A = \frac{a}{c} = \frac{\tan A}{\sec A} = \frac{1}{\operatorname{cosec} A} \quad . \quad . \quad . \quad (1)$$

$$\cos A = \frac{b}{c} = \frac{\cot A}{\operatorname{cosec} A} = \frac{1}{\sec A} \quad . \quad . \quad . \quad (2)$$

$$\tan A = \frac{a}{b} = \frac{\sin A}{\cos A} = \frac{1}{\cot A} \quad . \quad . \quad . \quad (3)$$

$$\cot A = \frac{b}{a} = \frac{\cos A}{\sin A} = \frac{1}{\tan A} \quad . \quad . \quad . \quad (4)$$

$$\sec A = \frac{c}{b} = \frac{\operatorname{cosec} A}{\cot A} = \frac{1}{\cos A} \quad . \quad . \quad . \quad (5)$$

$$\operatorname{cosec} A = \frac{c}{a} = \frac{\sec A}{\tan A} = \frac{1}{\sin A} \quad . \quad . \quad . \quad (6)$$

$$\sin^2 A + \cos^2 A = 1 \quad . \quad . \quad . \quad (7)$$

$$\sin A = \sqrt{1 - \cos^2 A} \text{ and } \cos A = \sqrt{1 - \sin^2 A} \quad (8)$$

$$\sec^2 A = 1 + \tan^2 A \text{ and } \operatorname{cosec}^2 A = 1 + \cot^2 A \quad (9)$$

$$\left.\begin{aligned} \sin A \times \operatorname{cosec} A &= \\ \cos A \times \sec A &= \\ \tan A \times \cot A &= \\ \sin^2 A + \cos^2 A &= \end{aligned}\right\} 1 \quad . \quad . \quad . \quad (10)$$

$$\sin (A + B) = \sin A . \cos B + \cos A . \sin B . \quad (11)$$

$$\sin (A - B) = \sin A . \cos B - \cos A . \sin B . \quad (12)$$

$$\cos (A + B) = \cos A . \cos B - \sin A . \sin B . \quad (13)$$

$$\cos (A - B) = \cos A . \cos B + \sin A . \sin B . \quad (14)$$

Functions of any two angles A *and* B.

$$\sin(A+B)+\sin(A-B)=2\sin A.\cos B \quad (15)$$
$$\sin(A+B)-\sin(A-B)=2\cos A.\sin B \quad (16)$$
$$\cos(A-B)+\cos(A+B)=2\cos A.\cos B \quad (17)$$
$$\cos(A-B)-\cos(A+B)=2\sin A.\sin B \quad (18)$$
$$\sin 2A = 2\sin A.\cos A \quad . \quad . \quad . \quad . \quad (19)$$
$$\cos 2A = \cos^2 A - \sin^2 A \quad . \quad . \quad . \quad . \quad (20)$$
$$1+\cos 2A = 2\cos^2 A \quad . \quad . \quad . \quad . \quad (21)$$
$$1-\cos 2A = 2\sin^2 A \quad . \quad . \quad . \quad . \quad (22)$$
$$\sin 3A = 3\sin A - 4\sin^3 A \quad . \quad . \quad . \quad (23)$$
$$\cos 3A = 4\cos^3 A - 3\cos A \quad . \quad . \quad . \quad (24)$$
$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A.\tan B} \quad . \quad . \quad . \quad (25)$$
$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A.\tan B} \quad . \quad . \quad . \quad (26)$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A} \quad . \quad . \quad . \quad . \quad . \quad (27)$$
$$\tan\tfrac{1}{2}A=\frac{\sin A}{1+\cos A};\ \cot\tfrac{1}{2}A=\frac{\sin A}{1-\cos A} \quad (28)$$
$$\tan^2\tfrac{1}{2}A=\frac{\tan\frac{1}{2}A}{\cot\frac{1}{2}A};\ \cot^2\tfrac{1}{2}A=\frac{\cot\frac{1}{2}A}{\tan\frac{1}{2}A} \quad . \quad (29)$$

Relations between the sides and angles of triangles.

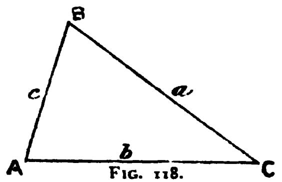


Fig. 118.

$$\cos A=\frac{b^2+c^2-a^2}{2bc} \quad . \quad . \quad . \quad . \quad . \quad (30)$$
$$\cos B=\frac{a^2+c^2-b^2}{2ac} \quad . \quad . \quad . \quad . \quad . \quad (31)$$
$$\cos C=\frac{a^2+b^2-c^2}{2ab} \quad . \quad . \quad . \quad . \quad . \quad (32)$$
$$2\sin^2\tfrac{1}{2}A=\frac{(a+b-c)(a+c-b)}{2bc} \quad . \quad . \quad (33)$$
$$2\cos^2\tfrac{1}{2}A=\frac{(a+b+c)(b+c-a)}{2bc} \quad . \quad . \quad (34)$$

Right-angled triangles.

Case 1. Given the hypotenuse c, and a side b.

$\sec A = R \times c \div b$ . . . . . . (35)

$a = \tan A \times b \div R$ . . . . . . (36)

$C = 90^\circ - A$ . . . . . . (37)

Case 2. Given a side c and one of the oblique angles A.

$C = 90^\circ - A$ . . . . . . (38)

$a = \sin A \times c \div R$ . . . . . . (39)

$b = \cos A \times c \div R$ . . . . . . (40)

Oblique-angled triangles.

Case 1. When two angles and a side opposite are given.

The sides are proportional to the sines of the opposite angles.

Let AB and a be given; then

$C = 180^\circ - (A + B)$ . . . . . . (41)

$b = \dfrac{a.\sin B}{\sin A}$ . . . . (42)

$c = \dfrac{a.\sin C}{\sin A}$ . . . . (43)

Case 2. When two sides and an angle opposite to one of them are given.

Let a, b, and A be given.

$\sin B = \dfrac{b.\sin A}{a}$ . . . . (44)

$\sin C = \dfrac{c.\sin A}{a}$ . . . . (45)

$B = 180^\circ - (A + C)$ . . . . . . (46)

or $C = 180^\circ - (A + B)$ . . . . . . (47)

$b = \dfrac{a.\sin B}{\sin A}$ . . . . (48)

Case 3. Given two sides and the included angle.

Let a, b, and C be given.

$A + B = 180^\circ - C$;

$\tan \tfrac{1}{2}(A - B) = \dfrac{(a - b).\tan \tfrac{1}{2}(A + B)}{a + b}$. (49)

$A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B)$ . . . . (50)

$B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B)$ . . . . (51)

$c^2 = a^2 + b^2 \pm 2\,ab.\cos C$ . . . . (52)

When C is obtuse the + sign, and when acute the − sign to be used.

Case 4. When the three sides are given.

Let a, b, and c be given and $s = \frac{1}{2}(a + b + c)$.

$$\sin^2 \tfrac{1}{2} C = \frac{(s - a)(s - b)}{a.\, b} \quad . \quad . \quad . \quad . \quad (53)$$

SPHERICAL TRIGONOMETRY

Right-angled Spherical Triangles.

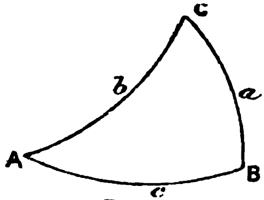


FIG. 119.

Case 1. Given the hypotenuse b and one of the angles C, to find the other parts.

$\tan a = \cos C \times R \div \cot b$ . . . . (54)

$\sin c = \sin b \times \sin C \div R$ . . . . (55)

$\cot A = \cos b \times R \div \cot C$ . . . . (56)

Case 2. Given the hypotenuse b and a side a.

$\cos C = \cot b \times \tan a \div R$ . . . . (57)

$\cos c = \cos b \times R \div \cos a$ . . . . (58)

$\sin A = \sin a \times R \div \sin b$ . . . . (59)

Case 3. Given the two sides a and c.

$\cos b = \cos c \times \cos a \div R$ . . . . (60)

$\cot A = \sin c \times R \div \tan a$ . . . . (61)

$\cot C = \sin a \times R \div \tan c$ . . . . (62)

Case 4. Given the two angles A and C.

$\cos b = \cot A \times \cot C \div R$ . . . . (63)

$\cos c = R \times \cos C \div \sin A$ . . . . (64)

$\cos a = R \times \cos A \div \sin C$ . . . . (65)

Case 5. Given a side a and its adjacent angle C.

$\cot b = R \times \cos C \div \tan a$ . . . . (66)

$\tan c = R \times \sin a \div \cot C$ . . . . (67)

$\cos A = \sin C \times \cos a \div R$ . . . . (68)

Case 6. Given a side c and its opposite angle C.

$\sin b = R \times \sin c \div \sin C$ . . . . (69)

$\sin A = R \times \cos C \div \cos c$ . . . . (70)

$\sin a = \cot C \times \tan c \div R$ . . . . (71)

This last is termed the 'ambiguous' case. See Chambers's 'Practical Mathematics,' p. 379.

R expressed logarithmically is 10·000000.

Oblique-angled Spherical Trigonometry.

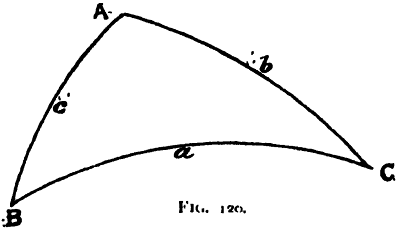


FIG. 120.

Rule 1. Given three sides to find the angles.

Let $s = \frac{1}{2}(a + b + c)$.

$$\text{Then } \sin \tfrac{1}{2} A = \sqrt{\frac{\sin (s - b) \times \sin (s - c)}{\sin b \times \sin c}} \quad . \quad (72)$$

$$\sin \tfrac{1}{2} B = \sqrt{\frac{\sin (s - a) \times \sin (s - c)}{\sin a \times \sin c}} \quad . \quad (73)$$

$$\sin \tfrac{1}{2} C = \sqrt{\frac{\sin (s - a) \times \sin (s - b)}{\sin a \times \sin b}} \quad . \quad (74)$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin s \times \sin (s - a)}{\sin b \times \sin c}} \quad . \quad . \quad (75)$$

and for B and C the formulæ are exactly analogous.

Rule 2. Given two sides and the included angle to find the other parts.

Let a and b be the sides and C the given angle.

$$\sin \tfrac{1}{2}(a + b) : \sin \tfrac{1}{2}(a \sim b) :: \cot \tfrac{1}{2} C : \tan \tfrac{1}{2}(A \sim B);$$

$$\cos \tfrac{1}{2}(a + b) : \cos \tfrac{1}{2}(a \sim b) :: \cot \tfrac{1}{2} C : \tan \tfrac{1}{2}(A + B);$$

$$\text{whence } A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A \sim B) \quad . \quad . \quad . \quad (76)$$

$$B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A \sim B) \quad . \quad . \quad . \quad (77)$$

To find c.

$$\sin \tfrac{1}{2}(A \sim B) : \sin \tfrac{1}{2}(A + B) :: \tan \tfrac{1}{2}(a \sim b) : \tan \tfrac{1}{2} c;$$

$$\therefore \tan \tfrac{1}{2} c = \sin \tfrac{1}{2}(A + B) \times \tan \tfrac{1}{2}(a \sim b) \div \sin \tfrac{1}{2}(A \sim B) \quad (78)$$

In every spherical triangle the sines of the angles are proportional to the sines of the opposite sides.

Rule 3. Given two sides and the angle opposite to one of them.

Let a, b, and A be given, then B is found by

$$\sin a : \sin b :: \sin A : \sin B \qquad (79)$$

and for c

$$\tan \tfrac{1}{2} c : \tan \tfrac{1}{2} (a \sim b) :: \sin \tfrac{1}{2} (A + B) : \sin \tfrac{1}{2} (A \sim B) \qquad (80)$$

and for C

$$\cot \tfrac{1}{2} C : \tan \tfrac{1}{2} (A \sim B) :: \sin \tfrac{1}{2} (a + b) : \sin \tfrac{1}{2} (a \sim b) \qquad (81)$$

TABLE LII.

Values of sin b × sin C *for Years* 1890 *to* 1900 *for Azimuth by* β *and* δ *Draconis in same Vertical.*

log sin Azim = tabular constant − log cos Lat.

1890	9·48568
1891	9·48559
1892	9·48549
1893	9·48540
1894	9·48531
1895	9·48521
1896	9·48512
1897	9·48503
1898	9·48494
1899	9·48484
1900	9·48475

TABLE LIII.

Values of sin b × sin C *for Years* 1890 *to* 1900 *for Azimuth by* β *and* ε *Ursæ Majoris* (*Merak and Alioth*) *when in same Vertical.*

log sin Azim = tabular constant − log cos Lat.

1890	9·72910
1891	9·72917
1892	9·72923
1893	9·72930
1894	9·72936
1895	9·72943
1896	9·72950
1897	9·72956
1898	9·72963
1899	9·72969
1900	9·72976

Table LIV.

Radii corresponding to Decimals of a Degree of Curvature per Chord of 100 feet.

Angle of deflection d	Radius in feet r	Angle of deflection d	Radius in feet r	Angle of deflection d	Radius in feet r	Angle of deflection d	Radius in feet r
0·05	114592	2·60	2204	8·20	699·3	20·00	287·9
0·10	57296	2·70	2122	8·40	682·7	21·00	274·4
0·15	38197	2·80	2046	8·60	666·8	22·00	262·0
0·20	28648	2·90	1976	8·80	651·7	23·00	250·8
0·25	22918	3·00	1910	9·00	637·3	24·00	240·5
0·30	19098	3·10	1848	9·20	623·4	25·00	231·0
0·35	16370	3·20	1791	9·40	610·2	26·00	222·3
0·40	14324	3·30	1736	9·60	597·5	27·00	214·2
0·45	12732	3·40	1685	9·80	585·4	28·00	206·7
0·50	11459	3·50	1637	10·00	573·7	29·00	199·7
0·55	10417	3·60	1592	10·20	562·5	30·00	193·2
0·60	9549	3·70	1549	10·40	551·7	31·00	187·1
0·65	8815	3·80	1508	10·60	541·3	32·00	181·4
0·70	8185	3·90	1469	10·80	531·3	33·00	176·0
0·75	7639	4·00	1433	11·00	521·7	34·00	171·0
0·80	7162	4·20	1364	11·20	512·4	35·00	166·3
0·85	6741	4·40	1302	11·40	503·4	36·00	161·8
0·90	6366	4·60	1246	11·60	494·8	37·00	157·6
0·95	6031	4·80	1194	11·80	486·4	38·00	153·6
1·00	5730	5·00	1146	12·00	478·3	39·00	149·8
1·10	5209	5·20	1102	12·50	459·3	40·00	146·2
1·20	4775	5·40	1061	13·00	441·7	42·00	139·52
1·30	4407	5·60	1024	13·50	425·4	44·00	133·47
1·40	4093	5·80	988·3	14·00	410·3	46·00	127·97
1·50	3820	6·00	955·4	14·50	396·2	48·00	122·93
1·60	3581	6·20	924·6	15·00	383·1	50·00	118·31
1·70	3370	6·40	895·7	15·50	370·8	55·00	108·28
1·80	3183	6·60	868·6	16·00	359·3	60·00	100·00
1·90	3016	6·80	843·1	16·50	348·5	65·00	93·06
2·00	2865	7·00	819·0	17·00	338·3	70·00	87·17
2·10	2729	7·20	796·3	17·50	328·7	75·00	82·13
2·20	2605	7·40	774·8	18·00	319·6	80·00	77·78
2·30	2491	7·60	754·4	18·50	311·1	85·00	74·01
2·40	2387	7·80	735·1	19·00	302·9	90·00	70·71
2·50	2292	8·00	716·8	19·50	295·3		

TABLE LV.

General Elements of the Decimal Spiral.

No. of point n	Curvature in one chord $n.c.$	Total curvature up to n s	Total curvature up to mid-chord or deflection of short chord from main tangent k	Tangential angle, or angle between long chord and main tangent i
	degrees	degrees	degrees	degrees
1	0·2	0·2	0·1	0·1
2	0·4	0·6	0·4	0·25
3	0·6	1·2	0·9	0·47
4	0·8	2·0	1·6	0·75
5	1·0	3·0	2·5	1·10
6	1·2	4·2	3·6	1·52
7	1·4	5·6	4·9	2·00
8	1·6	7·2	6·4	2·55
9	1·8	9·0	8·1	3·17
10	2·0	11·0	10·0	3·85
11	2·2	13·2	12·1	4·60
12	2·4	15·6	14·4	5·41
13	2·6	18·2	16·9	6·30
14	2·8	21·0	19·6	7·24
15	3·0	24·0	22·5	8·26
16	3·2	27·2	25·6	9·33
17	3·4	30·6	28·9	10·48
18	3·6	34·2	32·4	11·68
19	3·8	38·0	36·1	12·95
20	4·0	42·0	40·0	14·29
21	4·2	46·2	44·1	15·69

TABLE LVI.

Elements of No. 2 Spiral for Tramways.

n	r	x	y	L
	feet	feet	feet	feet
1	572·96	·003	2·00	2
2	286·48	·017	4·00	4
3	190·99	·049	6·00	6
4	143·24	·105	8·00	8
5	114·59	·192	10·00	10
6	95·49	·318	11·99	12
7	81·85	·488	13·98	14
8	71·62	·711	15·97	16
9	63·66	·993	17·95	18

TABLE LVI.—*Continued.*

Elements of No. 2 Spiral for Tramways.

n	*r*	*x*	*y*	*L*
	feet	feet	feet	feet
10	57·30	1·34	19·92	20
11	52·09	1·76	21·88	22
12	47·75	2·26	23·82	24
13	44·08	2·84	25·73	26
14	40·93	3·51	27·61	28
15	38·20	4·27	29·46	30
16	35·81	5·14	31·26	32
17	33·71	6·10	33·02	34
18	31·84	7·18	34·70	36
19	30·16	8·35	36·32	38
20	28·65	9·64	37·85	40
21	27·29	11·03	39·29	42

TABLE LVII.

Elements of No. 5 Spiral for Tramways and Light Railways.

n	*d s*	*r*	*x*	*y*	*L*
	degrees	feet	feet	feet	feet
1	4·00	1432·4	0·009	5·00	5
2	8·01	716·2	0·044	10·00	10
3	12·02	477·5	0·122	15·00	15
4	16·05	358·1	0·262	20·00	20
5	20·10	286·5	0·480	24·99	25
6	24·18	238·7	0·794	29·98	30
7	28·29	204·6	1·22	34·96	35
8	32·43	179·0	1·78	39·93	40
9	36·62	159·16	2·48	44·88	45
10	40·86	143·24	3·35	49·81	50
11	45·16	130·22	4·40	54·70	55
12	49·52	119·40	5·64	59·54	60
13	53·97	110·20	7·10	64·32	65
14	58·50	102·31	8·77	69·03	70
15	63·14	95·54	10·69	73·65	75
16	67·89	89·53	12·85	78·16	80
17	72·79	84·25	15·26	82·54	85
18	77·84	79·58	17·94	86·76	90
19	83·07	75·40	20·89	90·80	95
20	88·53	71·63	24·10	94·63	100
21	94·26	68·21	27·58	98·22	105

Table LVIII.

Elements of No. 10 Spiral for Light Railways.

n	d s	r	x	y	L
	degrees	feet	feet	feet	feet
1	2·000	2,865·0	0·017	10·00	10
2	4·001	1,432·0	0·087	20·00	20
3	6·003	954·9	0·244	30·00	30
4	8·006	716·2	0·523	39·99	40
5	10·012	573·0	0·960	49·98	50
6	12·022	477·5	1·588	59·96	60
7	14·034	409·3	2·442	69·93	70
8	16·052	358·1	3·556	79·87	80
9	18·074	318·3	4·96	89·77	90
10	20·102	286·5	6·70	99·61	100
11	22·136	260·4	8·80	109·39	110
12	24·197	238·6	11·28	119·08	120
13	26·226	220·4	14·19	128·65	130
14	28·283	204·6	17·55	138·07	140
15	30·350	191·0	21·37	147·31	150
16	32·427	179·1	25·69	156·32	160
17	34·514	168·5	30·53	165·08	170
18	36·614	159·2	35·88	173·53	180
19	38·726	150·8	41·78	181·61	190
20	40·852	143·3	48·20	189·27	200
21	42·992	136·4	55·16	196·45	210

Table LIX.

Elements of No. 15 Spiral for Light Railways.

n	d s	r	x	y	L
	degrees	feet	feet	feet	feet
1	1·33	4297·1	·02	15·00	15
2	2·67	2148·6	·13	30·00	30
3	4·00	1432·4	·37	45·00	45
4	5·33	1074·3	·78	59·99	60
5	6·67	859·6	1·44	74·98	75
6	8·01	716·2	2·38	89·95	90
7	9·34	613·9	3·66	104·89	105
8	10·68	537·2	5·33	119·80	120
9	12·02	477·5	7·45	134·65	135
10	13·36	429·7	10·05	149·42	150
11	14·71	390·7	13·20	164·09	165
12	16·05	358·1	16·93	178·62	180

TABLE LIX.—*Continued.*

Elements of No. 15 Spiral for Light Railways.

n	ds	r	x	y	L
	degrees	feet	feet	feet	feet
13	17·40	330·6	21·29	192·97	195
14	18·75	307·0	26·32	207·10	210
15	20·10	286·5	32·06	220·96	225
16	21·45	268·6	38·54	234·49	240
17	22·81	252·8	45·79	247·62	255
18	24·18	238·8	53·83	260·29	270
19	25·54	226·2	62·66	272·41	285
20	26·91	214·9	72·31	283·90	300
21	28·28	204·7	82·74	294·67	315

TABLE LX.

Elements of No. 25 Spiral for Narrow-Gauge Railways.

n	ds	r	x	y	L
	degrees	feet	feet	feet	feet
1	0·80	7161·9	0·043	25·00	25
2	1·60	3580·9	0·218	50·00	50
3	2·40	2387·3	0·610	75·00	75
4	3·20	1790·5	1·31	99·99	100
5	4·00	1432·4	2·40	124·96	125
6	4·80	1193·6	3·97	149·91	150
7	5·60	1023·1	6·10	174·82	175
8	6·40	895·2	8·89	199·66	200
9	7·20	795·8	12·41	224·41	225
10	8·01	716·2	16·75	249·03	250
11	8·81	651·1	21·99	273·48	275
12	9·61	596·9	28·21	297·69	300
13	10·41	551·0	35·48	321·61	325
14	11·22	511·6	43·86	345·16	350
15	12·02	477·5	53·43	368·26	375
16	12·83	447·7	64·23	390·81	400
17	13·63	421·4	76·31	412·70	425
18	14·44	397·9	89·71	433·81	450
19	15·24	377·0	104·44	454·01	475
20	16·05	358·2	120·51	473·16	500
21	16·85	341·1	137·90	491·11	525

TABLE LXI.

Elements of No. 50 Spiral for Narrow-Gauge Railways.

n	$d\,s$	x	y	L
	degrees	feet	feet	feet
1	0·4	0·087	50·00	50
2	0·8	0·436	100·00	100
3	1·2	1·221	149·99	150
4	1·6	2·62	199·97	200
5	2·0	4·80	249·92	250
6	2·4	7·94	299·82	300
7	2·8	12·21	349·64	350
8	3·2	17·78	399·33	400
9	3·6	24·83	448·83	450
10	4·0	33·51	498·07	500
11	4·4	43·99	546·96	550
12	4·8	56·42	595·39	600
13	5·2	70·96	643·23	650
14	5·6	87·73	690·33	700
15	6·0	106·86	736·53	750
16	6·4	128·47	781·62	800
17	6·8	152·63	825·40	850
18	7·2	179·42	867·61	900
19	7·6	208·88	908·01	950
20	8·0	241·02	946·32	1000
21	8·41	275·81	982·22	1050

TABLE LXII.

Elements of No. 75 Spiral for Trunk Lines.

n	$d\,s$	x	y	L
	degrees	feet	feet	feet
1	0·27	0·13	75·00	75
2	0·53	0·65	149·99	150
3	0·80	1·83	224·98	225
4	1·07	3·93	299·95	300
5	1·33	7·20	374·88	375
6	1·60	11·92	449·73	450
7	1·87	18·31	524·46	525
8	2·13	26·67	598·99	600
9	2·39	37·24	673·24	675
10	2·67	50·26	748·16	750

TABLE LXII.—*Continued.*

Elements of No. 75 Spiral for Trunk Lines.

n	*d s*	*x*	*y*	*L*
11	2·93	65·99	820·44	825
12	3·19	84·64	893·08	900
13	3·46	106·44	964·84	975
14	3·72	131·60	1035·49	1050
15	4·00	160·30	1104·79	1125
16	4·26	192·70	1172·43	1200
17	4·52	228·94	1238·08	1275
18	4·79	269·13	1301·41	1350
19	5·05	313·32	1362·01	1425
20	5·33	361·53	1419·47	1500
21	5·60	413·72	1473·32	1575

TABLE LXIII.

Elements of No. 100 Spiral for Trunk Lines.

n	*d s*	*C*	*x*	*y*	*L*
	degrees	feet	feet	feet	feet
1	0·2	100·00	0·17	100·00	100
2	0·4	199·99	0·87	199·99	200
3	0·6	299·98	2·44	299·98	300
4	0·8	399·97	5·23	399·94	400
5	1·0	499·94	9·60	499·85	500
6	1·2	599·86	15·88	599·65	600
7	1·4	699·70	24·42	699·29	700
8	1·6	799·45	35·56	798·66	800
9	1·8	899·04	49·65	897·66	900
10	2·0	998·39	67·02	996·14	1000
11	2·2	1097·44	87·98	1093·92	1100
12	2·4	1196·11	112·85	1190·78	1200
13	2·6	1295·71	141·92	1286·46	1300
14	2·8	1391·77	175·47	1380·67	1400
15	3·0	1488·52	213·73	1473·06	1500
16	3·2	1584·15	256·94	1563·24	1600
17	3·4	1678·85	305·27	1650·79	1700
18	3·6	1772·36	358·85	1735·22	1800
19	3·8	1863·42	417·77	1816·02	1900
20	4·0	1948·59	482·05	1892·63	2000
21	4·2	2040·48	551·64	1964·44	2100

TABLE LXIV.

For Ranging the Spiral from an Intermediate point.

The values of n, k, and i are common to all spirals: x and y are for No. 100 spiral, but for other spirals x and y are obtained by simple percentage. Thus for No. 1, spiral take 15 per cent.

Instrument at 1

n	k	x	y	i
	deg.	feet	feet	deg.
1	0	0	0	0
2	0·2	0·35	100·00	0·20
3	0·7	1·57	199·99	0.45
4	1·4	4·01	299·96	0·76
5	2·3	8·03	399·88	1·15
6	3·4	13·96	499·70	1·60
7	4·7	22·15	599·37	2·12
8	6·2	32·95	698·78	2·70
9	7·9	46·70	797·83	3·35
10	8·8	61·99	896·66	4·05
11	11·9	82·61	994·51	4·75
12	14·2	107·14	1091·45	5·61
13	16·7	135·88	1187·23	6·53
14	19·4	169·10	1281·56	7·52
15	22·3	207·04	1374·08	8·57
16	25·4	249·94	1464·41	9·69
17	28·7	297·96	1552·13	10·87
18	32·2	351·25	1636·75	12·11
19	35·9	409·88	1717·75	13·42
20	39·8	473·89	1794·58	14·79
21	43·9	543·23	1866·63	16·23

Instrument at 2

n	k	x	y	i
	deg.	feet	feet	deg.
2	0	0	0	0
3	0·3	0·52	100·00	0·30
4	1·0	2·27	199·98	0·65
5	1·9	5·58	299·93	1·07
6	3·0	10·82	399·79	1·55
7	4·3	18·32	499·51	2·10
8	5·8	28·42	599·00	2·72
9	7·5	41·47	698·14	3·40
10	9·4	57·81	796·80	4·15
11	11·5	77·74	894·79	4·97
12	13·8	101·60	991·90	5·85
13	16·3	129·66	1087·89	6·80
14	19·0	162·22	1182·44	7·81
15	21·9	199·52	1275·22	8·89
16	25·0	241·78	1365·85	10·04
17	28·3	289·10	1453·90	11·25
18	31·8	341·88	1538·89	12·55
19	35·5	399·96	1620·30	13·87
20	39·4	463·43	1697·57	15·27
21	43·5	532·26	1770·11	16·74

Instrument at 3

n	k	x	y	i
3	0	0	0	0
4	0·4	0·70	100·00	0·40
5	1·3	2·97	199·97	0·85
6	2·4	7·15	299·88	1·37
7	3·7	13·61	399·67	1·95
8	5·2	22·67	499·26	2·60
9	6·9	34·68	598·54	3·32
10	8·8	49·98	697·36	4·10
11	10·9	68·89	795·56	4·95
12	13·2	91·73	892·92	5·86
13	15·7	118·79	989·18	6·85
14	18·4	150·35	1084·07	7·90
15	21·3	186·68	1177·24	9·01
16	24·4	227·99	1268·31	10·19
17	27·7	274·47	1356·85	11·44
18	31·2	326·27	1442·38	12·75
19	34·9	383·49	1524·40	14·12
20	38·8	446·15	1602·33	15·56
21	42·9	514·22	1675·59	17·06

Instrument at 4

n	k	x	y	i
4	0	0	0	0
5	0·5	0·87	100·00	0·50
6	1·0	3·66	199·96	1·05
7	2·9	8·72	299·83	1·67
8	4·4	16·40	399·53	2·35
9	6·1	27·02	498·97	3·10
10	8·0	40·94	597·99	3·92
11	10·1	58·48	696·44	4·80
12	12·4	79·95	794·11	5·75
13	14·9	105·66	890·75	6·76
14	17·6	135·90	986·07	7·85
15	20·5	170·92	1079·74	9·00
16	23·6	210·96	1171·37	10·21
17	26·9	256·20	1260·55	11·49
18	30·4	306·80	1346·80	12·83
19	34·1	362·87	1429·61	14·24
20	38·0	424·43	1508·41	15·71
21	42·1	491·47	1582·61	17·25

TABLE LXIV.—*Continued.*

For Ranging the Spiral from an Intermediate point.

Instrument at 5

n	k	x	y	i
	deg.	feet	feet	deg.
5	0	0	0	0
6	0·6	1·05	99·99	0·60
7	1·9	4·36	199·94	1·25
8	3·4	10·29	299·76	1·97
9	5·1	19·18	399·37	2·75
10	7·0	31·37	498·62	3·60
11	9·1	47·18	597·36	4·52
12	11·4	66·95	695·39	5·50
13	13·9	90·97	792·46	6·55
14	16·6	119·54	888·29	7·66
15	19·5	152·92	982·56	8·85
16	22·6	191·35	1074·88	10·09
17	25·9	235·03	1164·83	11·41
18	29·4	284·12	1251·96	12·79
19	33·1	338·73	1335·73	14·23
20	37·0	398·91	1415·59	15·74
21	41·1	464·65	1490·95	17·31

Instrument at 6

n	k	x	y	i
	deg.	feet	feet	deg.
6	0	0	0	0
7	0·7	1·22	99·99	0·70
8	2·2	5·06	199·92	1·45
9	3·9	11·86	299·69	2·27
10	5·8	21·97	399·17	3·15
11	7·9	35·71	498·23	4·10
12	10·2	53·42	596·64	5·12
13	12·7	75·40	694·20	6·20
14	15·4	101·96	790·61	7·35
15	18·3	133·36	885·55	8·57
16	21·4	169·85	978·66	9·86
17	24·7	211·63	1069·51	11·20
18	28·2	258·89	1157·64	12·61
19	31·9	311·73	1242·53	14·08
20	35·8	370·23	1323·64	15·63
21	39·9	434·37	1400·36	17·23

Instrument at 7

n	k	x	y	i
7	0	0	0	0
8	0·8	1·40	99·99	0·80
9	2·5	5·76	199·89	1·65
10	4·4	13·43	299·60	2·57
11	6·5	24·75	398·96	3·55
12	8·8	40·05	497·78	4·61
13	11·3	59·64	595·84	5·72
14	14·0	83·84	692·87	6·90
15	16·9	112·91	788·55	8·15
16	20·0	147·11	882·52	9·46
17	23·3	186·66	974·37	10·84
18	26·8	231·75	1063·62	12·28
19	30·5	282·50	1149·79	13·80
20	34·4	339·00	1232·30	15·38
21	38·5	401·25	1310·56	17·02

Instrument at 8

n	k	x	y	i
8	0	0	0	0
9	0·9	1·57	99·99	0·90
10	2·8	6·46	199·87	1·85
11	4·9	15·00	299·50	2·87
12	7·2	27·53	398·71	3·95
13	9·7	44·38	497·28	5·10
14	12·4	65·85	594·95	6·32
15	15·3	92·24	691·41	7·60
16	18·4	123·80	786·29	8·95
17	21·7	160·78	879·21	10·36
18	25·2	203·36	969·69	11·84
19	28·9	251·69	1057·24	13·39
20	32·8	305·86	1141·29	15·00
21	36·9	365·90	1221·26	16·68

Instrument at 9

n	k	x	y	i
9	0	0	0	0
10	1·0	1·74	99·98	1·0
11	3·1	7·15	199·84	2·05
12	5·4	16·56	299·39	3·17
13	7·9	30·31	398·44	4·35
14	10·6	48·70	496·74	5·60
15	13·5	72·05	593·98	6·92
16	16·6	100·06	689·81	8·30
17	19·9	134·65	783·84	9·75
18	23·4	174·37	875·61	11·26
19	27·1	219·92	964·63	12·84
20	31·0	271·43	1050·35	14·49
21	35·1	328·93	1132·16	16·20

Instrument at 10

n	k	x	y	i
10	0	0	0	0
11	1·1	1·92	99·98	1·10
12	3·4	7·85	199·80	2·25
13	5·9	18·13	299·27	3·47
14	8·6	33·08	398·15	4·75
15	11·5	53·02	496·14	6·10
16	14·6	78·23	592·91	7·52
17	17·9	108·96	688·07	9·00
18	21·4	145·45	781·18	10·55
19	25·1	187·87	871·74	12·16
20	29·0	236·35	959·20	13·84
21	33·1	290·96	1042·97	15·59

Table LXIV.—*Continued.*

For Ranging the Spiral from an Intermediate point.

Instrument at 11					Instrument at 12				
n	k	x	y	i	n	k	x	y	i
	deg.	feet	feet	deg.		deg.	feet	feet	deg.
11	0	0	0	0	12	0	0	0	0
12	1·2	2·09	99·98	1·20	13	1·3	2·27	99·97	1·30
13	3·7	8·56	199·77	2·45	14	4·0	9·24	199·73	2·65
14	6·4	19·69	299·15	3·77	15	6·9	21·26	299·01	4·07
15	9·3	35·85	397·83	5·15	16	10·0	38·62	397·49	5·55
16	12·4	57·33	495·50	6·60	17	13·3	61·63	494·80	7·10
17	15·7	84·39	591·77	8·12	18	16·8	90·53	590·54	8·72
18	19·2	117·27	686·21	9·70	19	20·5	125·55	684·20	10·40
19	22·9	156·19	778·32	11·35	20	24·4	166·86	775·27	12·15
20	26·8	201·27	867·58	13·06	21	28·5	214·58	863·15	13·96
21	30·9	252·63	953·39	14·84					
Instrument at 13					Instrument at 14				
13	0	0	0	0	14	0	0	0	0
14	1·4	2·44	99·97	1·40	15	1·5	2·62	99·97	1·50
15	4·3	9·94	199·69	2·85	16	4·6	10·64	199·64	3·05
16	7·4	22·82	298·86	4·37	17	7·9	24·38	298·69	4·67
17	10·7	41·39	397·12	5·95	18	11·4	44·15	396·72	6·35
18	14·2	65·92	494·06	7·60	19	15·1	70·20	493·27	8·10
19	17·9	96·65	589·22	9·32	20	19·0	102·75	587·82	9·92
20	21·8	133·79	682·07	11·10	21	23·1	141·99	679·80	11·80
21	25·9	177·47	772·02	12·95					
Instrument at 15					Instrument at 16				
15	0	0	0	0	16	0	0	0	0
16	1·6	2·79	99·96	1·60	17	1·7	2·97	99·96	1·70
17	4·9	11·33	199·59	3·25	18	5·2	12·03	199·54	3·45
18	8·4	25·94	298·52	4·97	19	8·9	27·50	298·34	5·27
19	12·1	46·90	396·30	6·75	20	12·8	49·66	395·85	7·15
20	16·0	74·47	492·43	8·60	21	16·9	78·73	491·54	9·10
21	20·1	108·83	586·34	10·52					
Instrument at 17					Instrument at 18				
17	0	0	0	0	18	0	0	0	0
18	1·8	3·14	99·95	1·80	19	1·9	3·31	99·94	1·90
19	5·5	12·73	199·49	3·65	20	5·8	13·42	199·43	3·85
20	9·4	29·06	298·15	5·57	21	9·9	30·61	297·94	5·87
21	13·5	52·40	395·38	7·55					
Instrument at 19					Instrument at 20				
19	0	0	0	0	20	0	0	0	0
20	2·0	3·49	99·94	2·00	21	2·1	3·66	99·93	2·10
21	6·1	14·12	199·37	4·05					

TABLE LXV.—*Specific Gravity of Stones, Earths, &c. Water taken at* 62·3 lbs. per c. ft.

	G		G
Basalt	2·75 to 2·95	Glass	2·0 to 3·00
Bricks and brickwork	1·60 to 2·00	Gneiss and granite	average 2·65
Cement (American)	0·80 to 0·90	Limestone and marble	,, 2·65
,, (Portland)	1·35 to 1·45	Lime	,, 1·50
Concrete in lime	average 1·9	Masonry of dressed ashlar same G as the stone	
,, Portland cement	,, 2·2	Masonry of rough rubble	1·80 to 2·20
Coal, Newcastle	,, 1·25	Mortar	1·40 to 1·90
,, anthracite	,, 1·50	Mud	1·25 to 1·75
,, in bulk for stowage, 48 cubic feet per ton		Peat	average 0·40
Coke	,, 0·75	Pitch	,, 1·15
,, in bulk for stowage 80 to 100 cubic feet per ton		Sand	1·5 to 1·95
		Sandstones	2·10 to 2·50
		Sulphur	average 2·00
Chalk	,, 2·50	Tallow	,, 0·90
Earth	1·50 to 2·00	Tar	,, 1·00
Flint	average 2·60	Traprock	2·80 to 3·00

TABLE LXVI.—*Metals and Alloys.*

Aluminium	average 2·60	Iron (cast)	average 7·23
Antimony	,, 6·70	,, (wrought)	,, 7·78
Babbett or white metal	,, 7·30	Lead	,, 11·40
Bismuth	,, 9·80	Mercury	,, 13·60
Brass	,, 8·40	Platinum	21·50 to 23·00
Copper	,, 8·80	Silver	average 10·50
Gold	,, 19·00	Steel	7·75 to 8·00
Gun metal	,, 8·50	Tin	average 7·30
		Zinc	,, 7·00

TABLE LXVII.—*Timber.*

Acacia	0·70 to 0·80	Cork	average 0·24
Ash	0·70 to 0·76	Ebony	,, 1·19
Beech	0·70 to 0·80	Elm (English)	,, 0·56
Box	average 1·25	,, (American)	,, 0·72
Cedar (Lebanon)	,, 0·49	Fir	,, 0·51
,, (American)	,, 0·55	Hornbeam	,, 0·76
,, (West Indies)	,, 0·70	Ironwood	,, 1·15
Chestnut	,, 0·61	Larch	,, 0·55

TABLE LXVII.—*Timber.* (*Continued.*)

Lignum vitæ . .	average 1·33	Pine (white) . .	average 0·40
Mahogany (Honduras) . . .	,, 0·56	,, (yellow) . .	,, 0·55
Mahogany (Spanish)	,, 0·85	,, (red) . . .	0·57 to 0·65
Maple	,, 0·67	,, heart of long-leafed southern yellow	1·04
Oak (American) .	,, 0·80	Teak	0·74 to 0·86
,, (English) . .	0·78 to 0·93		

TABLE LXVIII.—*Liquids.*

Acetic acid	1·06	Sulphuric acid	1·84
Alcohol	0·80	Water, distilled at 62° Fahr., bar. 30 in., 62·355 lbs. per cubic foot	1·00
Ether	0·70	Water, at 212° Fahr.	0·957
Hydrochloric acid . . .	1·20	Sea	1·026 to 1·030
Nitric acid	1·22		
Oil (linseed)	0·94		
,, (olive)	0·92		
,, (petroleum)	0·88		
,, (whale)	0·92		

TABLE LXIX.—*Multipliers for reducing Specific Gravity to Weight of certain Volumes, Water taken at* 62·3 lbs.

Weight of 1 cubic centimetre in grammes .	=	G
,, 1 kilolitre or cubic metre in tonnes of 1,000 kilogrammes . . .	=	G
Weight of 1 decalitre in kilogrammes .	=	$G \times 10$
,, 1 cubic inch in lbs. . . .	=	$G \times 0{\cdot}036$
,, 1 cubic foot in lbs. . . .	=	$G \times 62{\cdot}30$
,, 1 cubic yard in tons . . .	=	$G \times 0{\cdot}751$
,, 1 Brit. imp. gallon in lbs. .	=	$G \times 10$
,, 1 Brit. imp. bushel in lbs. .	=	$G \times 80$
,, 1 U. S. liquid gal. in lbs. . .	=	$G \times 8{\cdot}322$
,, 1 U. S. struck bushel in lbs. .	=	$G \times 77{\cdot}467$
No. of cubic yards in one ton . . .	=	$\frac{1{\cdot}332}{G}$
No. of cubic feet in one ton	=	$\frac{35{\cdot}955}{G}$
No. of Brit. imp. gals. in one ton . .	=	$\frac{224}{G}$
,, ,, bushels ,, . . .	=	$\frac{28}{G}$
,, U. S. liquid gals. ,, . . .	=	$\frac{268{\cdot}42}{G}$
,, ,, struck bushels in one ton	=	$\frac{28{\cdot}92}{G}$

GLOSSARY

Account, By, a term used for either longitude or latitude when calculated from other data than observations.

Age of the Tide is the interval between the time of new or full moon and the time of the next spring tide, and varies from 1½ to 3 days.

Alioth, a star otherwise called ε Ursæ Majoris, in the constellation of the Great Bear (*see* Fig. 122).

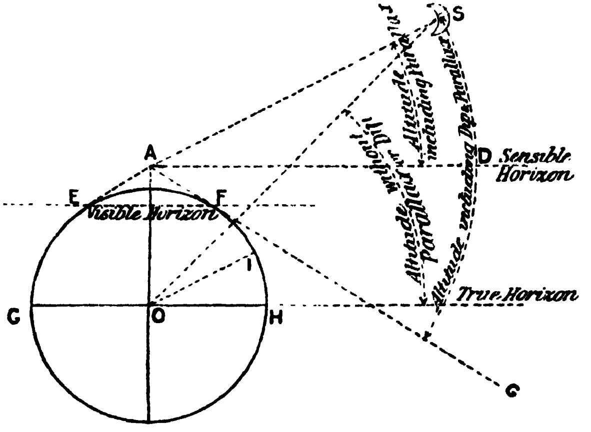


FIG. 121.

Altazimuth, an instrument for measuring at one adjustment of the line of sight the angle of altitude and of azimuth (*see* pp. 320, 322).

Altitude is the angular elevation of a heavenly body, or, in other words, the arc of a great circle passing through a heavenly body

TABLE LXVII.—*Timber.* (*Continued.*)

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Nitric acid	1·22		
Oil (linseed)	0·94		
,, (olive)	0·92		
,, (petroleum)	0·88		
,, (whale)	0·92		

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Weight of 1 cubic centimetre in grammes	=	G
,, 1 kilolitre or cubic metre in tonnes of 1,000 kilogrammes	=	G
Weight of 1 decalitre in kilogrammes	=	G × 10
,, 1 cubic inch in lbs.	=	G × 0·036
,, 1 cubic foot in lbs.	=	G × 62·30
,, 1 cubic yard in tons	=	G × 0·751
,, 1 Brit. imp. gallon in lbs.	=	G × 10
,, 1 Brit. imp. bushel in lbs.	=	G × 80
,, 1 U. S. liquid gal. in lbs.	=	G × 8·322
,, 1 U. S. struck bushel in lbs.	=	G × 77·467
No. of cubic yards in one ton	=	$\frac{1·332}{G'}$
No. of cubic feet in one ton	=	$\frac{35·955}{G}$
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,, ,, bushels ,,	=	$\frac{28}{G}$
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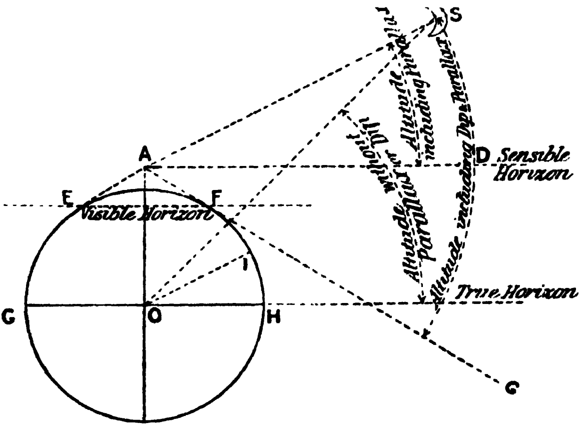

FIG. 121.

Altazimuth, an instrument for measuring at one adjustment of the line of sight the angle of altitude and of azimuth (*see* pp. 320, 322).

Altitude is the angular elevation of a heavenly body, or, in other words, the arc of a great circle passing through a heavenly body

measured from it to the true horizon (*see* Horizon). Let A, Fig. 121, be the observer's eye, S the object in altitude. SOH is the true altitude plus refraction, which has to be deducted (*see* Refraction). IO is drawn parallel to AS. So SOI represents the correction due to parallax in altitude (*see* Parallax). The sensible horizon is drawn for simplicity at the observer's eye, instead of at the earth's surface, because it only involves an inappreciable error of parallax, due to the increased length of radius of the earth at high elevations (*see* Parallax, Equatorial).

Amplitude is the spherical angle at the zenith contained between the plane of the prime vertical at the point of observation and that of the meridian of any celestial body observed on the horizon, rising or setting. It is measured by the arc of the horizon from the east or west point to the body; it is shown by AW in Fig. 133, or as AC in Fig. 127, and is therefore the complement of the azimuth. It is a very useful method of finding the variation of the compass at sea, but is of no use where there is no horizon. When a star is setting its true place is already under the horizon by about half a degree (*see* Refraction) and therefore more to the westward than it appears to be. If we take an amplitude when the star is 34′ above the horizon we shall be pretty near the truth; if we take it when actually setting it will appear too much to the west—that is, to the right of its true place. The greater the southern declination in the northern hemisphere—that is, the flatter the arc of the body's transit across the heavens—the greater the error will be. In the case of bodies with parallax (*see* Parallax) the error due to that cause makes them appear too low. The moon's parallax is nearly double the refraction; so she is still about half a degree above the horizon when she appears to set, and her true place is to the left of her apparent place.

Anallatism (Greek *a* privative, and *alasso*, to alter, unchangeableness). The centre of anallatism is that point in a distance-measuring telescope from which the distance of any object is proportional to the height intercepted upon the staff by two horizontal wires in the diaphragm. In ordinary telescopes it is situated at the anterior focus. In some tacheometers it is made to coincide with the vertical axis by means of an additional lens.

Aneroid (Greek *a* privative, and *neros*, wet), an instrument for measuring the pressure of the atmosphere (*see* p. 342).

Angle (Latin *angulus*, a corner) may be plane or spherical (*see* Spherical Angle). A plane angle is formed by the inclination of two straight lines to one another; it has been reckoned from 0 to 360

degrees all over the world until lately, when the centesimal system of dividing the circle into 400 degrees, and each degree into 100 minutes, has been coming into use for the purpose of simplifying calculations. An angle of 90° is called a right angle, one less than 90° an acute, and one greater than 90° an obtuse angle.

Angle Complement. The complement of an angle is the difference between it and 90°. The cosine is the sine of the complement of an angle, and the cotangent and cosecant are tangent and secant of the same complementary angle.

Angle Supplement, the difference between an angle and 180°.

Aphelion (Greek *apo*, from, and *helios*, sun), that point in the orbit of a comet or planet which is farthest from the sun.

Apogee (Greek *apo*, from, and *ge*, the earth), that point in the moon's orbit which is farthest from the earth.

Apparent (Latin *ad*, to, and *parere*, to appear), that which is opposite to the true or real. The apparent position of a celestial object is that which it appears to have to the observer with an instrument before being corrected for refraction, parallax, &c.

Apparent Time is the hour angle of the sun (*see* Hour Angle) reckoned westward from the meridian. It is the time shown by the sun-dial. The sun's apparent place in the heavens is constantly changing, owing to the earth's orbit, but this being elliptical, the movement is not uniform and is represented in the almanacs by daily changes of right ascension, with rate for one hour. A clock keeping apparent time would have to be altered every day, so the expedient of mean or average time is resorted to (*see* Mean Time). Apparent time is first found from observation and then reduced to mean time by the equation of time (*see* Equation of Time).

Arc of Excess, in sextants that part of the graduated arc behind the zero.

Aries, First Point of (*see* Right Ascension).

Argument (Latin *argumentum*, a thing taken for granted) means any mathematical datum or known quantity from which to determine others.

Ascension (*see* Right Ascension).

Astronomical Time (*see* Civil Time).

Augmentation of the moon's semi-diameter is the increase in angular dimension when in altitude above what it appears on the horizon, owing to its approach towards the observer, until, when in the zenith, it is closer by the amount of the earth's radius. The table is given in Chambers's 'Mathematical Tables.'

Axis (Greek *axon*, an axle), an imaginary line joining the north and

south poles of a celestial body upon which it is supposed to rotate. The imaginary line about which the vertical limb of a theodolite rotates. Is of very wide application (*see* also Optical).

Azimuth (Arabic *samatha*, to go towards) is the spherical angle at the zenith contained between the plane of the meridian of the place and that of the great circle of altitude passing through the object observed (*see* Fig. 133, p. 414). If Z be the zenith and NZS the meridian, Z ☽ A a great circle of altitude, NA is the azimuth. It is measured upon the horizon from the north or south point, whichever is nearest. (*See* also Course and Amplitude, of which the azimuth is the complement.)

Barometer, an instrument for measuring the pressure of the atmosphere (*see* p. 342 &c.).

Binary Stars are double stars which revolve round one another; when the motion appears to be rectilinear they are merely called double stars.

Circumpolar Stars, those whose polar distance is less than the latitude of the place, and which therefore do not set but culminate twice. At the North Pole the whole celestial hemisphere is circumpolar; at the Equator none. *See* Fig. 122.

Civil Time, like astronomical time, is a term having reference more to date than to time. Both are mean time (*see* Mean Time), but civil time begins its day from midnight, and astronomical time from the succeeding noon, so that January 1, 1890, at 6 A.M. by civil time, is December 31, 1889, 18 hours astronomical time. But January 1, 1890, 2 P.M. civil time, is the same date and time astronomically.

Co-altitude. *See* Zenith Distance.

Collimation, *Line of* (Lat. *cum*, with; *limes*, a limit), in telescopes is the axis of a pencil of light reaching the eye through the tube; or, which is the same thing, it is the straight line joining the two foci of the double-convex lens forming the object-glass and the focus of the eye-piece. The line of collimation is defined in levels and theodolites by two intersecting spider hairs or some such device, attached to a brass diaphragm which is placed in the optical axis by adjusting screws.

Colure (Gr. *kolouō*, I cut in the middle), two celestial meridians (*see* Meridian, Celestial) whose planes are at right angles to one another; whose line of intersection is terminated by the poles, and which cut the celestial sphere into quarters. One of these semicircles bisects the equator at the spring and autumn equinoxes, and the other at the summer and winter solstices.

Compass, Solar. *See* p. 328.

Compass, Variation of, is the angle between the astronomical meridian and the direction of the compass needle when at rest under the influence of terrestrial, but undisturbed by local, magnetism. It is

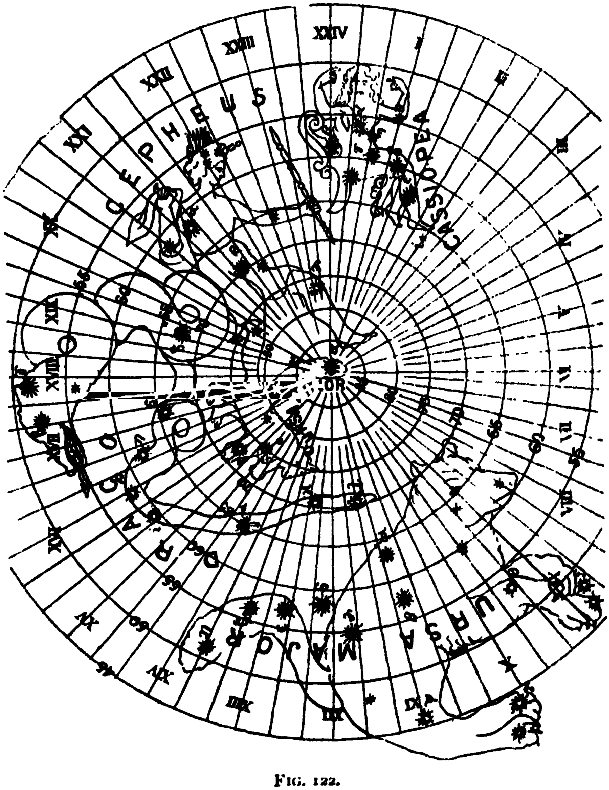

FIG. 122.

subject both to a diurnal oscillation and an annual variation as well as to local disturbances.

Constellation (Lat. *cum*, together and *stella*, a star), a portion of the heavens marked on globes and maps by dotted boundary lines

in which the main feature is a well-defined group of stars supposed to resemble some terrestrial object and accordingly designated.

Contraction of the semi-diameter of sun and moon are sensibly the same in amount, and arise from unequal refraction in the upper and lower limb. The table is given in Chambers's 'Mathematical Tables.'

Co-ordinates, Rectangular, a pair of straight lines locating any point in a plane by measuring its shortest distance from two fiducial lines at right angles to one another. Thus the rectangular co-

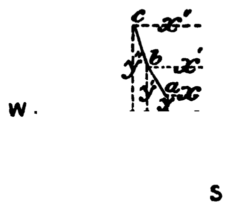

FIG. 123.

ordinates $x\ y$, $x'\ y'$, $x''\ y''$ determine the positions of the points a, b, and c relatively to the lines N.S. and E.W., and if N.S. be the meridian (*see* 'Meridian, Magnetic,' and 'Meridian, Celestial') y, y', and y'' are the latitude; x, x', and x'' the departure (*see* Latitude, Difference of).

Course in navigation and land-surveying is the direction of the line being travelled or measured with reference to the magnetic meridian or true meridian. It is the azimuth of the objective point. It is also called 'bearing' in land-surveying more frequently than course. It is reckoned in each quadrant separately, from the north eastwards and westwards, and from the south eastwards and westwards. This method is suitable to traversing with a large field compass, being ready for reduction to latitude and departure. Another method is to reckon clear round from 0° to 360°. Most countries reckon the 0° from the north point, some from the south. Theodolites are arranged to read from 0° to 360°, as the former method would not be suitable; but when working to latitude and departure the angles must first be reduced to azimuthal form. *See* Table XVII., p. 59.

Conjunction (Lat. *cum*, together; *jungere*, to join). Two bodies are said to be in conjunction when they appear in nearly the same part of the heavens. It is necessary, therefore, that one of them should

have an apparent movement through the heavens, such as the moon, which is in conjunction with a star when it has the same longitude or right ascension.

Culmination (Lat. *culmen*, the top) is the passage of a celestial body across the meridian of a place. In the northern hemisphere the sun and most of the stars used in observation culminate southwards; consequently the term 'to south' is used for southern culmiration. *See also* Circumpolar Stars.

Declination is the distance of a celestial body from the celestial equator measured north or south on the arc of a celestial meridian passing through it. Declination corresponds exactly with terrestrial latitude. In Fig. 127 SD is the north declination of a body S (*see also* Polar Distance). In some almanacs N. and S. declination are marked + and − respectively.

Degree (Lat. *degredior*, to go down: from *de*, down, and *gradus*, a step), a division of the circle which in the sexagesimal is $\frac{1}{360}$ or in the centesimal is $\frac{1}{400}$ of the total circumference.

Departure in a traverse means the easting or westing from a known point which is taken as the origin of rectangular co-ordinates. It is equal to the distance run multiplied by the sine of the angle, azimuth, or course (*see* Course, Azimuth).

Depression or *Dip of the Horizon* is the angle of depression of the apparent horizon, due to elevation of the eye above the level of the sea. If we direct a levelling instrument in good adjustment towards the horizon from the top of a high cliff we shall at once perceive that there is a depression, and with a large-sized transit theodolite we can measure that angle with sufficient accuracy to know the height of the cliff within ten to twenty feet. Depression arises from the curvature of the earth. The values are given for different elevations of the eye in Table L., p. 288. These answer for correcting an altitude taken at sea, or for estimating the elevation of a cliff in the manner just described. The depression is always deducted from the observed altitude. A simple way to keep it in memory is 'Dip makes me see too much, and therefore I deduct it.'

Diameter (Gr. *dia*, through, and *metron*, a measure) in its ordinary use is limited to the circle and the sphere of which it is double the distance from centre to circumference. It also means the breadth of anything.

Diaphragm, in telescopes is an annular brass plate fixed in the focus by adjusting screws and forming both a passage to confine the light and a frame to hold the cross hairs.

Dip of the Horizon. *See* Depression.

Diurnal Inequality of Heights is, in irregular tides, the difference between the height of high water of each successive tide.

Diurnal Inequality of Time is, in irregular tides, the difference between the lunitidal intervals of each successive tide.

Eclipse (Gr. *ekleipsis*, a disappearance), the phenomenon occurring in the heavens from the disappearance of one body in the shadow of another. The following diagram illustrates the theory of the various forms of eclipses of sun and moon, from which it will be seen that an eclipse of the sun (which is more correctly an occultation, *see* Occultation), can only take place when the sun and moon

FIG. 124.

FIG. 125.

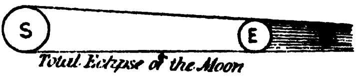

FIG. 126.

are in conjunction, or, in other words, at new moon, and an eclipse of the moon, which is really an eclipse, can only take place at opposition or full moon.

An eclipse of a satellite of Jupiter is due to similar causes. When entering the shadow its immersion is said to take place, and on leaving it is said to emerge. The idea of obtaining the longitude by observing eclipses of Jupiter's satellites originated with Galileo.

Ecliptic, the great circle of the heavens which the sun appears to describe in the year; it derives its name from the fact that eclipses can only take place when the moon is also on the ecliptic. It is commonly called the sun's annual path, to distinguish it from the sun's diurnal path, due to the earth's axial rotation.

Ecliptic, Obliquity of, the inclination of the plane of the ecliptic to that of the celestial equator, producing the phenomena of the seasons. The angle is about 23° 27′, and is very gradually dimi-

nishing. It is measured by the declination at the solstitial points, June 21 and December 21.

Elongation (Lat. *longe*, afar off), the angular distances of the pole star eastward or westward of the true pole; also the similar distances of a planet from the sun or a satellite from its primary.

Equation of Time, the daily correction at mean noon to be added to or deducted from the apparent time ascertained by observation, in order to obtain mean time (*see* Mean Time). It is sometimes expressed as sun 'after clock' or 'before clock,' meaning that the equation is to be added to or deducted from the apparent time. Thus: when the sun is on the meridian it is noon of apparent time; if in the table the equation is marked 'sun after clock' 5 min. 6 sec. it will be then 0 hr. 5 min. 6 sec. astronomical, or 12 hrs. 5 min. 6 sec. civil time. If the table had it 'sun before clock' 11 min. 3 sec., when the sun culminated it would be 11 hrs. 48 min. 57 sec. civil time, and 23 hrs. 48 min. 57 sec. astronomical time of the previous day's date (*see* Civil Time, Astronomical Time; *see also* Sidereal Time).

Equator, Celestial, is the intersection of the plane of the terrestrial equator with the celestial sphere (EQ in Fig. 127).

Equator, Terrestrial, is the great circle of the earth's surface whose plane is midway between the poles and at right angles to the earth's axis. AD in Fig. 129.

Equatorial instrument, a telescope which is made to move, by hand or by clockwork, in the equator, or, in other words, in which the axis of rotation, instead of being set vertical, as in an ordinary transit theodolite, points to the celestial pole.

Equinox. *See* Equinoctial.

Equinoctial (Lat. *æquus*, equal; *nox*, night), another name for the celestial equator (*see* Equator), because when the sun is in it the nights are equal all over the world. It will be obvious in looking at a celestial globe that whatever angle the pole makes with the horizon the equator always intersects with the prime vertical (*see* Prime Vertical) at the horizon; consequently the equator is in every latitude half above and half below the horizon. If an observer could be stationed precisely at the north or south pole, the equator would then be coincident with the horizon, and he would see a half-sun going clear round the horizon at the equinox. The sun is in the equinoctial on March 21 and September 21 (*see* Right Ascension).

Establishment, Vulgar, is the lunitidal interval when the time of moon's meridian passage is 0 hr. 0 min. or 12 hrs. It is termed by Raper the tide-hour, and defined as the apparent time of the first high

water that takes place in the afternoon of the day of full or change. In German *Hafenzeit*, or harbour-time.

Establishment, Mean, is the mean of all the lunitidal intervals in a semi-lunation, and is often less by ten to forty minutes than the vulgar establishment.

Fiducial, any point, line, or arc which is known, fixed, or may be otherwise *relied upon* for locating others, such as the meridian of Greenwich; the datum line of a profile or cross section; an ordnance benchmark, &c. &c.

Focal Length, the distance from the 'centre' of the lens to the focus. The 'centre' of a double-convex lens is that point in the axis which is midway between the two surfaces.

Focus, the common meeting-point of all the converging rays passing through a lens. In the double-convex lens of a telescope the focus inside the tube is termed the focus, that outside the tube is termed the anterior focus. In German *Brennpunkt*, or burning-point.

Geographical Mile, or *Admiralty Knot*, is 1·15152 statute mile. It slightly differs from that of the United States, which is 1·15157 statute mile and adopted by the Coast Survey as being the linear distance in the arc of 1 minute of a great circle of a true sphere whose surface area is equal to that of the earth at sea level. It also equals 1·85324 kilometres.

Gibbous (Lat. *gibbus*, convex-bunched), when the moon is rather more than half but less than full.

Great Circle of a sphere is any circle described about it whose plane passes through its centre, as HZRN or PSDO in Fig. 127.

Great Circle Sailing is sailing between two points on the earth's surface upon the arc of a great circle (*see* Spherical Distance).

Horary Angle. *See* Hour Angle.

Horizon. *The sensible horizon* is a plane parallel to that of the true horizon, but touching the surface of the earth at the point of observation.

The true horizon is the intersection of the celestial sphere by a plane GH (Fig. 121) passing through the centre of the earth and at right angles to a diameter of the earth at the observer's standpoint.

The visible or *apparent horizon* is the intersection of a conical surface, of which the apex is the observer's eye, with the sphere (*see* EF, Fig. 121). The dip of the horizon is equal to the complement of half the angle of the cone.

Hour Angle is the angle at the pole contained between the meridian of the place and the celestial meridian passing through any celestial

body. SPZ, Fig. 127, is the hour angle; it is measured by the arc DE of the celestial equator. The calculation of the hour angle is reduced from arc to time by the proportion of the total time of a revolution to the hour angle; thus, as 360° : 24 hours : : hour angle in arc : hour angle in time. The tables of log. sines, cosines, &c., in Raper give the horary values of all angles, in addition to which tables for converting arc into time and *vice versâ* are given. For formulæ adapted to slide-rule *see* p. 136.

The calculation of the hour angle of the sun is the commonest method of obtaining apparent time at place, from which by equation of time (*see* Equation of Time) and a chronometer registering Greenwich mean time the longitude is easily calculated (*see also* Apparent Time, Longitude). The hour angle of a star is sidereal time, which can be reduced to mean time by rule (p. 410).

Hypsometric, 'height-measuring,' is used for observations with the aneroid or boiling point thermometer to determine the approximate elevation above the sea.

Integral, consisting of entire numbers, as contrasted with fractions.

Kilometre, a distance of one thousand metres (*see* Metre).

Knot. *See* Geographical Mile.

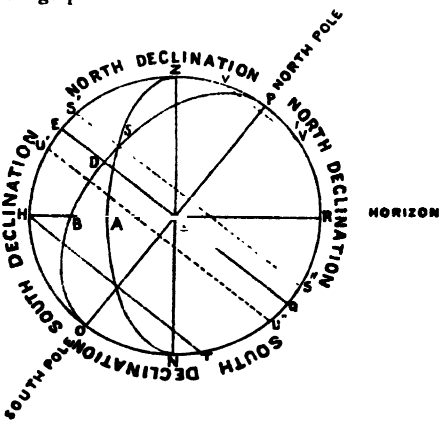

FIG. 127.

Latitude, Difference of, in traversing called latitude for shortness, is the northing or southing of the base line (*see* Co-ordinates and Traverse).

Latitude (*Terrestrial*), the spherical distance (*see* Spherical Distance)

between the equator and the position of the observer, measured north and south upon a meridian of longitude. It is represented by BC, Fig. 129.

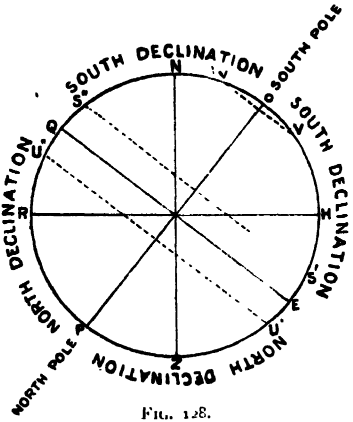

FIG. 128.

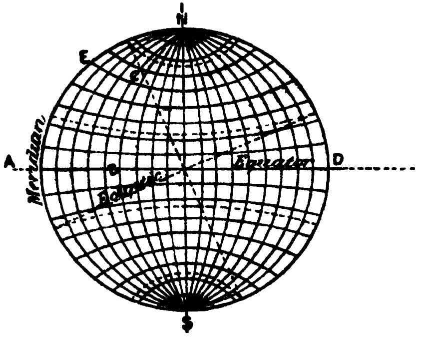

FIG. 129.

Lens, in telescopes a circular glass, truly ground to a surface of that curvature which causes the rays of light passing through it to be refracted at angles which meet in a common centre.

Limb of a celestial body means the extreme edge of the circumference, upon which the observation is taken.

Limb of a theodolite is the vertical or horizontal portion of the instrument (*see* Chapter IX.).

Longitude (*Terrestrial*) is the spherical angle at the pole (*see* Spherical Angle) between the plane of the meridian of Greenwich (*see* Meridian, Terrestrial) and that of the meridian of the place of observation. Thus the angle ANC in Fig. 129 is the longitude of C west of Greenwich when AEND is the meridian of Greenwich, and is measured on any of the parallels of latitude in angular measure. Notice in the figure that though the small circles of latitude give the same arc of a circle as the equator, and consequently the difference of longitude, they do not give the true spherical distance—that is (*see* Spherical Distance), the arc of a great circle which is the flattest circle, and consequently the shortest distance between any two points.

Lunitidal Interval is the time that elapses each day between the transit of the moon over the meridian and high water.

Mean Distance of a planet from the sun is the mean of the perihelion and aphelion distances, which see.

Mean Time. Instead of correcting the watch daily to keep apparent time (*see* Apparent Time) the average length of a solar day throughout the year is calculated and termed a mean solar day. Both civil and astronomical time are kept in this way (*see* Civil Time and Astronomical Time). The corrections by which to obtain it from observed apparent time are given in the almanacs and called equation of time. Mean time is the only possible method of regulating time by the sun and yet keeping it uniform (*see* Standard Time).

Meridian, Celestial. The observer's celestial meridian is the great circle of the celestial sphere passing through the zenith—the pole, the north and south points of the horizon, and the south pole. Its plane is therefore at right angles to that of the prime vertical, and is shown by ZHNR in Fig. 127.

The celestial meridian of a star is a semicircle of the heavens which passes through it.

Meridian (*Distance*). *See* Hour Angle.

Meridian, Magnetic, is the line of direction of the compass needle (*see* Compass) produced by the magnetic polarities of the earth. The lines of equal magnetic variation do not even approximate to great circles of the earth's surface, although the magnetic poles approximate in position to the terrestrial poles.

Meridian, Terrestrial, is a great circle of the earth's surface passing through the north and south poles and the place of observation.

Every place has its meridian, but certain ones are chosen by the several sea-going nations as the basis of their calculations of longitude, such as the meridian of Greenwich, of Paris, of Washington, of Lisbon. A conference was held two years ago with a view to adopting one for the world, but no agreement was come to.

Metre, the French standard measure of length = one ten-millionth of the distance from the pole to the equator as measured by earlier astronomers = 39·37079 British and American inches. It is about ½ inch longer than the seconds pendulum.

Micrometer (Gr. *mikros*, small; *metron*, a measure), an instrument for measuring small angles or distances (*see* p. 317, Chapter IX.).

Mile (*see* Statute, Geographical).

Moon-culminating Stars are certain stars which lie close to the moon's path through the heavens, and from this cause furnish a ready means of obtaining, at their culmination, the difference of local time at any two places, and hence the longitude. The interval between the culminations at Greenwich is obtained from the 'Nautical Almanac' by the difference in right ascension of the moon and star. The interval at place is found by watch, and the difference between the two intervals is proportional to the difference of longitude.

Nadir, the point in the celestial sphere at the opposite extreme to the zenith.

Nautical Mile. *See* Geographical Mile.

Nodes (Lat. *nodus*, a knot) are the points of intersection of a planet's orbit with the sun's path or ecliptic. The 'ascending node' is where it crosses the ecliptic from south to north, and the descending node the opposite point.

Obliquity of the Ecliptic. *See* Ecliptic.

Occultation is the disappearance or hiding of a celestial body by the intervention of another. Thus the stars in the moon's path are occulted by her, and the satellites of a planet by the body of the planet.

Optical Axis in instruments is the line joining the centres of the true spherical surfaces of the lenses.

Orientation, the general direction of a chain of triangles, or the placing of a plane-table or similar instrument so that it will preserve the same line of direction.

Parallax (*see* Fig. 121, where the parallax in altitude is shown as angle SOI. It is the difference of altitude which would exist between two simultaneous observations of the same star by two observers, one stationed at the earth's surface and the other at its centre). All calculations of celestial bodies are reduced to the earth's

centre, and therefore parallax is always to be added. A way to keep it in memory is to think of the earth's centre as our *true* place of observation, so that being *raised* up too high by an amount equal to the earth's radius we see the body too low.

When the body is in the zenith the angle of parallax, as will be seen by inspection of Fig. 121, is eliminated. It is at a maximum when the body is on the horizon, where it is termed horizontal parallax, and the table for its values for different bodies and at different seasons is given in the almanacs together with a table for the sun's parallax in altitude.

Parallax, Equatorial. In the 'Nautical Almanac' the equatorial or longest radius of the earth is used for computing parallax; so when great accuracy is sought the equatorial parallax must be again reduced by a correction for latitude. Only the sun, moon, and planets have parallax. The fixed stars, being at too vast a distance, have no appreciable parallax except in a few instances, where the parallax is measured by assuming as a base not the earth's radius, but the diameter of the earth's orbit round the sun.

Parallax in Altitude. Given the horizontal parallax of a celestial body and its altitude, to find its parallax in altitude. The sun's parallax in altitude is given in a table, because the variation is sensibly constant, but the horizontal parallax of the moon is deduced by the following rule:

Let P = the horizontal parallax
P' = the parallax in altitude
A = the apparent altitude

$$R : \cos A :: \sin P : \sin P'$$

$$\therefore \sin P' = \frac{\cos A . \sin P}{R}$$

Or by logarithms:

$$\log \sin P' = \log \cos A + \log \sin P - 10.$$

Parallax in telescopes, the apparent dancing about of the cross hairs when the eye is shifted about during observation. It arises from the eyepiece not being correctly in focus. The cross hairs should be clearly defined by moving the eyepiece out or in.

Perigee (Gr. *peri*, near; *ge*, the earth), the converse of apogee (*see* Apogee).

Perihelion (Gk. *peri*, near; *helios*, the sun), the converse of aphelion (*see* Aphelion).

Pointers, the stars α and β in the Great Bear, Ursa Major (*see* Fig. 122), or familiarly the Waggon and Horses. The former of them is also called Dubhé. The seven principal stars of this constellation are

all that are commonly known as belonging to it, but there are many more. These appear to revolve round the pole without in our latitudes even touching the horizon, the pointers maintaining all the while their direction towards the pole star.

Polar Distance is the arc of a celestial meridian passing through a celestial body measured from the pole to the body. In the northern hemisphere it is the complement of the declination when that is north, and equals 90° + the declination when it is south. In the southern hemisphere the polar distance, *i.e.* distance from the *south* pole, is *vice versâ*.

Pole (Gr. *poleō*, I turn) (*Celestial*), the intersection of the earth's axis when produced to the celestial sphere. The apparent unchangeableness of this point renders it the basis of all astronomical measurements. The point does actually change from year to year, but not with sufficient rapidity to enter into daily calculations of latitude and longitude.

Pole (*Terrestrial*), the two points north and south forming the apices of the axis of the earth's rotation (N, S, Fig. 129).

Pole Star is a star of the second magnitude near the celestial pole in the end of the tail of the Little Bear; it is called either α Ursæ Minoris or Polaris; its right ascension for 1890 is 1 hr. 18 min. 29·1 sec., its declination 88° 43′ 18″ N. Fifty years ago it was R.A. 1 hr. 2 min. 10·683 sec. and declination 88° 27′ 21″·94 N.; it is therefore travelling (in appearance) slowly towards the celestial pole.

Precession of the Equinoxes is a slow retrograde movement of the equinoctial points, due to the attraction of the sun and planets (*see* Aries, First Point of).

Prime Vertical is the great circle of the celestial sphere passing through the zenith, east and west points of the horizon, and the nadir. Its plane is therefore at right angles to that of the meridian. It is shown as ZCN in Fig. 127.

Primitive, the great circle upon the plane of which a stereographic projection is made.

Quadrant (Lat. *quadrans*, a fourth part), the fourth part of a circle. An instrument so named was used in taking altitudes before the introduction of the sextant.

Radius Vector (Lat. *radius*, a sunbeam; *vector*, a bearer), the shortest distance from the centre of the earth to the centre of the sun at any point of the earth's orbit. Has also the meaning of radius of curvature to any curve other than a circle at any particular point in the curve, such as the distance from the centre of the earth to any point upon its spheroidal surface.

Range of Tide is the difference between the height of high- and low-water levels of *any one tide* without any reference to datum. Is also termed height of tide.

Refraction (Lat. *refrangere*, to bend) is the bending of the ray of light proceeding from a celestial body when passing at an angle from the rarer ether, or whatever the medium may be, into our denser atmosphere. It makes bodies appear higher than they are, and the correction for it is given in Chambers's 'Mathematical Tables,' Whitaker, &c. It is always to be deducted. In consequence of this law all bodies appear to rise earlier and to set later than they really do, with the sole exception of the moon (*see* Amplitude). If the ray of light were passing from a dense into a rarer medium it would be bent the opposite way. A simple but entirely unphilosophical and somewhat grotesque way of remembering the *direction* in

FIG. 130.

which the ray is bent is to imagine the pencil of light as a long thin wand bending down with the weight of the star at the end of it. When in a vertical position it will not deflect either way, but the more the angle of depression the greater the deflection. The direction of the ray as it reaches the eye, or line of sight, is where the eye sees the star; but its true position is below that, at the end of the curved pencil of light; so that we must deduct the correction for refraction from the observed altitude. It is just the same for all bodies, at whatsoever distance they may be. It depends entirely upon the angle and density of the medium, and the tables give corrections for difference of barometric pressure and temperature where close calculation is required. It varies from 0′ when the body is in the zenith to 34′ on the horizon.

Right Ascension of a celestial body is analogous to the longitude of a terrestrial position. It is the arc of the celestial equator measured from a meridian passing through the first point of Aries to a meridian passing through the celestial body. The first point of Aries has nothing in particular to mark it in the heavens; it is the vernal equinoctial point which in the times of the ancient astronomers

was actually situated in Aries, but, owing to the precession of the equinoxes, is now in another constellation altogether. It is quite an imaginary point upon the celestial equator, chosen, like the observatory of Greenwich in terrestrial calculations, as a fiducial point from which to map the stars. Fortunately for astronomers there has not been the same display of national feeling in the selection of a celestial meridian as there has been about the choice of a common terrestrial meridian, so there is but one. The right ascension is reckoned from west to east, and is expressed in hours and minutes, the 360° of the equator being 24 hours (*see* Sidereal Time). The celestial semicircle which crosses the poles and the first point of Aries is called the vernal equinoctial colure, and its opposite semicircle the autumnal equinoctial colure, because on March 21 and September 21 or thereabouts the sun's path (*see* Ecliptic) intersects the celestial equator. The sun has then no R.A. and no Decl., and day and night are equal all over the world (*see* Equinoctial).

Rise of a tide is the height of the high-water level above the low spring datum.

Satellites (Lat. *satelles*, a companion), the little bodies which revolve round the planets.

Sea Mile. *See* Geographical Mile.

Semi-diameter of sun and moon is half the angle subtended by the diameter of the visible disc; it varies according to the bodies' distance from the earth, and values are given in the almanacs; by it observations of the upper or lower limb are reduced to the centre.

Sextant (Lat. *sextans*, a sixth part), an angular reflecting instrument for making celestial observations (*see* Chapter IX.).

Sidereal Time. If a star is watched passing any fixed point, such as the line between two perfectly straight vertical rods, on successive evenings by a correct watch, it will be seen to pass them 3 min. 56 sec. (more correctly 3 min. 55·91 sec.) earlier each evening. This movement is perfectly regular, and means simply the time of one complete revolution of the earth upon its axis. Sidereal time is needed to find the time when any star will culminate, and to correct watches or chronometers, which may be done by a transit instrument to a fraction of a second.

It would be of no use as civil time, because the time would keep dropping back. Sidereal time commences when the first point of Aries is on the meridian of the place (*see* Right Ascension), and is counted through 24 hours until the same point comes round again. It is a shorter measure of time than mean time.

• 24 hrs. of sidereal time = 23 hrs. 56 min. 4·0906 sec. of mean time, and 24 hrs. of mean time = 24 hrs. 3 min. 56·5554 sec. of sidereal time.

Sidereal time at mean noon is the heading of a column in the Nautical Almanac Ephemeris, and also in Whitaker.

In the American Nautical Almanac the same thing is termed sidereal time, or right ascension of mean sun.

In Chambers's 'Mathematics' it is called the sun's mean right ascension at mean noon.

All these names are sufficiently reasonable, but such a difference of nomenclature is confusing to the beginner, who has some difficulty in grasping the thought of a 'mean sun.'

We will confine ourselves to the first-mentioned expression and endeavour to put it in popular language.

Everybody knows that the sun appears to go round the heavens once a day and once a year, owing to the earth's daily rotation and annual orbit.

The sun's yearly path is indicated by different stars appearing at sunset at one time of the year from another. At sunset in March the brilliant constellation of Orion is nearly overhead. In June it is the sun's bedfellow, so we do not see it at all.

The starting-point of star-measurement (*see* Right Ascension) is an arbitrary point in the heavens called the first point of Aries, situated in a semicircle which is termed the vernal equinoctial colure, because the sun is always there at spring-time.

This starting-point is marked 24 hrs. or 0 on the celestial globes, and it is also the commencement of sidereal time at any place.

When the first point of Aries is on the meridian of any place it is sidereal noon, just as when the sun is on the meridian it is apparent noon. Hence in the American Nautical Almanac there is a column headed 'Mean Time of Sidereal Noon.' In the British Nautical Almanac it is termed 'Mean Time of Transit of First Point of Aries.'

Sidereal time is a perfectly regular measure of time like mean time, but it is not the same measure, since we see it gains about 4 minutes a day.

Why, then, do we use it at all? Because the sidereal time at mean noon given in the almanac enables us to tell when any star will culminate, as will be presently shown.

Why do we not use it exclusively? Because it does not keep with the sun. We should have to put our breakfast hour on

every morning if we kept sidereal time, and say Monday at 8, Tuesday at 8·04, &c., or else we should soon be breakfasting at midnight.

If a star has a R.A. of 24 hours, like the second star of Cassiopeia nearly (*see* Fig. 122), it will culminate at sidereal noon. If its R.A. is 1 hour it will culminate 1 sidereal hour afterwards, and so on. Hence we can find the mean time of any star's culmination by adding its right ascension reduced to mean time to the mean time of sidereal noon given in the almanac (*see* Figs. 131, 132).

Thus: Find the mean time of the culmination of a star in 4 hrs. R.A. on May 18, 1889. 4 hrs. sidereal = 3 hrs. 59 min. 20·7 sec. mean time, which is the interval between the passing of the first point of Aries and the star across the meridian.

But sidereal noon by the almanac was at 20 hrs. 15 min. 10·46 sec. mean time on May 17. Adding the R.A. in mean time, we have 0 hr. 14 min. 31·16 sec. as the mean time on the 18th of the star's culmination.

Or we may do it in another way. The sidereal time at mean noon on May 18 is given as 3 hrs. 45 min. 26·47 sec. This represents the interval of sidereal time between the culmination of the first point of Aries and mean noon. But the sidereal interval between the first point of Aries and the star is 4 hours. Therefore, if we deduct the one from the other we get the sidereal interval 14 min. 33·53 sec. from mean noon. This reduced to mean time is the same as we had before, 14 min. 31·16 sec.

The two rules are therefore as follows:—*To find the mean time of any star's culmination at any meridian.*

Rule 1. To the mean time of sidereal noon on the previous day or the given day add the star's right ascension reduced to mean time.

(If the two quantities make more than 24 hours take the previous day, if less than 24 hours take the day itself.)

Rule 2. From the star's right ascension, increased if necessary by 24 hours, deduct the sidereal time at mean noon: result will be a sidereal interval which reduced to mean time will be the answer.

When the meridian is not the same as Greenwich the mean time, apparent time, or sidereal time have all to be corrected for the difference of longitude reduced to time, as explained on p. 136.

It is, no doubt, with the object of making the matter clearer that sidereal time at mean noon is expressed in some books as the mean right ascension of mean sun. One is told to imagine a sun which

keeps mean time in its movements, and whose right ascension will therefore be the sun's right ascension plus or minus the equation of time.

Since the right ascension of any body is synonymous with the sidereal time of its culmination, the right ascension of this imaginary sun when on the meridian—that is, at mean noon—is the same thing as sidereal time at mean noon.

There is not, however, any such thing really as apparent right ascension or mean right ascension ; it is only a hyperbole for conveying the twofold idea of the real sun keeping apparent time and an imaginary sun keeping mean time.

It is important not to confound the expression 'the sun's mean right ascension at mean noon' with that of 'the sun's right ascension at mean noon.' The latter only differs by one or two seconds from its right ascension at apparent noon—that is to say, it is the difference of right ascension which the sun has made during the interval represented by the equation of time—whereas the sun's mean right ascension, or sidereal time, at mean noon is the position of an imaginary sun whose right ascension always differs from that of the true sun by the equation of time itself.

This explanation has only been added because of the difference of nomenclature. The term 'sidereal time at mean noon' is sufficiently intelligible for our present purpose without introducing the idea of two kinds of right ascension.

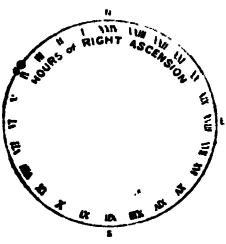

FIG. 131.—Position of the Celestial Equator at Sidereal Noon, May 17, 20 hrs. 15 min. 10·46 sec. Mean Time.

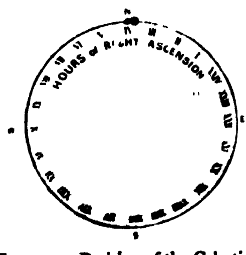

FIG. 132.—Position of the Celestia Equator at Culmination of Star at 0 hr. 14 min. 31·16 sec. Mean Time, May 18, and 4 hrs. Sidereal Time.

The 'mean sun' of the foregoing example is shown in the figure close to the star ; but 14 min. 31·6 sec. ahead.

Note. The position of the heavens in these two figs. is as they would appear in the southern hemisphere, where the sun's culmination

is to the north. The primitive is the ecliptic. The only reason for this arrangement was that the north point and first point of Aries might both be at the top of the paper, as being perhaps more readily understood.

When the equation of time at mean noon is marked as sun $\frac{\text{after clock}}{\text{before clock}}$, mean time is that much $\frac{\text{faster than}}{\text{slower than}}$ apparent time; and sidereal time at mean noon is that much $\frac{\text{less than}}{\text{more than}}$ the sun's right ascension at mean noon. In the illustration just given the equation of time was 3 min. 46·69 sec. 'before clock,' and the sun's right ascension at mean noon was 3 hrs. 41 min. 39·78 sec., which added together make 3 hrs. 45 min. 26·47 sec., which is the sidereal time at mean noon.

To reduce sidereal to mean time intervals by slide rule:

Place the right-hand 1 of the slide over the 9·83 of the rule, and for hours and decimals of sidereal time on the slide read off seconds and decimals on the rule, which are to be *deducted* from the sidereal interval to obtain the mean-time interval; thus 2·50 hrs. sidereal time = 2 hrs. 30 min. is opposite to 24·6 seconds to be deducted.

To reduce mean to sidereal time intervals by the slide rule:

Place the right-hand 1 of the slide over the 9·86 of the rule, and for hours and decimals of mean time on the slide read off seconds and decimals on the rule, which are to be *added* to the mean-time interval; thus 6·50 hours mean time are opposite to 64·1 secs., or 1 min. 4·1 secs. to be added.

To reduce sidereal time at mean noon at Greenwich to sidereal time at local mean noon by slide rule:

Express the difference of longitude in hours and decimals.

Adjust the rule with a 1 of the slide over 9·86 on the rule, and for hours of difference of longitude on the slide read the correction in seconds and decimals of sidereal time on the rule.

Example. What will be the sidereal time at mean noon in New York, lon. 74° W., it being 16 hrs. 41 min. 11 sec. at Greenwich?

The lon. in time is $74 \times 4 = 296$ min., or 4·93 hrs., which being W. is the amount *behind* Greenwich.

Sid. time, mean noon, Greenwich .	16^h	41^m	11^s
4·93 hrs. × 9·86 sec. . . .	− 0	0	48·6
Sid. time at mean noon, New York .	16	40	22·4

Signs of the Zodiac are twelve symbols denoting the constellations successively traversed by the sun in his apparent annual circuit of the heavens. They are as follows with the sun's position in them:

Aries, the Ram	♈	March 20 to April 20
Taurus, the Bull	♉	April 20 to May 21
Gemini, the Twins	♊	May 21 to June 21
Cancer, the Crab	♋	June 21 to July 22
Leo, the Lion	♌	July 22 to August 23
Virgo, the Virgin	♍	August 23 to September 23
Libra, the Scales	♎	September 23 to October 23
Scorpio, the Scorpion	♏	October 23 to November 22
Sagittarius, the Archer	♐	November 22 to December 21
Capricornus, the He-Goat	♑	December 21 to January 20
Aquarius, the Waterman	♒	January 20 to February 18
Pisces, the Fishes	♓	February 18 to March 20

The signs of the zodiac are supposed to be Chaldean or Egyptian hieroglyphics, intended to represent some occurrences peculiar to the month in which the sun occupied each of the constellations at that time. Thus the spring signs show productiveness of nature. When the sun is in Libra the autumnal equinox takes place, whence the origin of that title is evident. Explanations more or less likely are given to all the rest of them.

Small Circles of a sphere are those whose planes do not pass through its centre, such as the parallels of latitude (Fig. 129).

Solstice (Lat. *sol*, the sun, and *stare*, to stand), the two periods, June 21 and December 21, when the sun's declination is temporarily constant.

South, To. *See* Culmination.

Sphere, Celestial, is the apparent vault of the heavens supposed to be viewed by an observer at the centre of the earth, in which the heavenly bodies appear to be situated and upon which their relative positions or movements are determined by measurements of spherical distances taken from arbitrary but fiducial circles and points supposed to be drawn upon the surface of the sphere like the meridians of longitude and parallels of latitude upon the terrestrial maps.

Spherical Angle is that formed at any point upon a sphere by two great circles intersecting there. It is measured by the inclination of their planes or by the angle between the tangents to the circles at the point of intersection. Thus the spherical angle NZA (Fig. 133) may be measured by the angle NOA between the planes, or by the angle TZT between the tangents, or by the arc NA of the great circle whose plane is at right angles to those of the two intersecting planes.

Spherical Distance is the arc of a great circle (*see* Great Circle) passing through two points which is intercepted between them, as SD,

PZ in Fig. 127 or as NA Fig. 133. It is the shortest distance upon the spherical surface between any two points.

Standard Time on the continent of America is a form of keeping mean time (*see* Mean Time) by which there are no variations except those of an even hour at a time, since 15° of lon. correspond with one hour's difference of time. The time of every fifteenth meridian, beginning at New York with the 75th and ending with San Francisco on the 120th, rules the belt for about $7\frac{1}{2}$° on each side of it. The convenience of railway systems causes in some cases the overlapping of the

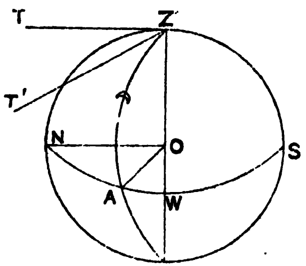

FIG. 133.

times, so they are distinguished by the following terms: Intercolonial time, Eastern time, Central time, Western or Mountain time, and Pacific time. The first mentioned is time of the 60th meridian and is used in Nova Scotia.

Statute Mile, the British and American standard of long measure; it is equal to 8 furlongs, or 80 chains (Gunter), or 320 rods, or 1,760 yards, or 5,280 feet, or 63,360 inches. It is also equal to 0·868719 of a knot (*see* Geographical Mile) and 1·609315 kilometre.

'Semi-mensual inequality of heights' is the difference between the heights of spring and neap tides above mean water-level.

'Semi-mensual inequality of time' is the difference between the greatest and smallest lunitidal interval.

Supplement, the difference between any angle and a semicircle.

Tacheometer (Gr. *tacheōs*, swiftly, and *metreo*, I measure), same as telemeter, but exclusively applied to instruments furnished with telescopes.

Telemeter (Gr. *tele*, far off, and *metreo*, I measure), an instrument for measuring distance without chaining.

Time. *See* Apparent, Astronomical, Civil, Mean, Sidereal.

Traverse in land-surveying is used in contradistinction to triangulation (*see* Triangulation). It is the method of surveying by measuring base lines in length and angular direction continuously forward, whereas in triangulation the lengths are computed by trigonometry or graphic construction.

A closed traverse is one in which the base lines box the compass back to the starting-point (*see* Course).

'Working a traverse' is the reduction of the angular base lines to rectangular co-ordinates of latitude and departure (*see* Latitude and Departure). The term is also largely used in navigation.

Triangulation in land-surveying is the determination of points by the intersection of rays taken from the ends of a base of known length. It is the foundation of geodetic operations of large extent as well as the most cursory field-sketching with a sketch-board. It is the root principle of all range-finders and telemeters and of the whole science of surveying. It is hardly ever used without traversing as well. In primary triangulation for geodetic survey the detail is filled in by traversing (*see* Traverse), and upon a route survey traverse the detail is sketched from triangulation. So the two principles dovetail into one another, taking alternative forms according as accuracy or despatch is the main point aimed at.

Zenith is that point in the heavens which is directly overhead. It would be the celestial pole if the observer were standing at the earth's pole, and on the celestial equator if he were crossing the 'line.'

Zenith Distance is the coaltitude or complement of the altitude. In Fig. 127 it is indcated by SZ.

Zodiac (Gr. *zonē*, a girdle), a belt of the heavens extending 8° on either side of the ecliptic, within which the sun, moon, and the major and many of the minor planets perform their annual revolutions (*see* Signs of the Zodiac).

INDEX

INDEX TO TABLES